GRAPHIC STATICS

Part II.

BY
R. NANJUNDAYYA, M. I. E. T. (LON.), A. M. I. S. E. (India)

Head of the Department of
STRUCTURAL ENGINEERING DRAWING
ENGINEERING COLLEGE

B. H. U.

WITH ~~[illegible]~~ FIGURES
AND
174 WORKED EXAMPLES.

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PREFACE TO THE REVISED EDITION.

In this edition all the chapters have been carefully revised and all the defective diagrams are redrawn with an up-to-date and improved methods of working to some of the examples whose solutions were lengthy. In addition to the above, the University question papers in the subject of Practical Geometry up to the year 1937 have also been added with diagrams and the entire volume brought up to date.

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After thoroughly revising this book for a second time I found some mathematical errors and some more defective drawings that escaped my notice. These have been replaced by fresh ones and University Question Papers up to the year 1940 have also been added.

Engineering College
B. H. U.
November, 1940.

R. N.

PREFACE.

In the first volume the first three chapters on Forces, Moments, Couples, Centres of Gravity, Moments of Inertia, Reinforced Concrete Sections and Roof Trusses are dealt with in brief and this volume contains fourteen chapters commencing from chapter four. The items of the subjects treated in these fourteen chapters are briefly stated under the heading of Contents in the next few pages. All the diagrams are drawn very carefully and accurately to the scale by me and all of them are neatly inked and finished by my friend Mr. Sukhdev Prasad, the Architectural Draughtsman of the Engineering College B. H. U. and I thank him very much. This book contains full of numerical examples with their solutions mostly graphical and the following books have been very often consulted for verification.—Andrew's Design of Structures, Structural Engineering by Husband and Harby, Charnock's Graphic Statics and Applied Mechanics by D. A. Low.

A chapter on Influence Lines is written mainly for a beginner to have a fair knowledge of the subject as it is written from the very first principle. In Chapter XV cams are elaborately written and profusely illustrated with diagrams from questions mainly selected from the University Examination papers and all of them are solved without any exception.

Chapter XVI deals with miscellaneous examples where some, of the typical problems are selected and solved in detail for the benefit of the students, and in the last chapter examples on velocity and acceleration diagrams of simple mechanisms are added with their graphical solutions.

At the end of this book series of questions connected to fourteen chapters, (that is from Chapter IV to XVII) in the order are selected mostly from the University Examination papers for the students to solve themselves, and in addition University question papers in the subject of Practical Geometry up to date have also been printed. Answers to above questions where necessary have been workd out carefully and entered.

There may be some mistakes which escaped my notice and I shall be highly thankful to those who point them out to me.

In the end I hope that this book will be useful for the engineering students who prepare themselves for the examination, and to a very great extent to young structural engineers and draughtsmen who are engaged in Structural work.

Engineering College,
B. H. U.
4 - 7 - 1932.

R. NANJUNDAYYA.

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GRAPHIC STATICS.

PART II.

Chapter IV.

BEAMS-BENDING & SHEAR IN A BEAM.

BENDING.—First take a beam of L feet long supported on both ends and let it be loaded centrally with a concentrated load W. Naturally the effect of this load is to make the beam bend, see fig. 1.

The beam selected here is of a flanged section shown in picture view in fig. 2.

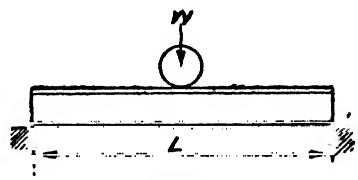


FIG. 1.

Owing to the load W the top flange is compressed and the bottom flange is elongated and the web is stiffening the top and bottom flanges; see fig. 3.

Fig. 4 represents the same beam as shown in fig. 1. Cut off the beam in EF at a distance D from the left support, and provide a hinge at E.

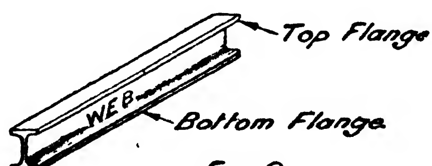


FIG. 2.

The reaction R of the left support would turn the portion of the beam AEFG about E and its turning effect being R × d which is called the Bending Moment at the section EF, but before it is cut this tendency to bend the beam is resisted by the horizontal tension Ht in the bottom flange; see fig. 5.

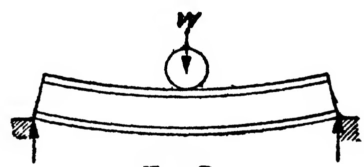


FIG. 3.

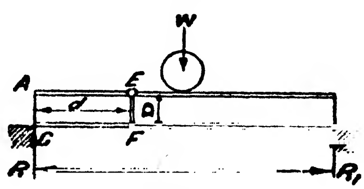


FIG. 4.

Now AEFG of fig. 4. is kept in equilibrium by three forces viz. reaction R, horizontal tension Ht, and the reaction R_2 at the hinge. Selecting E as moment centre we have $R \times d = Ht \times D$ where D is the depth from centre to centre of flanges. Here $R \times d$ is the Bending Moment at the section EF, and $Ht \times D$ is called the Resisting Moment at the same section EF.

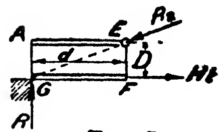


FIG. 5.

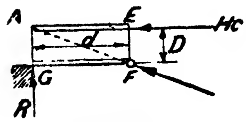


FIG. 6.

Hence at any section in a beam the Bending Moment is equal to the Resisting Moment or in symbols B. M. = R. M.

$$\text{Again } R \times d = Ht \times D$$

$$Ht = \frac{R \times d}{D} = \frac{B.\ M.}{D}$$

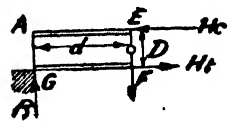


FIG. 7.

Therefore the flange stress in any beam or girder (Plate or Braced Girder) is equal to the bending moment divided by the depth of the beam or girder. D is the depth from the centres of gravity of both the flanges.

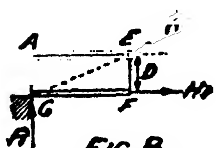


FIG. 8.

The same result we will get by selecting F as moment centre see fig. 6.

$$Hc \times D = R \times d. \therefore Hc = \frac{R \times d}{D} = \frac{BM}{D}$$

It has been shown above that $Ht = \frac{BM.}{D}$ $\therefore$ $Ht = Hc$

Hence it is known that the compressive flange stress is equal to the tensile flange stress.

Now let us select a moment centre midway between E & F as shown in fig. 7. Then the direction of reaction at the hinge is vertical and is equal to R. Then the beam AEFG is kept in equilibrium by the two balancing couples viz. $R \times d$ and $Ht \times D$ or $Hc \times D$.

We have then $R \times d = Ht \times \frac{D}{2} + Hc \times \frac{D}{2}$ or $Ht \times D$ or $Hc \times D$ as before.

Hence for equilibrium the Bending Moment at any section in a beam must be equal to the Resisting Moment at that section.

SHEAR—The load on the beam is vertical and the reaction is vertical; see fig. 8. The turning effect of reaction R about the point E

is resisted by Ht. The hinge is therefore to resist the vertical stress viz. the vertical shearing stress.

The vertical shearing resistance is nothing but the vertical component of hinged reaction R2, the horizontal component of which is Ho. Therefore Ho & Ht in the flanges resist the turning effect of R, and these being horizontal cannot balance R which is vertical.

In a complete beam therefore nearely the whole of the horizontal stress is borne by the flanges and nearly the whole of the vertical stress (viz. shear) by the web.

STRESSES IN BEAMS.

In a solid beam the resisting force is provided by the tensions and compressions in all the longitudinal fibres in a cross section, as proved previously. We will now find where the resultants of these forces act. By this, the effective depth of the beam may be ascertained.

Before proceeding to investigate further, students should know Hook's Law and Modulus of Elasticity.

HOOK'S LAW—This law states that within the limit of elasticity, the stress is proportional to strain.

To illustrate this let us take a bar of wrought iron 10 inches effective length and 1 square inch in cross section and load it in a testing machine with a pull of 6 tons.

Within the limit of elasticity, if the total elongation be 005 inch then per inch length of the specimen elongation would be $\frac{\cdot 005}{10} = \cdot 0005$.

If e represents the elongation per inch of length; L the original length of the bar; l = total elongation and P = the stress per □″ producing this elongation; then $\frac{l}{L} = e$. ∴ $e = \frac{\cdot 005}{10} = \cdot 0005$.

·0005 is the strain per inch length of the specimen. Hook's law states that within the limit of elasticity $\frac{P}{e}$ is constant, that is $\frac{\text{stress per □}''}{\text{strain per inch.}}$

This constant $\frac{\text{stress per □}''}{\text{strain per inch.}}$ is known as the Modulus of Elasticity.

Hence the Modulus of Elasticity of the wrought iron specimen is equal to $\frac{\text{stress per square inch}}{\text{strain per inch}} = \frac{6}{\cdot 0005} = 12000$ tons per square inch.

Now going back to the first paragraph, let us take a beam and load it with W, assuming the load W is within the limit of elasticity of the beam.

The stresses in the fibres are then proportional to their strains. *Fig. 9 represents a beam supported at the ends, but as yet unloaded and* let it be divided into a number of equal short lengths.

Now load the beam with W at the centre and the effect of this load is to bend the beam as shown in Fig 10. The upper surface is compressed and the lower is elongated.

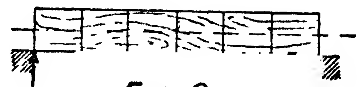


FIG. 9.

Any two adjacent lines of division originally vertical, become inclined to each other and their directions, if produced meet at the centre of curvature. Take one of the divisions to a bigger scale as shewn in fig. 11.

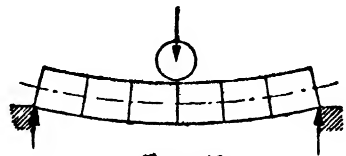


FIG. 10.

Line LL originally vertical assumes the position $L_1 L_1$ after bending. The ordinates of the triangles LOL_1 & LOL_1 are the actual strains in the different fibres and therefore represent their stresses. As we observe the stress is zero at O and increasing in proportion to the distance of any fibre from O and maximum at the extreme fibres viz. at the top and bottom of the beam.

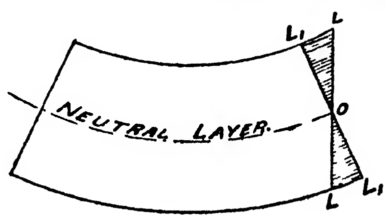


FIG. 11.

The longitudinal layer in a beam which is neither contracted nor extended is called the Neutral Layer and the intersection of this layer with a cross section LL is called the Neutral Axis of the section.

It is evident then that the longitudinal stress per unit of area is a maximum at the extreme fibres of a cross section and uniformly diminishes to zero at the neutral axis.

Since only the extreme fibres are offering their full resistance, it will be a great saving, if material is removed from the region of the neutral axis and placed as far as possible from it.

This explains why in steel sections such as standard beams and similar sections, the material is mostly concentrated at the flanges as seen in fig. 12.

In ordinary steel beams the web is usually very thin and is not taken into consideration while calculating to resist a given bending moment and yet gives a very satis factory result for most practical purposes.

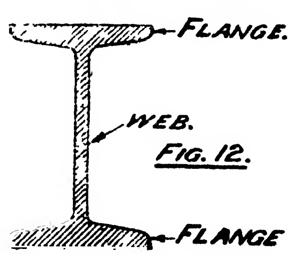


FIG. 12.

RECTANGULAR SECTION.

Wooden beams cannot be conveniently made flanged sections owing to the very low value of working stress of 1200 lbs. per square inch in compression and in tension. The breaking load per square inch is 8400 lbs only. Consequently timber beams are generally rectangular in section.

Take a portion of a rectangular beam supported at both ends. The neutral axis is at mid depth as shown in figure 13

The ordinates of the triangle OLL, are the actual strains and therefore represent the stresses at different distances from the neutral axis, viz. compressive and tensile stresses above and below the neutral axis respectively.

The resultants Rc & Rt of these stresses must pass through the centres of gravity of these two triangles that is at 2/3 the perpendicular height OL. The resisting moment at the section LL is $Rc \times dc + Rt \times dt = (Rc \text{ or } Rt) \times 2/3\ D$, where D is the depth of the beam.

Now Rc or Rt is equal to sum of the ordinates of the triangle OLL, or the area of the triangle OLL,. But LL, represents the extreme fibre stress.

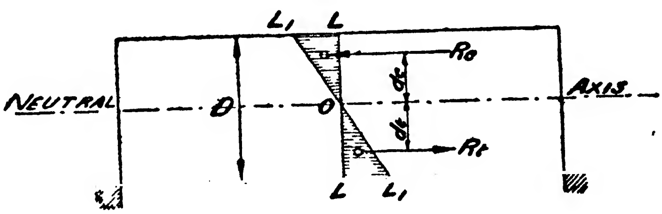


FIG. 13.

If we call f as the extreme fibre stress, then Rc or Rt$=\frac{1}{2} f \times \frac{D}{2}$ but resisting moment$=\frac{2}{3} D \times Rc$ or $Rt = \frac{1}{2} f \times \frac{D}{2} \times \frac{2}{3} D = \frac{fD^2}{6}$.

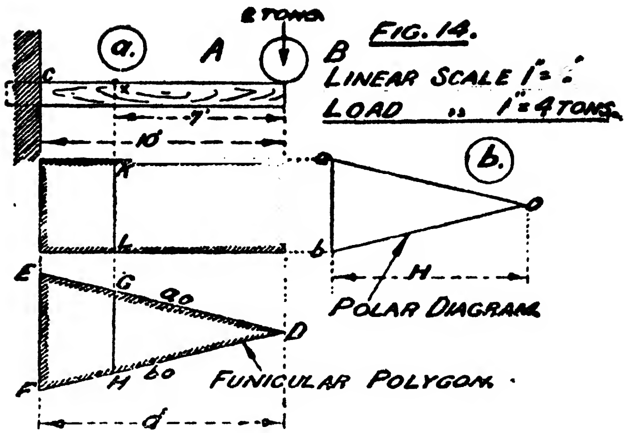
If you assume some breadth B to the beam then Resisting Moment is equal to $\frac{1}{6} f\ BD^2$ or in symbols:—

$$R\ M. = \frac{1}{6} f\ BD^2$$

N.B. To have an advanced knowledge of the beam theory, students are advised to read some standard books on the subject.

CANTILEVERS.

EXAMPLE 1—Take a cantilever 10′ long loaded with 2 tons at the extreme end (A beam having one end fixed and the other free is called a cantilever). see fig. 14.



The moment of this load about the point C is equal to $2 \times 10 = 20$ tons feet, because the moment about any point in the beam is equal to the load multiplied by the perpendicular distance to the point from the load. Since the load tries to bend the beam towards the point C, the value of 20 tons feet is called the Bending Moment at that point.

We can draw the Bending Moment diagram graphically as follows:—

Plot the load AB to some convenient scale, see fig. 14 (b) and select a pole O at a distance of some convenient whole number of inches from *ab* to facilitate easy multiplication.

Produce the load line AB fig. 14 (a) downwards and select any point D on it and draw rays parallel to *ao* & *bo*, so that they may touch the wall line in E & F.

Now the triangle DEF is the Bending Moment diagram.

PROOF:—The two triangles *abo* & DEF are similar $ab : H :: EF : d$. $\therefore ab \times d = EF \times H$, but $ab \times d$ is the moment of the force AB about the point C.

Therefore the intercept EF multiplied by the pole distance H is the moment of the force AB about the point C. At any other point of the beam say X the B. M. is equal to the intercept GH multiplied by pole distance H.

SHEARING FORCE:—The load AB has another effect on the beam, that is to shear the beam vertically at any section equal in amount to load AB, and this is plotted in the figure as shown.

Note:—(1) To measure the shearing force at any point X in a beam, draw a vertical line from the point on to the shearing force diagram, the intercept KL in load scale gives you the shear at the point X.

(2) While calculating the bending moment take intercept in linear scale and pole distance in load scale. This may be calculated by Moment Scale thus—Linear Scale × H tons = 8×1×4 = 32.

∴ 1" = 32 tons feet

RELATION BETWEEN BENDING-MOMENT & SHEARING-FORCE.

The *Bending Moment* at any-point in a beam is equal to the area of the *Shearing Force* diagram between the free end of the beam and that point. Thus in fig. 14 the B. M. at the point X, 7' from free end is equal to the area of the shearing force diagram between the free end and the point X and this is equal to 2 × 7 = 14 tons feet.

B. M. at the point X is equal to the intercept GH multiplied by pole distance H in load scale. GH = 3·5 feet pole dist. 1" = 4 tons, Therefore 3·5×4 = 14.0 tons feet as above.

Similarly B. M. at C = 2×10 = 20 tons feet and this is equal to the area of the shearing force diagram between the free end of the beam and the point C.

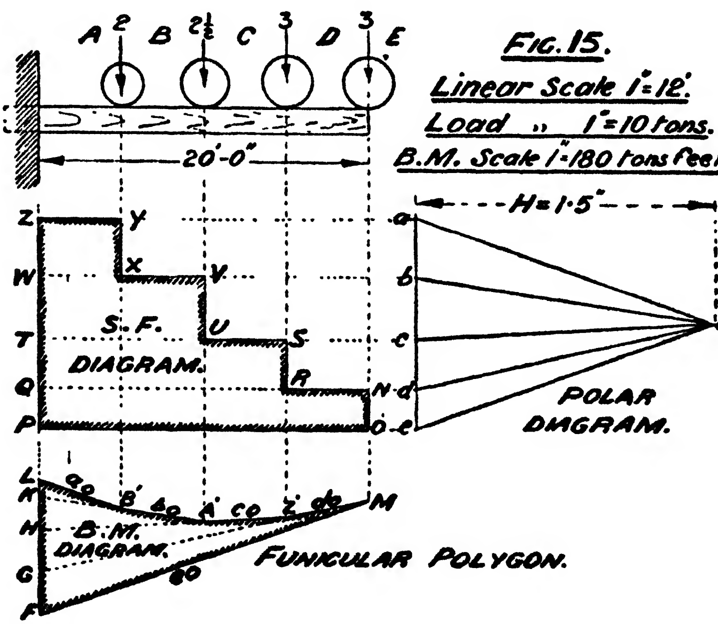


FIG. 15.
Linear Scale 1" = 12'.
Load " 1" = 10 tons.
B.M. Scale 1" = 180 tons feet.

EXAMPLE 2 :—A cantilever 20' long carries loads as indicated in fig. 15. Draw Bending Moment and Shearing Force diagrams graphically.

SOLUTION :—Draw the load line *abcde* and connect the same with a pole O and a polar distance H equal to some whole number of

inches. Draw the corresponding Funicular Polygon. This Funicular Polygon is the Bending Moment diagram. (see fig. 43 Part 1). Maximum B. M. is at the fixed end of the beam and at any point, B. M. is equal to the ordinate in the funicular polygon multiplied by polar distance H.

Note :—Bending moment diagram for load DE, is M G F, for loads CD, DE is M Z' A' H G F M ; for loads BC, CD, DE is M Z' A' B' K H G F M ; and for loads AB, BC, CD & DE is M Z' A' B' L K H G F M.

Shearing Force diagram for one load DE, is NO'PQN ; for loads CD DE is, S R N O'PQTS; for loads BC, CD, DE is, VUSRNO' PWV and for loads AB, BC, CD & DE is YXVUSR NO'PZY.

A uniformly distributed load is a load which is applied with equal intensity along the whole length of the beam, as in a beam supporting a wall of uniform height of equal thickness or a beam supporting the floor slab of uniform thickness.

EXAMPLE 3 :—A cantilever 20' long is loaded uniformly with 2 tons per foot run. Draw the B. M. & S. F. diagrams.

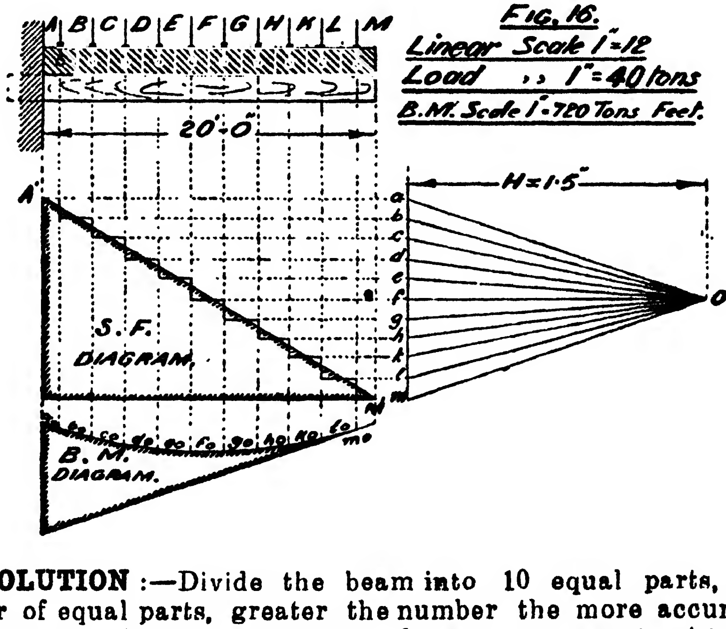


FIG. 16.
Linear Scale 1"=12'
Load „ 1"=40 tons
B.M. Scale 1"=720 Tons Feet.

SOLUTION :—Divide the beam into 10 equal parts, or any number of equal parts, greater the number the more accurate the result will be. In this case each division represents 4 tons and this load is assumed to act at the center of gravity of each division as shown here.

Bending Moment and Shearing Force diagrams are drawn in the same way as pointed out in the previous example. The top line in the B. M. diagram is almost a parabola.

In the Shearing Force diagram the top line is of stepped form, because we converted the uniformly distributed load into a number of

concentrated loads and the stepped S. F. diagram is correct for concentrated loads, but as this is really a distributed load every point in the beam is loaded and hence the shear should decrease gradually from the fixed end to the free end. Now if we double the number of divisions, we get twice the number of smaller steps in the same space; and by taking innumerable divisions in the beam the stepped form of the shearing force diagram will almost end in a straight line. Therefore one straigt line A′ M′ is drawn and this passes exactly through the centres of these steps. The projected portion above this mean line may be neglected, but it shows the construction for the number of loads we took.

CANTILEVER WITH A UNIFORMLY INCREASING DISTRIBUTED LOAD.

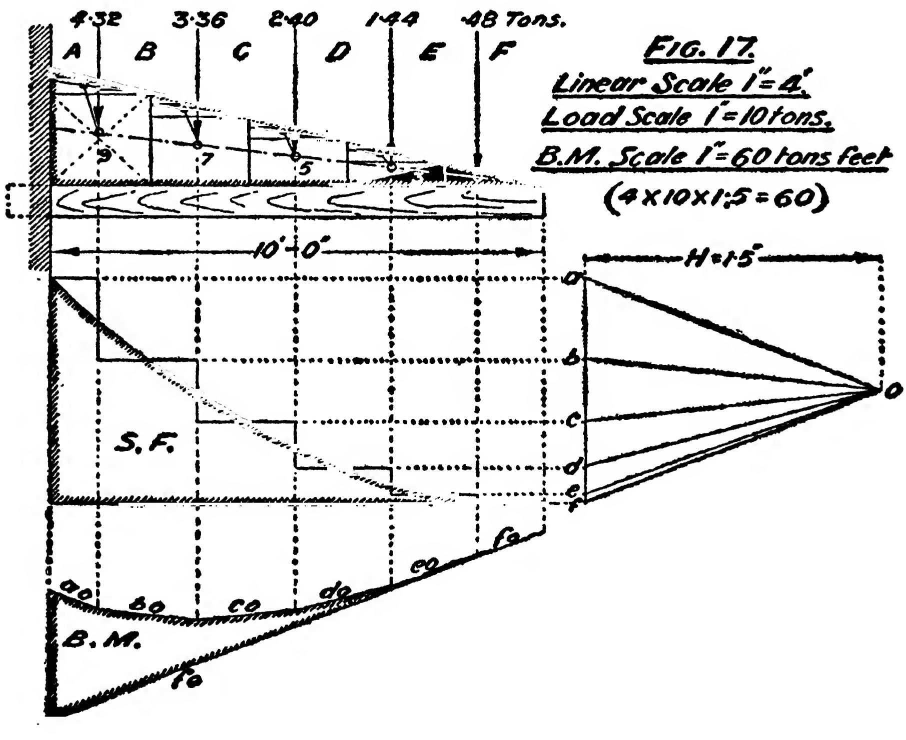


FIG. 17.
Linear Scale 1″=4′.
Load Scale 1″=10 tons.
B.M. Scale 1″=60 tons feet
(4×10×1.5=60)

EXAMPLE 4.—A cantilever 10′ long is loaded with a uniformly increasing daitributed load starting from the free end. Total load is equal to 12 tons. Draw the bending moment & shearing force diagrams.

Solution.—First draw a triangle representing the given load

over the cantilever by selecting any suitable height over the fixed end of the cantilever and joining that point to the free end as shown in fig. 17.

This triangle represents the load diagram and the area of this *triangle represents 12 tons. Divide this triangle into* any number of equal parts say at two feet intervals as shewn here.

Except the first division on the right, the rest of the divided portions are in the form of trapeziums. Find the centres of gravity of these portions as follows:—Divide the trapezium into a rectangle and a triangle and get the centres of gravity of these two. Connect these two points with a straight line and the intersection point of this straight line with the centre line of the trapezium is the Centre of Gravity of the trapezium.

The method of calculating the load is as follows.—The first division at the free end of the cantilever is a triangle, the second division is a trapezium and this contains three such triangles to the size of the first. These three triangles are shewn in the figure. By similar construction it can be shewn that the third division contains five, fuorth seven, and the fifth nine such triangles.

In all there are 25 such triangles like the one represented in the first division. Total load equals 12 tons and therefore each triangle will have the magnitude of the load equal to $\frac{12}{25} = \cdot 48$ton.

The left hand first division contains 9 small triangles and the magnitude of the load is equal to $\cdot 48 \times 9 = 4 \cdot 32$ tons. Similar calculations for the rest of the divisions are made.

These loads are assumed to act at the centres of gravity of the corresponding divisions. The polar diagram, together with the corresponding funicular polygon, which is the B. M. diagram and the shearing force diagram are drawn as usual.

Note:—The shearing force diagram will end in a parabolic curve for the following reasons:—We observed in the previous sheet example 3, that a meam line drawn in the shearing force diagram exactly cuts the mid point of each step; consequently the mean line drawn from the fixed end to the free end in this figure, must also cut the mid point of each step; the line joining these mid points will naturally take the form of a parabolic curve.

Since every point in the beam is loaded the B. M. & S. F. diagrams are to be continued as far as the extreme end of the beam, as shown in the diagrams.

BEAM SUPPORTED ON BOTH ENDS & CARRYING A CONCENTRATED LOAD AT THE CENTRE.

EXAMPLE 5—The span of a supported beam is 16′ and the beam is loaded at the centre with 10 tons. Draw the B. M. & S. F. diagrams graphically. See fig. 18.

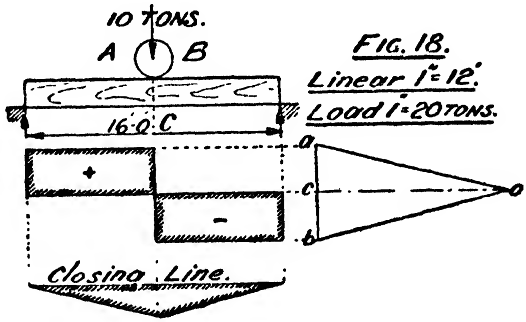


FIG. 18.

SOLUTION:—Draw the polar diagram for the load given and draw the corresponding funicular polygon. This funicular polygon represents the B. M. diagram. See fig. 43 part I.

From pole O draw a ray OC parallel to the closing line of the funicular polygon. Then $b\,c$ represents the magnitude of reaction at the right support and $c\,a$ the reaction at the left support.

After determining the reactions you are to draw the shearing force diagram. Now the shear at the left end is equal to the reaction at that point$=ca$ and this is constant till it meets the load AB. Load AB is acting downwards and therefore the shear at that point is equal to $ca-ab=-cb$ and this is constant till it meets the right reaction and it is neutralized by the upward reaction of 5 tons and consequently reduces to zero.

Again if you start from the right end, you see the force BC acting upwards and this is the shear at that point and is constant till it meets the load AB which acts downwards. Shear at this point$=bc-ab=ac$ which acts downwards and this is constant till it meets the left reaction ca which is acting upwards and is then reduced to zero. At the centre of the beam there exists a positive shear of 5 tons and a negative shear of 5 tons and so the total shear there, is zero.

Note:—Immediately to the left & right of the load at the centre, the shear suddenly increases to the value of $+5$ tons ann -5 tons respectively on the assumption that the load is applied at a single point. Actually this is not the case, but every load has a few inches bearing and the S. F. diagram at the load point then appears as shown in fig. 24 page 15.

IDENTIFICATION OF POSITIVE & NEGATIVE SHEAR IN A BEAM.

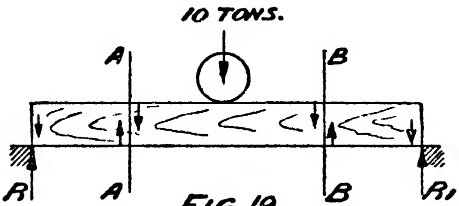


FIG. 19.

The left end reaction shears the beam upwards at that section and to the right of that section the shear is downwards as shown in fig. 19. Also at any section such as AA between the left hand support and the central load, the shear is downwards to the right of that section and upwards to the left of that section. The downward shear to the right at the section has clockwise moment about the left support, and hence the shear from the left support to the central load is positive.

Again coming to the right support, the reaction shears the beam upwards at that section and to the left of that section, the shear is downwards. Also at any section such as BB between the right end and the central load, the shear is upwards to the right of the section and downwards to the left of the section as shown in the fig. 19.

The downward shear has anti-clockwise moment about the right support and therefore the shear is negative.

POSITIVE & NEGATIVE BENDING MOMENT.

In a supported beam as in the above example, the central load tends the beam to bend concave upwards as shown in fig. 20; whereas in a cantilever with a load at the free end the concave side is downwards, as in fig. 21. It has become the usual practice to distinguish these bending actions by designating the bending shown in figure 20 as positive and that of 21 as negative.

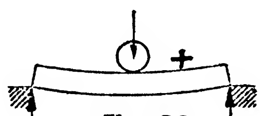


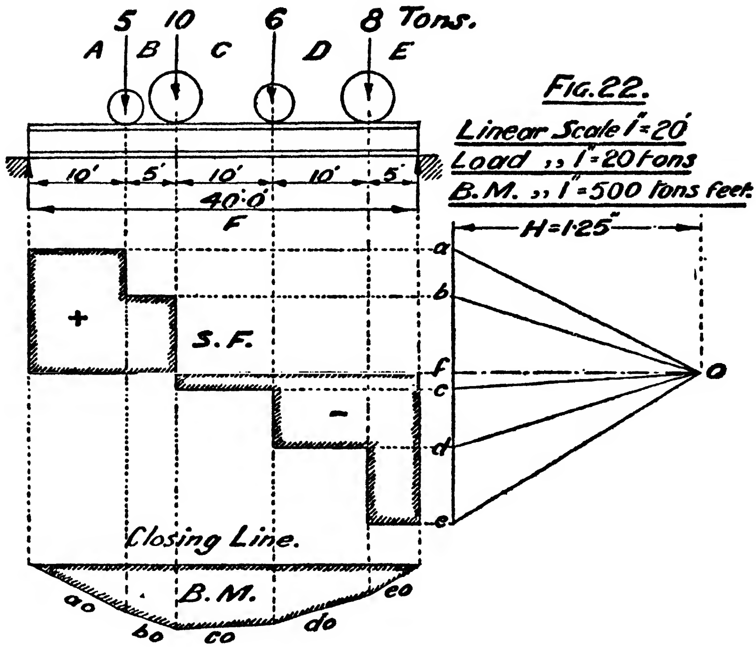
FIG. 20.

FIG. 21.

Note:—In graphical method, the S. F. diagrams are projected from the load line in the polar diagram and the positive & negative shear will be disignated by + and − sign respectively.

SUPPORTED BEAM CARRYING ANY NUMBER OF CONCENTRATED LOADS

EXAMPLE 6—A beam of 40 feet span carries loads as shewn and it is required to draw the B. M. & S. F. diagrams graphically.



SOLUTION—Draw the polar diagram for the loads given and from it the corresponding equilibrium polygon as usual. From pole O draw a ray parallel to the closing line of the equilibrium polygon and let it intersect the load line at f. Then lines ef & fa represent the magnitudes of reactions at the right and left support.

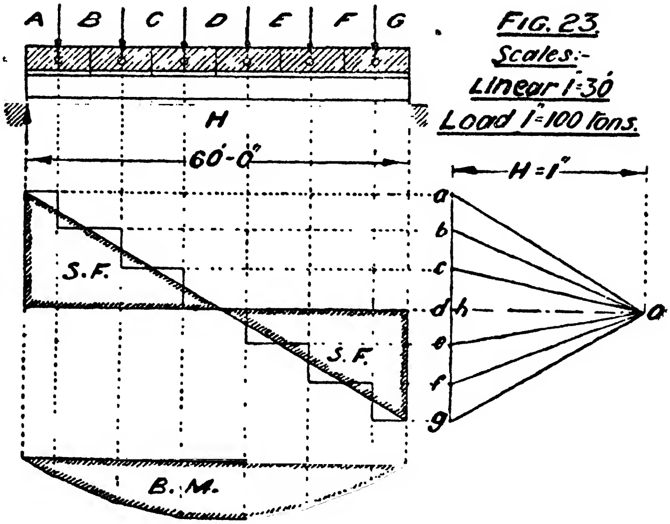
Equilibrium polygon is the Bending Moment diagram.

The shear at the left end is equal to the reaction fa and is constant till it meets the load AB. At AB the shear is equal to $fa-ab=fb$ and this is constant as for as it meets the load BC and at BC the shear is equal to $fa-ab-bc=-fc$, this is constant as far as the load CD. At CD the shear is equal to $fa-ab-bc-cd=-fd$. This fd is constant as far as the load DE and at DE the shear is equal to $fa-ab-bc-cd-de=-fe$ and this is neutralized by the upward reaction ef at the right support. If the load has a few inches bearing the shear under the load is reduced by half the amount of the load in the negative shear and in the positive shear it will increase by half of that load. For example shear under the load A B is equal to $fb+\frac{1}{2}\,ba$ and under the load $DE=(-fd)+\frac{1}{2}(-de)$.

SUPPORTED BEAM CARRYING A UNIFORMLY DISTRIBUTED LOAD OVER THE WHOLE SPAN.

EXAMPLE 7.—Let the beam (fig. 23) be of 60 feet span and carry a uniformly distributed load of 2 tons per foot run.

Draw B M. & S.F. diagrams for the same.



SOLUTION:—Divide the load into any number of equal parts as shown, and name these loads according to Bow's Notation and proceed in the same way as shown in the example 3.

Note:—If w = the total load & L = span in feet. Then reaction $= \frac{W}{2}$. B. M. at centre $= \left(\frac{W}{2} \times \frac{L}{2}\right) - \left(\frac{W}{2} \times \frac{L}{4}\right) = \frac{WL}{4} - \frac{WL}{8} = \frac{2WL - WL}{8} = \frac{WL}{8}$. Hence for a uniformly distributed load, the maximum bending moment $= \frac{WL}{8}$. Similarly for a concentrated load at the centre the maximum B. M. is equal to $\frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$.

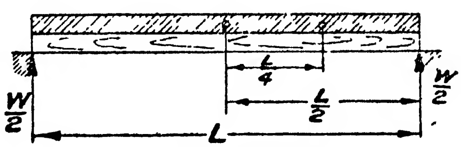


FIG. 23. A

SUPPORTED BEAM WITH A PARTIALLY DISTRIBUTED LOAD.

EXAMPLE 8.—Beam shewn in fig 24 is to carry a load of 2 tons per foot run over a length of 40 feet. Draw B. M. & S. F. diagrams.

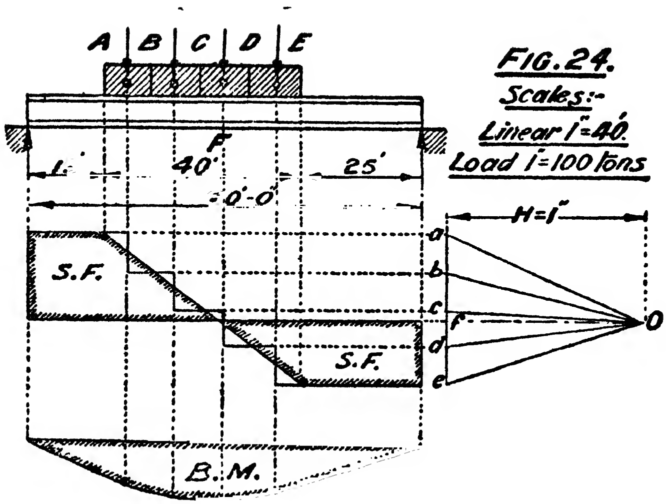


FIG. 24.
Scales:-
Linear 1"=40'.
Load 1"=100 Tons

SOLUTION:—First draw the B. M. diagram and after finding the reaction point *f*, in the polar diagram you can draw the S. F. diagram as usual.

EXAMPLE 9—Figure 25 is a balanced cantilever with concentrated loads as shown. Draw the B. M. & S. F. diagrams.

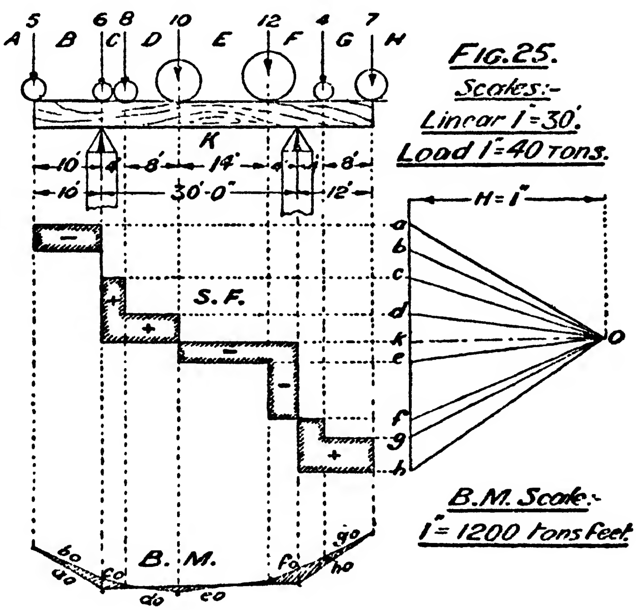


FIG. 25.
Scales:-
Linear 1"=30'.
Load 1"=40 Tons.

B.M. Scale:-
1"= 1200 Tons Feet

SOLUTION:—In funicular polygon the closing line is to touch the reaction lines. Therefore the first and the last ray of the funicular polygon are to be produced to meet the reaction lines as shown. This polygon is the B. M. diagram.

Shearing Force Diagram:—Shear at the left end of the beam is equal to *ab* and this is constant as far as the left support. At the

support the shear is equal to ka acting upwards and just over this point, in opposite direction there is a load of 6 tons BC acting downwards. Therefore the shear at the support$=ka-ab-bc=kc$ and this is constant till it meets the load CD. At this point the shear$=kc-cd=kd$ and this continues on till it meets the load DE. At DE the shear$=kd-de=-ke$ and this is constant till it meets the load EF. At EF the shear$=(-ke)+(-ef)=-kf$ and this is the same as far as the right support. At this right support the shear is equal to the reaction hk. Out of hk which is acting upwards you are to deduct the downward shear kf and the balance hf is constant till it meets the load FG. At this point the shear$=hf-fg=hg$ and this continues till the right free end.

EXAMPLE 10.—Given the balanced cantilever, 52 feet long, supported at 12 & 10 feet from either end and loaded with a uniformly distributed load of 2 tons per foot run together with the concentrated loads as shewn, draw the B. M. & S. F. diagrams.

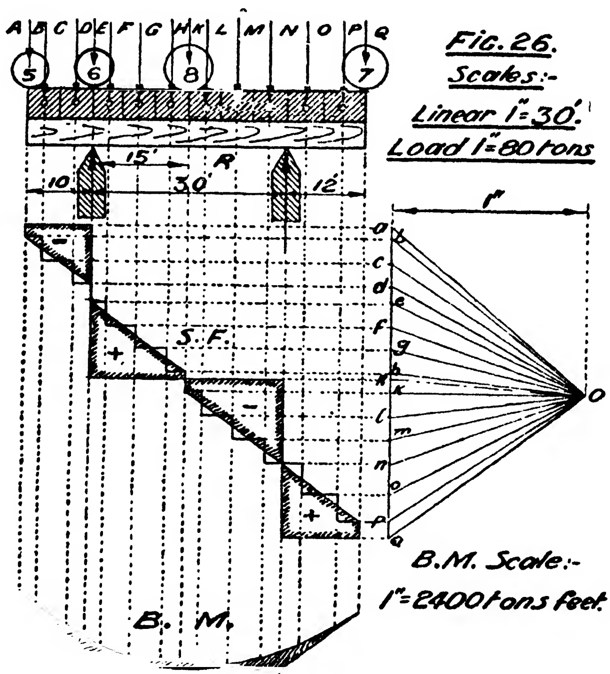


Fig. 26.
Scales:-
Linear 1"=30'
Load 1"=80 tons

SOLUTION:—Divide the distributed load into any number of parts so that the divisional line may exactly coincide with the line of action of the concentrated load. In this way, concentrated loads are not mixed up with the distributed loads. From the left end of the beam to

the left support, divisions are of 5 feet intervals and from the right support to the right end of the beam they are of 6 feet intervals.

B. M. diagram for these loads may be drawn without any difficulty, but care must be taken that the first and the last ray of the funicular polygon must touch the reaction lines at the supports.

Shearing force diagram:—Left free end shear is equal to the concentrated load AB and from that point throughout the length of the beam there is a distributed load. Where there are concentrated loads, there the vertical lines equal to their magnitudes are to be drawn; and the rest of the stepped form of the S. F. diagram is to be connected with mean straight lines as shewn here.

EXAMPLE 11:—A beam 40′ long carries a wall of the shape shown in fig. 27 and it is of uniform thickness of one foot. One cubic foot of wall weighs 112 lbs. Draw bending moment and shearing force diagrams.

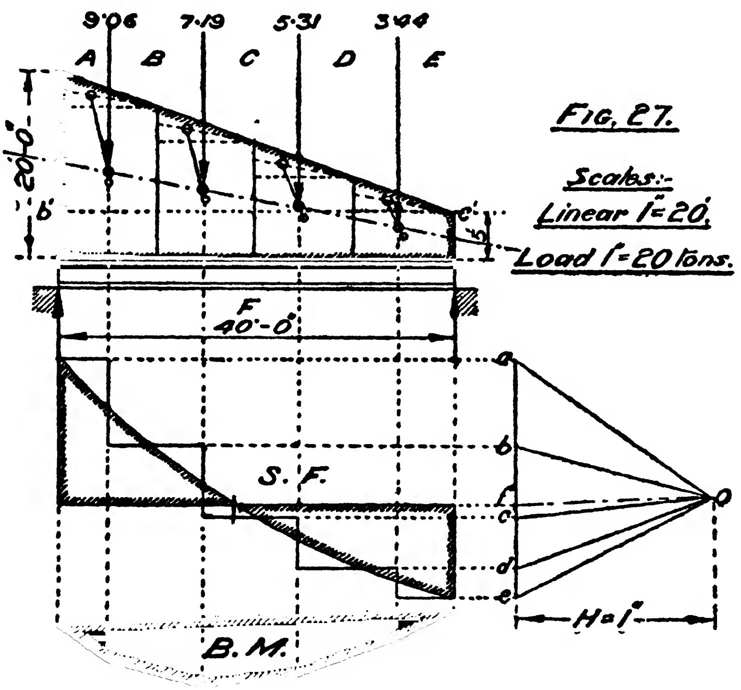


FIG. 27.

SOLUTION:—Weight of wall $= \frac{20+5}{2} \times 40 \times 112$ lbs. $\div 2240 = 25$ tons. Rectangular portion of 5 feet of wall weighs $40 \times 5 \times 112 \div 2240 =$ 10 tons. Then the triangular portion of the wall must weigh $25 - 10 =$ 15 tons. This big triangle as a whole contains 16 smaller triangles to the size shown in the right hand first division (see example 4 page 10).

Therefore each triangle weighs *15/16* tons or in decimal ·9375 ton. In this way loads are calculated. For example load DE = 2·5 tons + ·9375 = 3·437 or say 3·44 tons. 2·5 tons is the weight of rectangular portion of the wall of one division.

Bending moment diagram is drawn as usual. Shearing force diagram is drawn in a stepped form as if the loads were concentrated ones but since the load is uniformly increasing, every mid point of each step is to be connected with a mean curve as shewn in the diagram.

Note:—Before proceeding to plot the load in the load diagram, you are to find the centre of gravity of each divided strip as shewn in example 4 page 9 fig. 17.

EXAMPLE 12:—A beam 16 feet long is loaded uniformly with 2 tons per foot-run and supported in the centre. Show how you draw the B. M. & S. F. diagrams.

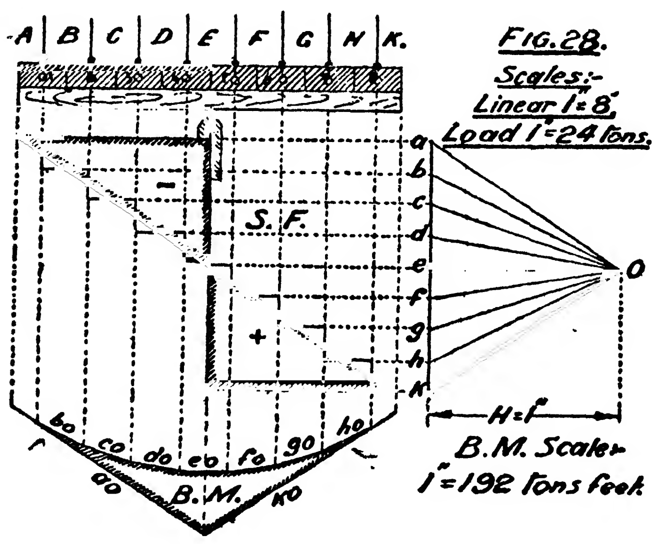


FIG. 28.
Scales:-
Linear 1"=8'.
Load 1"=24 Tons.

SOLUTION:—Since there is only one support the total load is equal to the reaction at the centre.

Therefore in B. M. diagram the first and the last ray of the funicular polygon are to meet at the reaction line as shewn here. Shear at the centre is equal to the reaction, which equals the total load *ka*. At the extreme left and right end, the shear is zero and gradually increasing on either side towards the support as shown.

EXAMPLE 13:—A beam is in equilibrium by the action of 3 external loads and two reactions. The magnitudes of loads and one reaction are known. Determine the span and the other reaction.

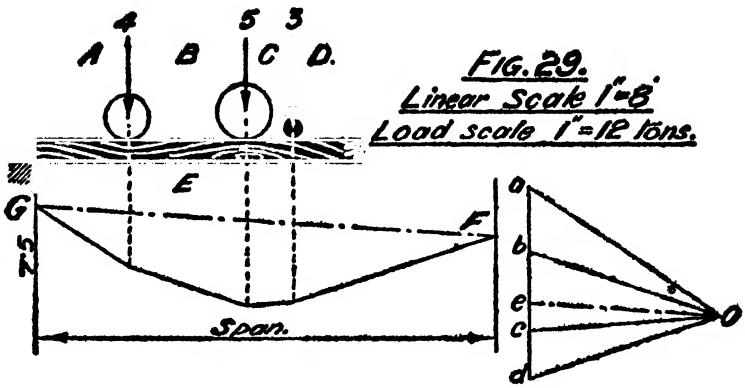


FIG. 29.
Linear Scale 1" = 8'
Load scale 1" = 12 tons.

SOLUTION.—AB = 4, BC = 5, & CD = 3 tons. Reaction EA = 7·5 tons. Draw the polar diagram for the given loads. Plot the given reaction *ea* on the load line equal to 7·5 tons. Join *e* with the pole O. Now *eo* should be parallel to the closing line of the equilibrium polygon, because the closing line determines the magnitudes of the reactions at the supports. From the starting point G of the equilibrium polygon draw the closing line and get it intersected at F. This fixes the span.

EXAMPLE 14:—Draw the bending moment and shearing force diagrams for the balanced cantilever with concentrated loads as indicated in figure 30.

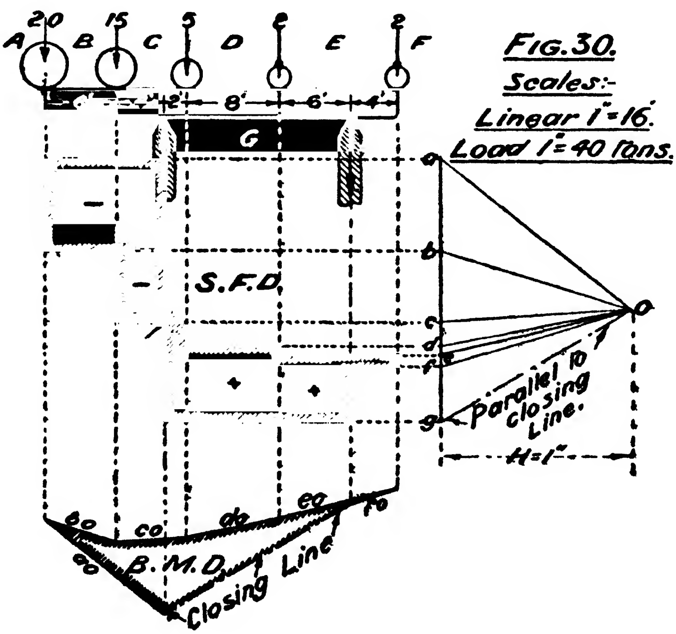


FIG. 30.
Scales:-
Linear 1" = 16'.
Load 1" = 40 Tons.

SOLUTION:—*Draw the B. M. diagram. The first and the last ray* should touch the reaction lines. From pole O draw a line parallel to the closing line and since this line goes a little down of the load line of the polar diagram, the load line is to be produced downwards to meet it at *g*. Hence the reaction at the right support is acting downwards and its magnitude is *fg*. Magnitude of the reaction at the left support is *ga* acting upwards.

Shearing Force—Shear at the left free end is *ab* and when it comes to the load BC, the shear is further added by the load BC. This remaining the same as far as the left support. At this support shear is equal to *ga* acting upwards and AB, BC, act downwards. Therefore the balance *gc* is constant as far as the load CD. Shear at this point $=gc-cd=gd$, and this is constant till it reaches the other load DE. At this point the shear $=gd-de=ge$ still acting upwards till it reaches the right support. The reaction at this support is downwards and the shear at this point $=ge-fg=fe$ and this continues till the right free end is reached and this equals the load EF.

FLOATING BEAMS.

Suppose a beam of uniform thickness and density is thrown into the water, then the beam floats as it is lighter bulk for bulk than water, and the water exerts a total upward pressure just equal to the weight of the beam.

At every point in the bottom surface of the beam there is an upward pressure just sufficient to resist the downward pressure at that point as shown in the diagram of fig. 31.

The beam therefore will be in a state of perfect balance and consequently there is no bending moment or shearng force. When there is an external load on the beam as shewn in fig.32 water will have to exert an additional upward pressure equal to the load; and this pressure will be uniformly distributed along the length of the beam.

FIG. 31.

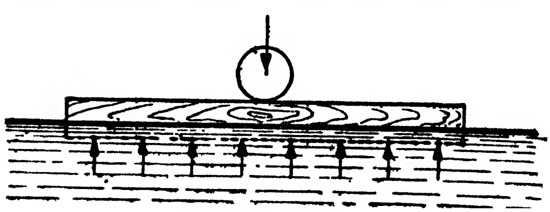


FIG. 32.

The pressure that balances the weight of the beam produces no bending moment or shearing force. The only load that produces the bending moment is the external load placed on the floating beam.

EXAMPLE. 15.—A beam of uniform thickness and density is 20 feet long and floats in water. A boy weighing 80 lbs. sits on it 8 feet from one end. Draw B. M. & S. F. diagrams.

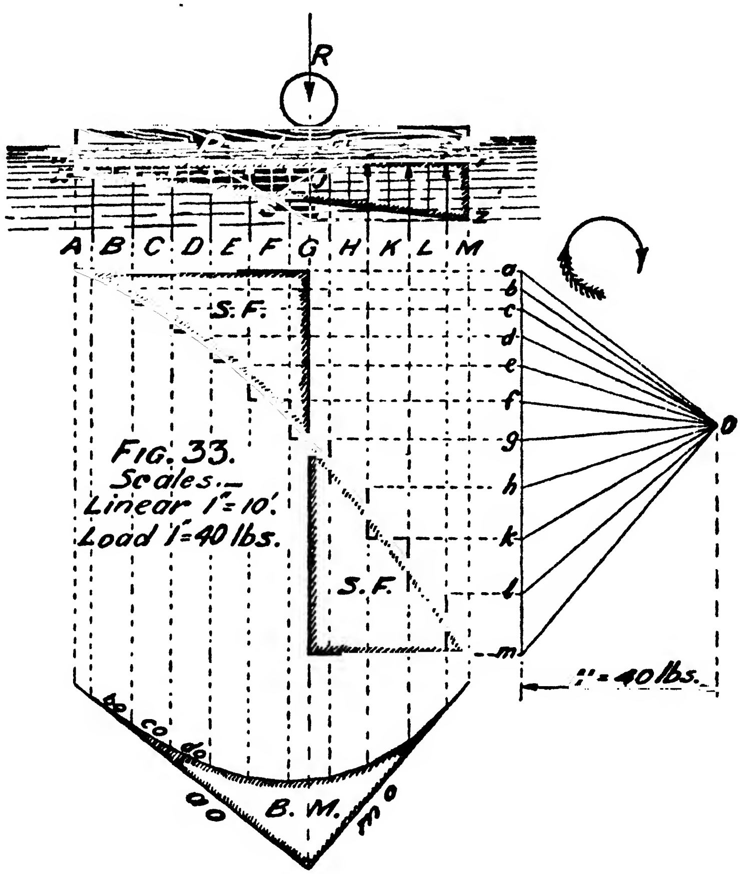


FIG. 33.
Scales.—
Linear 1" = 10'.
Load 1" = 40 lbs.

SOLUTION :—Since the boy sits a little bit right to the centre of the beam the upward pressure of the water is not uniform, and therefore the pressure figure is to be drawn as shown in the diagram. The oridinate VS is equal to $\frac{R}{L} = \frac{80}{20} = 4$ lbs. and is taken exactly in the centre of the beam. Points P and Q are at one third the length of the beam and are joined to the point S. The resultant load R is produced downwards to intersect the line QS at U and PS produced at T. Then UX and TZ are drawn horizontally to meet the vertical lines drawn from the ends of the beam. Then WYZX is the pressure figure. For the explanation of this pressure figure see the chapter on miscellaneous examples at the end of this book. The pressure figure is divided equally into 10 parts and the bending moment and shearing force diagrams are drawn as usual. (See example 12 also)

EXAMPLE 16 ;—A timber beam 25' long and 15" square floats in sea water. The weight of the timber is 40 lbs. per cubic foot and of the water 63·5 lbs. per cubic foot.

Two weights just sufficient to immerse are placed upon the beam 7′ from each end. **Draw B. M. and S. F diagrams and state the value of the maximum B. M At what distance from the ends should the weights be placed so that the greatest B. M is as small as possible. ? (B. Sc. Lond).**

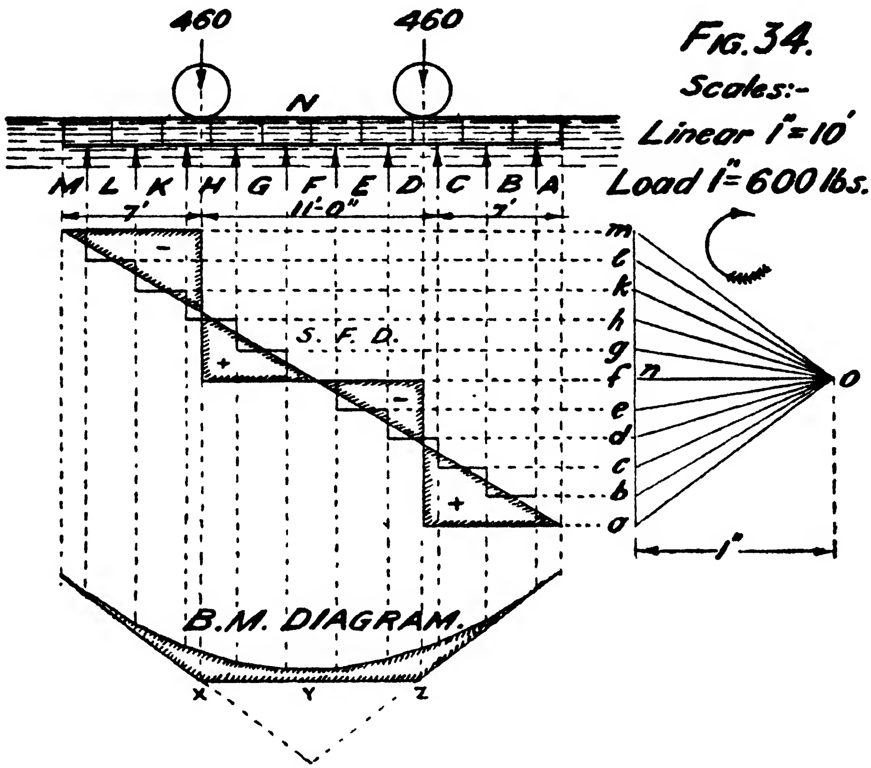


FIG. 34.

SOLUTION :—Weight of beam$=\left(\frac{15''\times15''}{12\times12}\right)\times 25\times 40=1562$ lbs, weight of water displaced$=39\cdot06\times62\cdot5$ lbs$=2480$ lbs Then the magnitude of the weight$=2480-1562=918$ lbs, which is to be placed upon the beam in two points. Each weight is therefore equal to $918/2=459$ or say 460 lbs. Upward pressure of water per foot length of the beam$=918/25=36\cdot72$ lbs.

The beam has been divided into 10 equal parts and each part is equal to 2·5 feet with an upward pressure of $2\cdot5\times36\cdot72$. lbs. $=91\cdot8$ lbs. assumed to act at the centre of gravity of each division as shown.

The external two loads applied at 7 feet from each end serve as supports of the beam. This beam may be taken as a balanced cantilever with a uniformly distributed load of 36·72 lbs per foot run

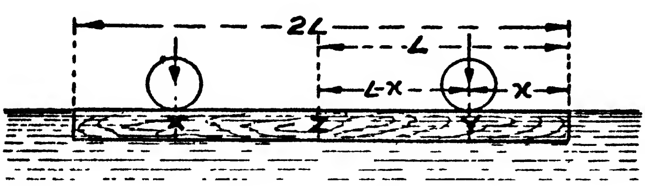


FIG. 35.

Bending moment and shearing force diagrams may be drawn as shewn in fig. 34.

Again if the bending moment at XYZ (See fig, 34) were equal to one another, then the maximum B M. will be as small as possible.

Call W the upward pressure per foot run of the beam, 2L the span, L half the span and x the distance from the end of the beam to the external load. See fig. 35.

Now B. M. at Z $= -WL \times \frac{L}{2} + WL\,(L-x)$ changing the minus sign we have $WL \times \frac{L}{2} - WL\,(W-x)$. This must be equal to the B. M. at X or Y, but the B. M. at Y is equal to $-Wx \times \frac{x}{2}$. Therefore $WL \times \frac{L}{2} - WL\,(L-x) = -Wx \times \frac{x}{2}$; eliminating W from both the terms we have $\frac{L^2}{2} - L^2 + Lx = -\frac{x^2}{2}$. Multiplying this by 2 we get $L^2 - 2L^2 + 2Lx = -x^2$. Transposing we have $x^2 + 2Lx + L^2 = 2L^2$

$$(x+L)^2 = 2L^2;$$
$$x + L = \times\sqrt{2L^2} = L\sqrt{2}$$
$$\therefore\ x = L\sqrt{2} - L = 12{\cdot}5 \times 1{\cdot}4142 - 12{\cdot}5 = 5{\cdot}17 \text{ feet.}$$
$$\therefore\ x = 5{\cdot}17 \text{ feet.}$$

Hence the external weights must be placed at 5·17 feet from either end of the beam to get the maximum B. M. as small as possible.

BEAM HINGED AT ONE END & SUPPORTED NEAR THE OTHER END.

EXAMPLE 17:—A beam 30′ long is hinged at the left end and supported at a point 10′ from the right end. Draw B. M. & S. F. diagrams for a load of 10 tons placed at the right end.

SOLUTION :—

AB is the load, BC and CA are the reactions at the right and left support respectively.

Load AB is to be connected with the rays ao & bo, right reaction BC with bo & co and left reaction CA with ao & co

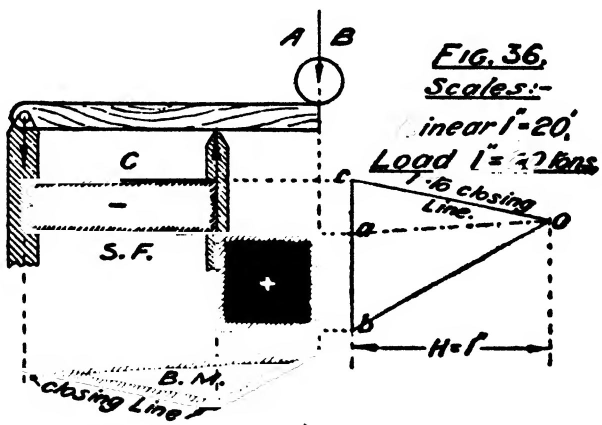


FIG. 36.
Scales:-
Linear 1"=20'
Load 1"=?tons

If from pole O, a line parallel to the closing line were to be drawn it intersects the load line ab above a in the polar diagram. Left end

reaction *ca* in the polar diagram reads down from *c* to *a* and hence the direction of reaction at the hinged end is downwards. After this the S. F. diagram may be drawn as usual.

EXAMPLE 17:—Draw the bending moment and shearing force diagrams for the same beam of example 16 with an additional load of 20 tons at 10 feet from the left end.

SOLUTION:—The procedure is similar to the example 16 and there is no difficulty for the student to follow this; the only difference in this example is, that the reaction at the hinged end is upwards.

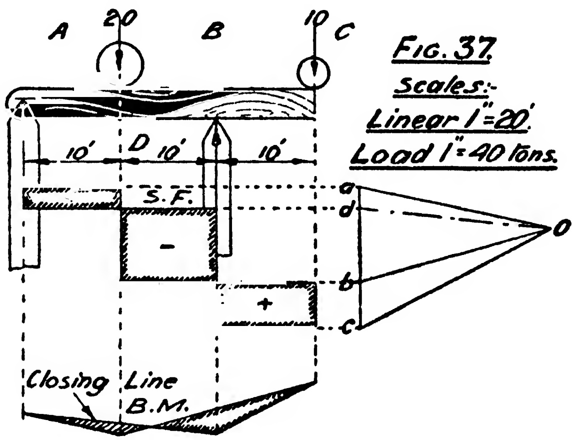


FIG. 37
Scales:-
Linear 1" = 20'
Load 1" = 40 tons.

EXAMPLE 18:—A cantilever is loaded and supported as shown in the figure 38. Draw diagrams of shearing force and bending moment and measure the maximum values of these quantities. State the magnitude of the supporting forces at H. & K. (I. Sc. Eng. Part II. 1928)

SOLUTION:—Directions of reactions are given; uniformly distributed load extends to a distance of 4 feet. You are to divide that at least into 4 equal parts and the magnitude of each bit is 120 lbs. as the total magnitude of the load is 480 lbs.

Name these loads as shown and proceed as usual. Care must be taken that the first ray *ao* is to touch the left reaction line and the last ray *go* is to touch the right reaction line. A ray *oh* is to be drawn from pole O parallel to the closing line and you observe that it goes right up and gets itself intersected at h.

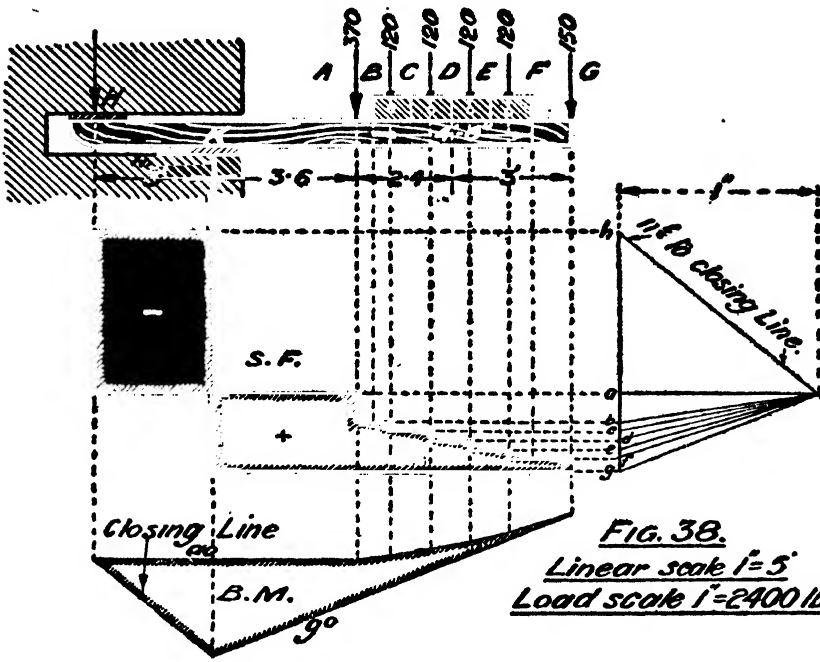


FIG. 38.
Linear scale 1" = 5'
Load scale 1" = 2400 lbs.

Now *gh* is the magnitude of the reaction at the right support and *ha* which acts downwards is the magnitude of the reaction at the left support.

Maximum bending moment is exactly at the right support and the magnitude of which may be measured to the B. M. scale.

Maximum shear is at the left reaction and this may be measured to the load scale.

Shearing force diagram—Shear at the left support is equal to the reaction *ha* acting downwards and is constant as far as the right support. Shear at the right support $= gh$ acting upwards, therefore the balance $gh - ha = ga$ acting upwards is constant till it meets the load AB which is acting downwards, and must therefore be deducted. Similarly every other load is to be deducted as shown.

EXAMPLE 19:—A beam 40 feet long is supported at 10 feet from each end and is loaded uniformly with one ton per foot length on the projected portion beyond the supports. Draw S. F. and B. M. diagrams.

A.

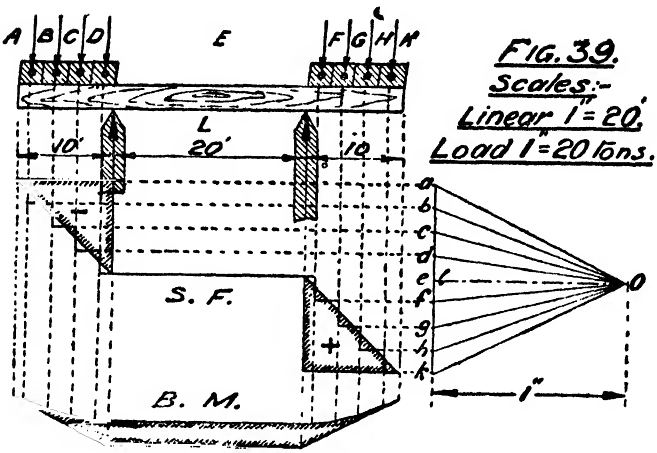


FIG. 39.
Scales:-
Linear 1" = 20'.
Load 1" = 20 tons.

SOLUTION:—Bending moment diagram is drawn as usual and bending moment is uniform between the supports as shown in the diagram.

Shearing force is nil at the extreme free end and increases uniformly towards the support, but at each support the reaction is equal to the downward load and shear is neutralized and reduced to zero. Hence the shear between the supports is zero.

HORIZONTAL BEAMS WITH INCLINED LOADS.

When there is a vertical load on the horizontal beam, it acts vertically down and reactions on the supports are vertical and when the same beam is exposed to an inclined load, this inclined load tries to push the beam either to the right or to the left depending on the inclination of the load. For example load AB (fig. 40) is inclined towards the right and it tries to push the beam towards left, therefore the beam should either be fixed or hinged to prevent its motion towards the left end of the beam. Hence the direction of reaction at the fixed end is not vertical; since the right support is free to move, the direction of reaction is vertical.

This inclined load and the right reaction intersect at D and the reaction at the hinged end must also meet at the same point.

Hence the direction of reaction at the hinged end is determined. The triangle of forces drawn on top of the figure determines the magnitudes of these forces.

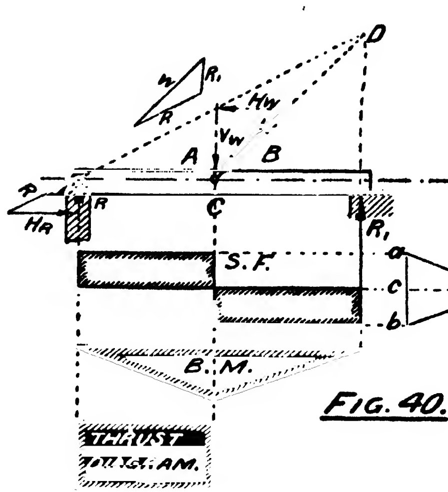


FIG. 40.

Bending Moment.—Resolve this load W into its horizontal and vertical components HW & VW. Vertical component produces bending moment and shear in the beam, and horizontal component produces thrust in the beam. Hw= HR. Bending moment diagram is drawn in the same way as in the case of an ordinary beam.

Thrust at the point of application of the load is equal to Hw and it is plotted vertically in the diagram and this is constant as for as the left support and at this support HR which is the horizontal component of R, thrusts the beam in the opposite direction to Hw and neutralizes the thrust.

Fig. 41 shows two inclined loads; one inclined at 45° to the right and another at 60° to the left. Horizontal components of these two loads are acting in opposite directions.

The resultant thrust is equal to the differance of these two and is equal to T_1. Hence the left reaction is to resist this thrust

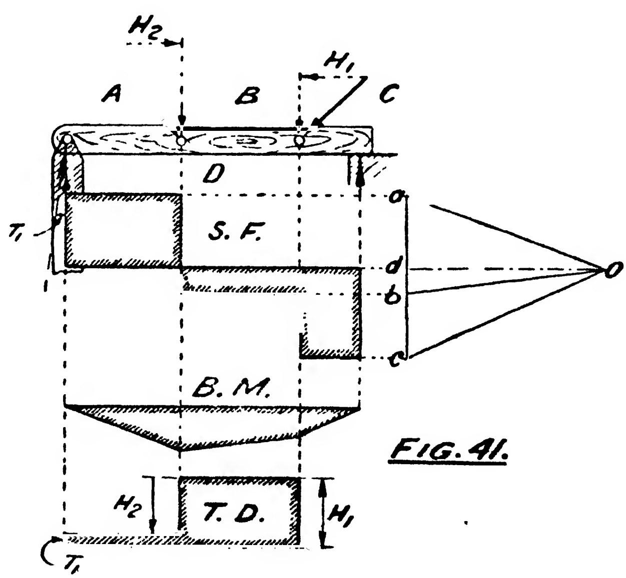


FIG. 41.

T_1 and from this the direction of *reaction* is known. This method of finding the direction of reaction is easier than shewn in fig. 40.

EXAMPLE 20:—A beam supported on a span of 16 feet is exposed to the action of three forces with different inclinations. Draw the bending moment, shearing force and thrust diagrams and also find the direction of the reaction at the hinged end.

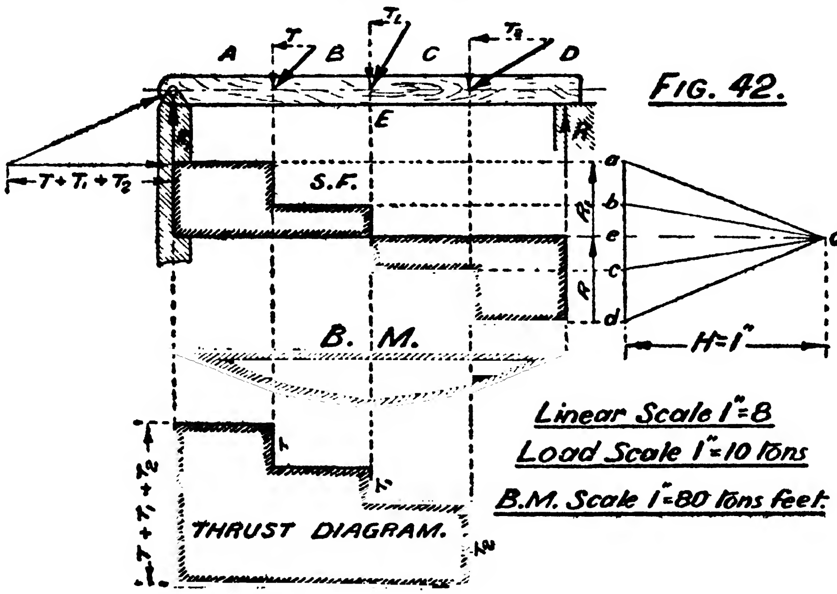


FIG. 42.

SOLUTION:—First resolve the forces into their horizontal and vertical components.

Take only the vertical components for bending moment and shearing force diagrams and draw them as usual.

All the horizontal thrusts are of the same direction and the starting point of the thrust is from the point of application of the third force.

Vertical reaction at the left end of the beam is equal to $ea = R_1$. Since the left end of the beam is hinged it is to resist T, T_1 & T_2 the horizontal components of the given loads. The final reaction would be the resultant of R_1 and $(T+T_1+T_2)$ and they are combined in the diagram above.

Thrust Diagram:—At the left end of the beam the thrust is equal to $T+T_1+T_2$ and plot this amount in a vertical line as shown. This amount of thrust which is acting towards the right is constant till it

meets the load AB; and at this point the thrust is equal to $(T+T_1+T_2) - T = T_1+T_2$; this is constant till it meets the load BC. At BC the thrust $= (T+T_1+T_2)-(T+T_1) = T_2$. Similarly at CD the thrust $= (T+T_1+T_2)-(T+T_1+T_2) = O$.

EXAMPLE 21:—Beam shewn in fig. 43 is loaded with inclined forces as shown. One end is fixed and the other is free Draw bending moment, shearing force and thrust diagrams and find the direction of reaction at the fixed end.

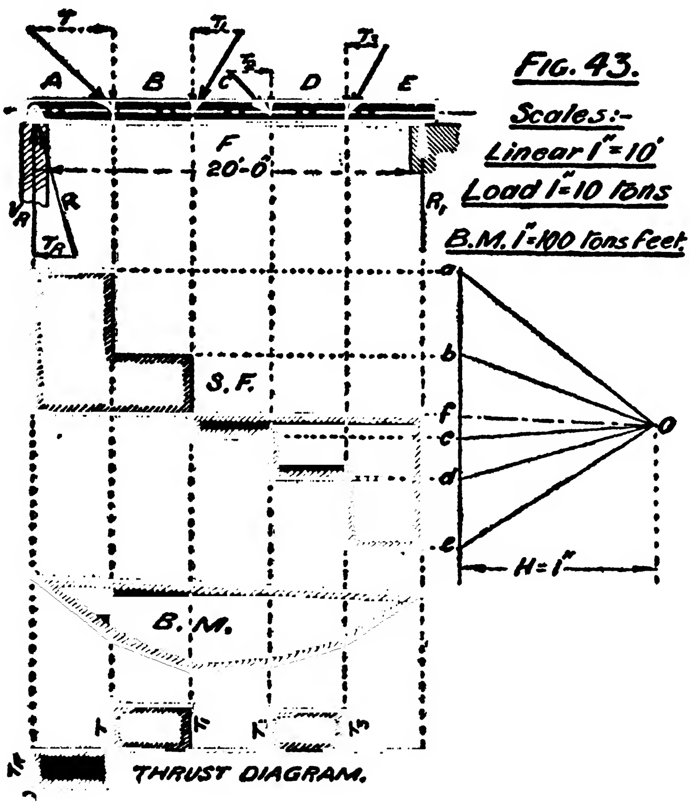


FIG. 43.

Scales:-

Linear 1" = 10'

Load 1" = 10 tons

B.M. 1" = 100 tons feet

SOLUTION—Draw vertical and horizontal components of each load as shown. Take only vertical components for drawing B. M. & S. F. diagrams as usual. Horizontal components only are to be taken to draw the thrust diagram.

Thrust at DE $= T_3$ acting towards left and it is constant as far as the load CD; at CD thrust T_2 acts towards right and it is to be deducted from T_3. The balance of thrust which is very little acts further on till it reaches the load BC; and at BC the thrust T_1 is to be deducted from this and with similar deduction till the left support is reached. The

resultant thrust is acting towards right, the hinged end is to resist this pull and consequently the direction of reaction is as shown in the diagram.

Note:—Direction of reaction, may also be found by plotting the given loads as they are, in a load line and drawing polar diagram and a corresponding funicular polygon, as generally done in roof truss diagrams. See part I.

EXAMPLE 22:—Suppose the overhanging beam shown in fig. 44 is exposed to the action of inclined loads as shown. Draw the bending moment, shearing force, thrust diagrams and also the direction of reaction at the fixed point of the beam.

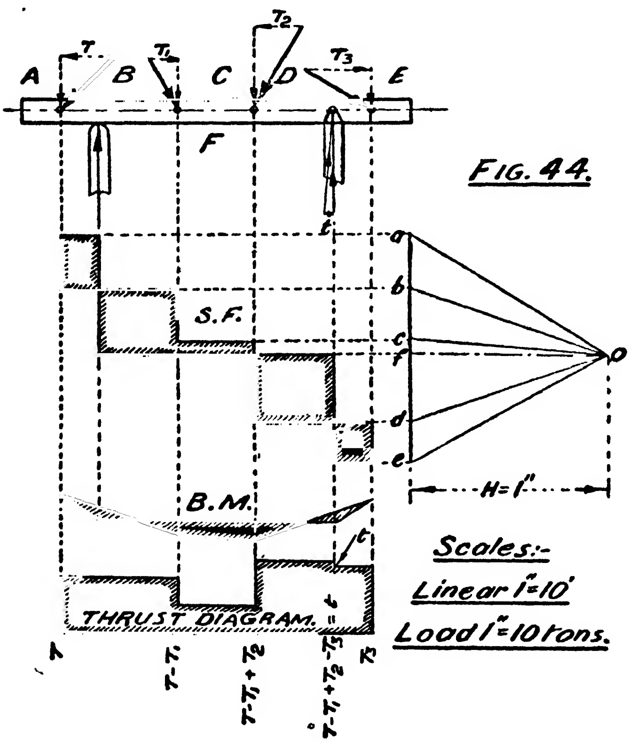


FIG. 44.

SOLUTION:—Bending moment and shearing force diagrams may be drawn as usual by taking only the vertical components of the inclined loads.

Thrust diagram—From the load AB to load BC the thrust is equal to T which pushes the beam towards the left. At BC the thrust T_1 pushes the beam towards the right; hence the thrust from this point to the load CD is equal to $(T-T_1)$. At CD the thrust T_2 is acting towards the left of the beam and therefore the thrust is equal to $T-T_1+T_2$ and this is constant as for as the fixed point of the beam. At this fixed

point the thrust is equal to $(T-T_1+T_2)-T_3$ and from the right of this point to the point of application of the load DE, the thrust is equal to T_3.

Reaction at the fixed point is found as follows. Thrust at the fixed point $=(T+T_2)-(T_1+T_3)=t$. Vertical reaction at the fixed end is equal to *ef*. Plot these two forces at this point and get their resultant and this resultant gives you the magnitude as well as the direction of the final reaction as shown in the figure.

EXAMPLE 23:—A beam 30 feet long hinged at the left end and supported at 10 feet from the right end is loaded as shown. It is required to draw the bending moment, shearing force and thrust diagrams.

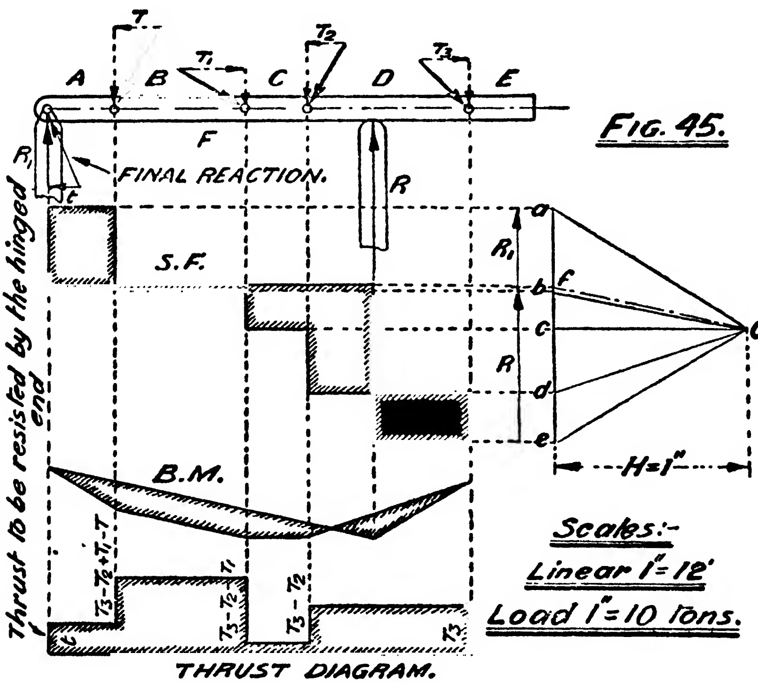


FIG. 45.

SOLUTION:—Since the right hand support is free the reaction is vertical. At this point it appears as if there is a thrust on the right hand support, but really the thrust is on the beam and does not act at the support as it is free.

The thrust diagram is as shown in the diagram. The resultant thrust *t* which pulls the beam to the right is to be resisted by the hinged end of the beam.

The final reaction at the hinged end of the beam is to be determined by plotting the vertical reaction R_1 and the resultant thrust t at this end. Then the resultant of these two is the final reaction as shown in the diagram. It gives you the direction as well as the magnitude.

In drawing bending moment shearing force and thrust diagrams for inclined beams with vertical loads, we are to resolve the vertical loads into components parallel and perpendicular to the beam. The perpendicular components are to be considered in drawing the bending moment and shearing force diagrams, but in drawing thrust diagrams the parallel compenents are to be taken.

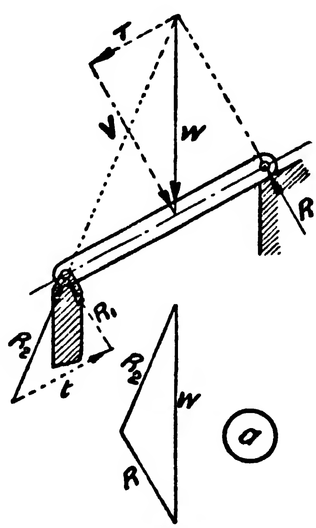
Bending moment diagrams may also be drawn for vertical loads without resolving those forces into components parallel and perpendicular to the inclined beam; for reasons which will be proved presently in the example 24.

Before proceeding to draw the diagrams for inclined beams, students should be well acquainted with the possible practical methods of supports.

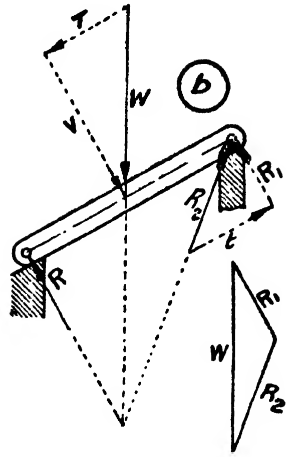
PRACTICAL METHODS OF SUPPORTS.

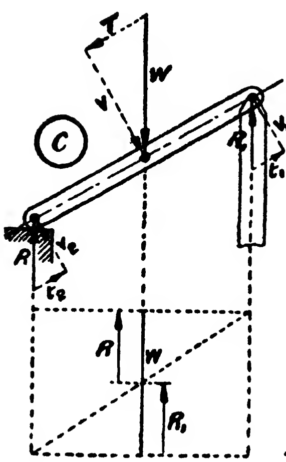
In fig. 46 (a) the lower end is hinged and the upper end is freely supported. The reaction R is normal to tho beam. The reaction R_2 is drawn through the intersection point of W and R. The triangle of forces gives you the magnitudes of R_2 and R and the parallel component of R_2 equals T which is the parallel component of W. The hinged end of the beam is to resist the whole thrust. There is no thrust at the upper free end of the beam. $R+R_1=V$.

Note:—The direction of reaction at the free end is always perpendicular to the bearing surface.



In fig. 46 (b) the lower end is free to move and the upper end is hinged. Reaction R is normal, and t the parallel component of R_2 equals T. The whole thrust goes to the upper joint. $R_1 + R = V$.





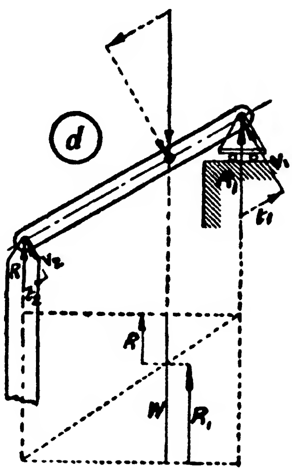


FIG. 46.

In fig. 46 (c) the upper end is hinged and the lower end is free to slide horizontally. R is therefore vertical. $R + R_1 = W$. Magnitudes of R and R_1 can be determined as shown in the lower portion of the figure; t_2 and v_2 are respectively the parallel and perpendicular components of R and similarly v_1 and t_1 are the components of R_1. If W is exactly in the centre of the beam $t_1 = t_2 = \frac{1}{2}$ T and $V_1 = V_2 = \frac{1}{2}$ V. Here we get the thrust at the free end of the beam as well.

In fig. 46 (d) the top end is free to move and the lower end is fixed. The same relations as shown in fig 46. (c) hold good for this as well.

Note:—In these two figures the bearing surfaces at the free ends are horizontal and the directions of reactions are therefore vertical. See note of fig. 46 (a).

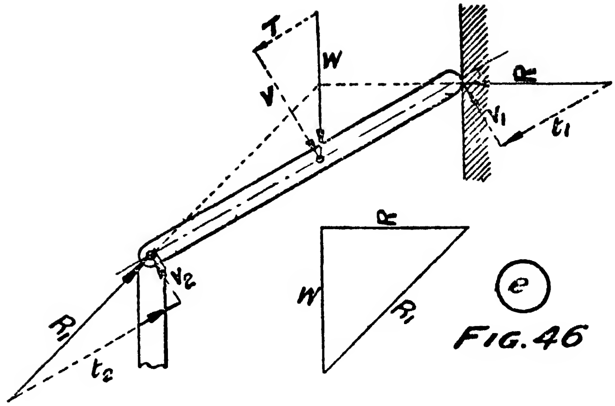


FIG. 46

In fig 46. (e) the lower end is fixed and the upper end is resting freely against a smooth wall and the reaction R is therefore horizontal. Reaction R_1 is drawn through the intersection point of R and W. Magnitudes of R and R_1 are determined by the triangle of forces shown in the figure. $t_2 - t_1 = T$; $V_1 + V_2 = V$. In this case there is a greater thrust at the hinged end.

EXAMPLE 24:—A beam 30 feet long is inclined at 30° to the horizontal and is exposed to the vertical forces as shown in fig. 47. One end of the beam is hinged and the other end is resting freely on the masonry. The supporting surface at the free end is horizontal. Draw the bending moment, shearing force and thrust diagrams.

SOLUTION:—Since the bottom end of the beam is free to move horizontally the reaction R is vertical. Reaction R_1 also is vertical as the given loads are vertical. The magnitudes of R and R_1 can be obtained by polar and bending moment diagrams as shown in the figure. Bending moment diagram is drawn for the given vertical loads or it may be drawn by taking their normal components, but both give us the same value thus —Select any point Y in the beam. B M. at $Y = V_5 \times X$. B. M. at Y is also $= R_1 \times L$. Proof:—Two triangles M Y N & M O P are similar. MP: NY :: MO : MY. $\therefore MP \times MY = NY \times MO$ Here $MP = V_5$, $MY = X$, $NY = L$, $MO = R_1$ hence $V_5 \times X = R_1 \times L$.

After determining the magnitudes of R and R_1 resolve them parallel and perpendicular to the beam; perpendicular components only are to be taken in drawing shearing force diagram; and for thrust diagram the parallel components are to be taken. Shearing force and thrust diagrams are clearly shown in fig. 47. and may be studied.

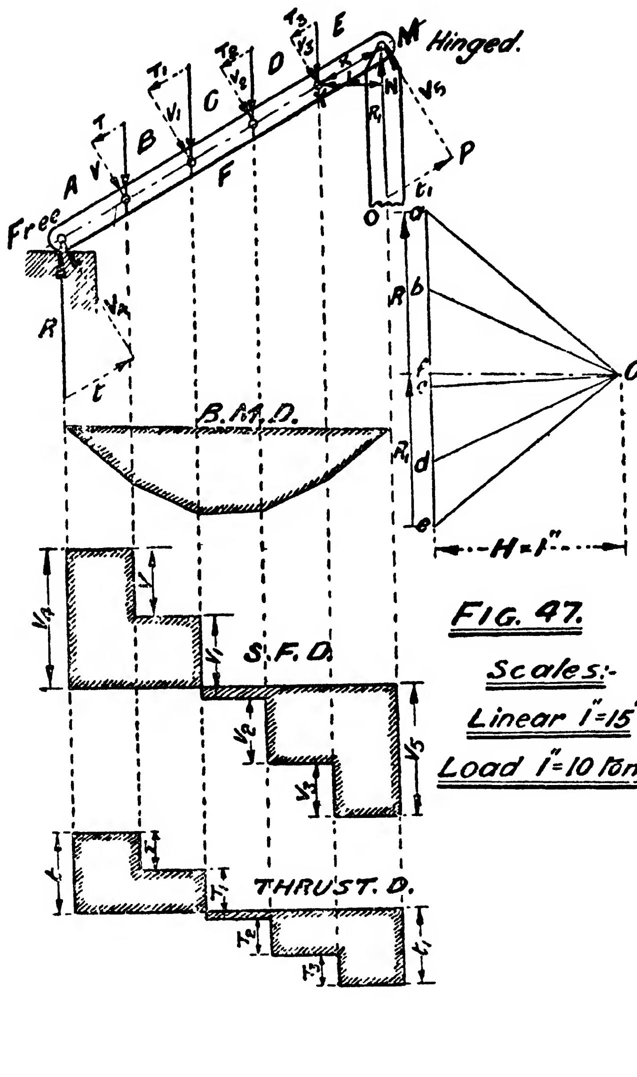


FIG. 47.

Scales:-

Linear 1"=15'

Load 1"=10 Tons.

Note:—There is a thrust at the free end of the beam, only because the supporting surface is not parallel to the beam but if the supporting surface is parallel to the beam there will not be any thrust as you will see in the following example.

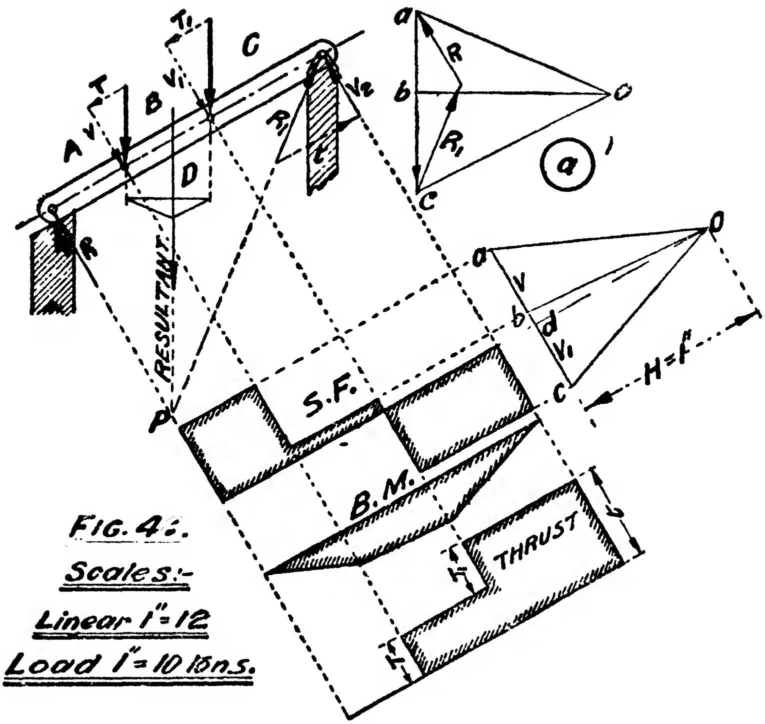


FIG. 48.
Scales:-
Linear 1"=12
Load 1"=10 tons.

EXAMPLE 25:—Take a similar beam of example 24 and load it as shown in fig. 48. Let the supporting surface at the free end be parallel to the beam Draw the bending moment, shearing force and thrust diagrams.

SOLUTION:—AB, BC are the vertical loads. Right end of the beam is hinged and the left end is free and is kept on a base parallel to the beam. The reaction is perpendicular to the beam. The line of action of the resultant of the two given loads determined as shown above, intersects the free end reaction R at P and the reaction at the hinged end must therefore pass through P. Magnitudes of R and R_1 can be obtained by the triangle of forces shown in fig. 48 (a). Bending moment and shearing force diagrams are drawn by taking vertical components as usual but thrust diagram is drawn by taking parallel components.

EXAMPLE 26:—A beam 25 feet long is hinged at one end and the other end is resting against a smooth wall. Inclination of the beam to the wall is 60° and the loads are as shown in fig. 49. Draw the bending moment, shearing force and thrust diagrams.

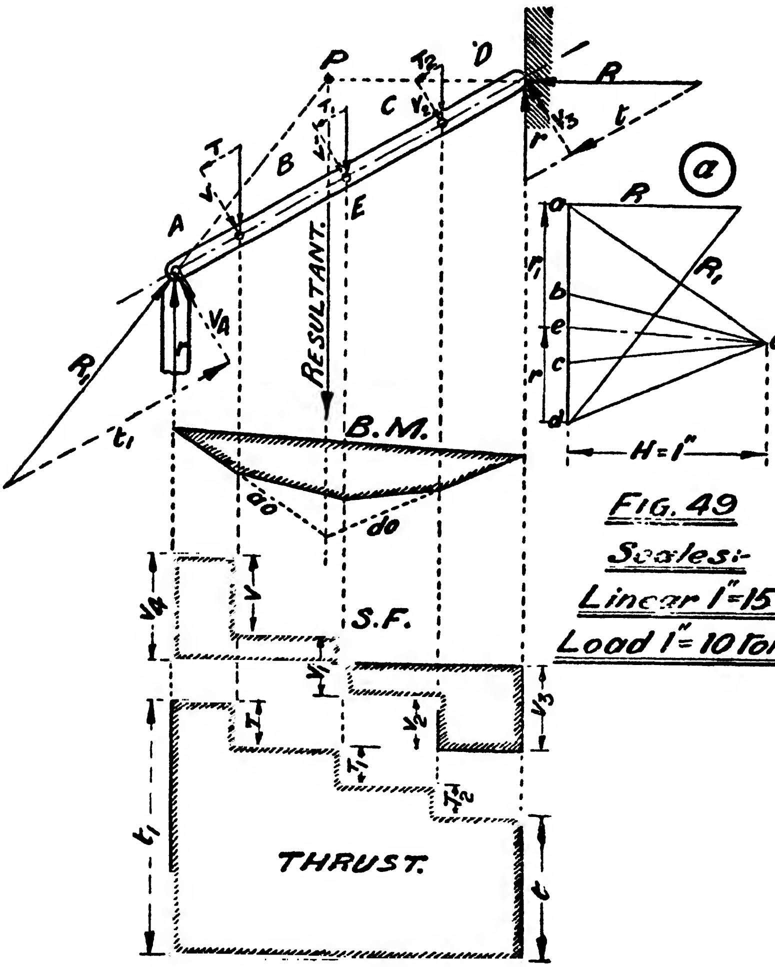


FIG. 49

Scales:-

Linear 1" = 15'

Load 1" = 10 Tons.

SOLUTION:—Reaction at the right end of the beam is horizontal as it is resting against the smooth wall.

The resultant of the vertical loads passes through the intersection of the first and the last ray of the funicular polygon. The horizontal reaction and the resultant intersect at P and therefore the reaction at the hinged end must pass through the point P. Magnitudes of these two reactions are obtained by the triangle of forces as shown in fig. 49. (a).

Bending moment diagram is drawn for the vertical loads, shearing force diagram is drawn by taking the normal components of the vertical loads and thrust diagram is drawn by taking the parallel components of the same loads as shewn above.

Note:—r_1 and r are the vertical reactions, V_4 and V_3 are normal reactions and R_1 and R are the final reactions. Thrust $t_1 - t = T + T_1 + T_2$.

EXAMPLE 27:—A 30 feet girder is hinged at B and simply supported at A. The loading is as shown in the figure. Draw the B. M. and S. F. diagrams. (I. Sc. Eng. Part II. 1926)

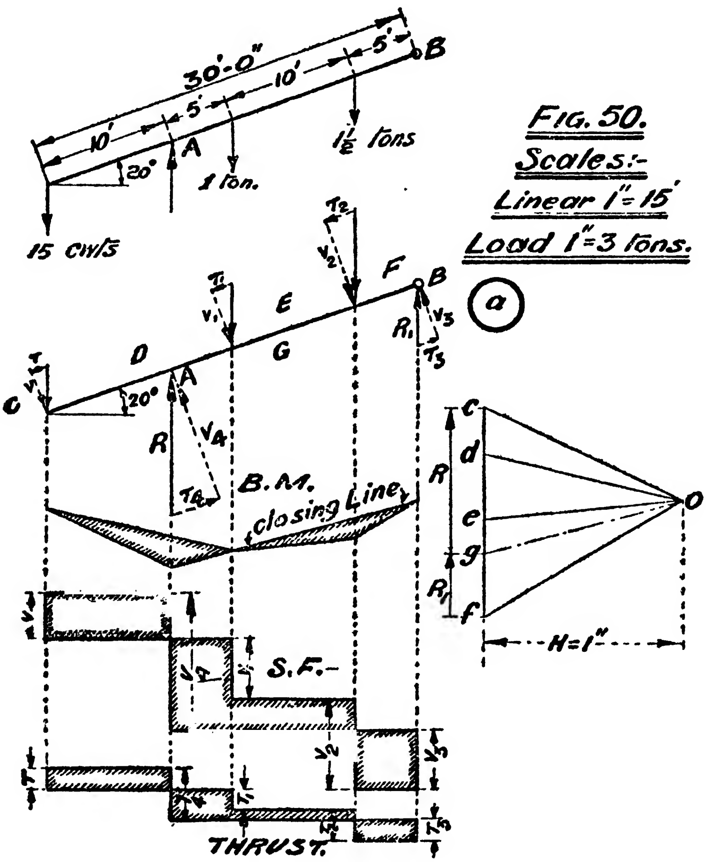


FIG. 50.
Scales:-
Linear 1" = 15'
Load 1" = 3 tons.
(a)

SOLUTION:—Draw the given figure as shown in fig. 50 (*a*). This will facilitate the naming of the forces according to Bow's Notation.

B. M. diagram is drawn for vertical loads, but normal components of these vertical loads are to be taken to draw the S. F. diagram.

Parallel components are to be taken in drawing thrust diagram. For vertical loads, vertical reactions R_1 and R are determined from polar diagram and funicular polygon. Each reaction R and R_1 is resolved parallel and perpendicular to the beam. These perpendicular components $(V_4 + V_3) = (V + V_1 + V_2)$ and parallel components $(T_4 + T_3) = (T + T_1 + T_2)$. Study the diagrams carefully.

EXAMPLE 28:—A beam 32 feet long is hinged at 8 feet from one end and simply supported at 8 feet from the other end. The load is uniformly distributed with 1 ton per foot run. Draw the bending moment, shearing force and thrust diagrams.

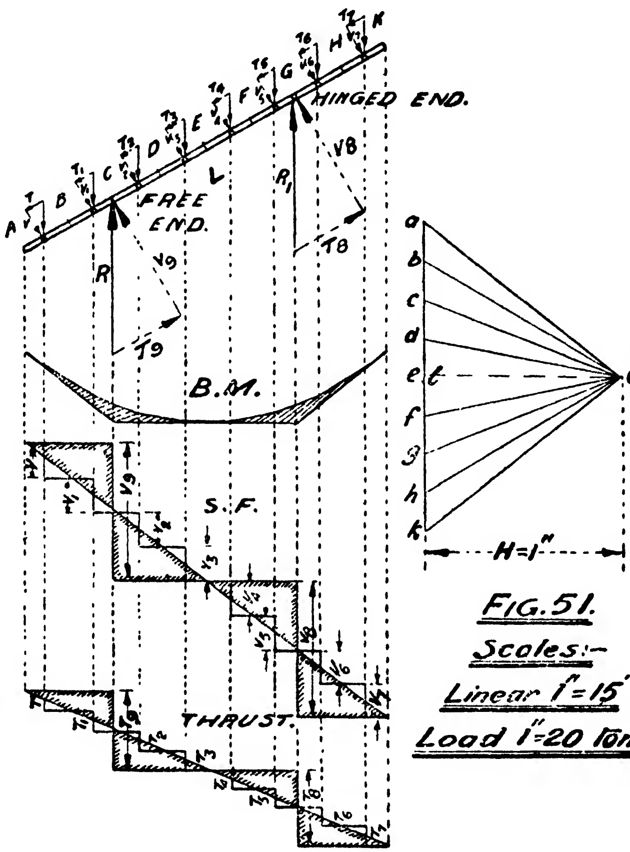


FIG. 51.
Scales:—
Linear 1" = 15'
Load 1" = 20 Tons.

SOLUTION:—The beam has been divided into 8 equal parts and each part equals 4 feet. Load which is equal to 4 tons acts at the centre of gravity of the divided portion. Each load is resolved parallel and perpendicular to the beam. B. M. diagram is drawn for the beam taking vertical loads. S. F. diagram is drawn by taking normal components of the loads. Thrust diagram is drawn by taking parallel components of the loads.

In shearing force and thrust diagrams mean lines are to be drawn, as the load on the beam is a uniformly distributed one.

Note:—(1) one thick line is to be drawn to represent the beam.

(2) Free end reaction is vertical and the beam is allowed

to move and hence this end also is to resist the thrust T_9 as shown in the figure. If the beam were allowed to rest freely on a bearing surface parallel to the beam at the free end, the direction of reaction were then be perpendicular to the beam and consequently there would have been no thrust.

EXAMPLE 29:—A cantilever 6 feet long is inclined to the vertical wall at an angle of 70°. It carries a uniformly distributed load of ½ a ton per foot run. Construct bending moment, shearing force and thrust diagrams graphically.

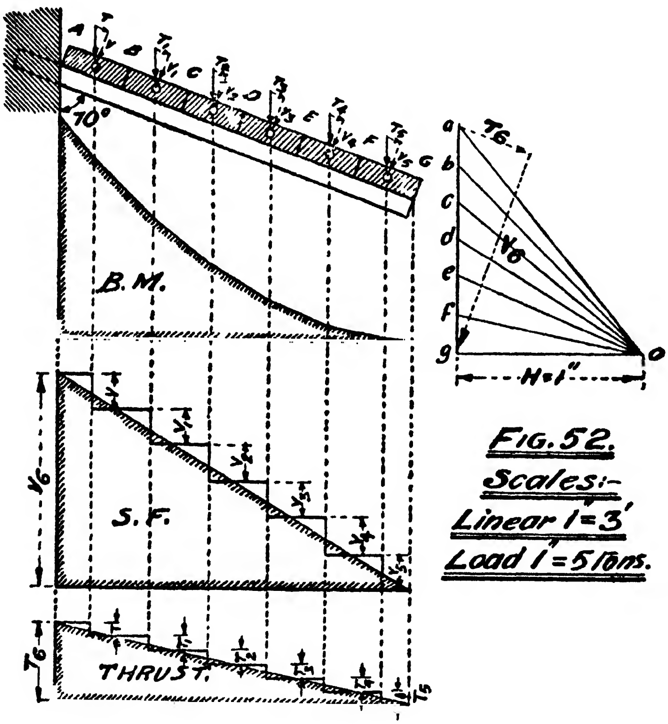


FIG. 52.
Scales:-
Linear 1"=3'
Load 1"=5 Tons.

SOLUTION:—There is no difficulty in following this, as every thing is made clear in the diagrams and no explanation is needed.

ROLLING LOADS.

BENDING MOMENT AND SHEARING FORCE DIAGRAM FOR ROLLING LOADS.

EXAMPLE 30:—A beam supported at both ends on a clear span of 20 feet carries a single concentrated rolling load over it. Draw the bending moment and shearing force diagrams graphically.

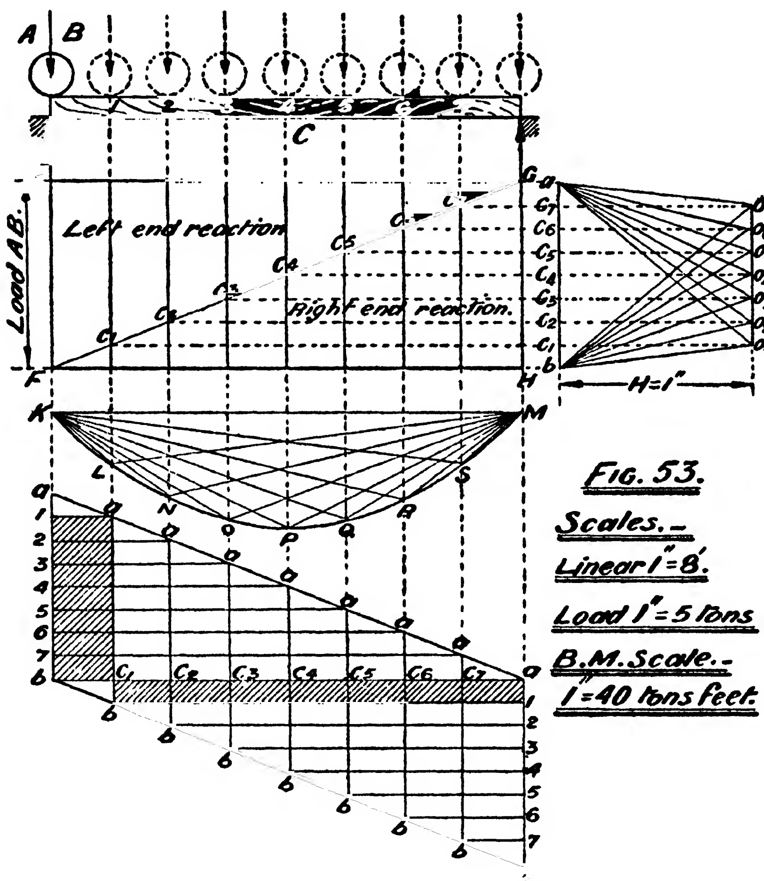


Fig. 53.
Scales.—
Linear 1" = 8'.
Load 1" = 5 tons
B.M. Scale.—
1" = 40 tons feet.

SOLUTION:—Divide the span into any number of convenient equal divisions as shown. Draw EF and GH equal to the travelling load AB and complete the rectangle. Draw the diagonal FG and from each division of the beam draw vertical ordinates as shown.

Now when the load is at the abutment the reaction is epual to the load AB and when the load moves to point 1 the reaction at the left abutment is equal to the ordinate below the point in the left traingle GEF and at the right abutment reaction is equal to the ordinate just below the point 1 in the right triangle GFH. Similarly at every position of the load in the beam the magnitude of reaction can easily be determined.

Method of drawing the Bending Moment Diagram—You know the magnitude of reaction for every position of the moving load and mark these points on the load line successively as c_1, c_2, c_3.........c_7. Exactly at right angles to these points mark poles o_1, o_2, o_3.........o_7 with a common pole distance H.

The object of selecting a pole exactly opposite to the reaction point is to get the closing line of the funicular polygon parallel to the beam. Bending moment diagram for the first position of the load is the triangle KLM, drawn with a polar diagram $a\ b\ o_1$, for the second position KNM with polar diagram $a\ b\ o_2$, for the third position KOM with polar diagram $a\ b\ o_3$, and so on for all the positions. Here you observe all the closing lines are coincident in one line. The apexes of thesd triangles lie in a parabola.

Shearing Force Diagram:—When the travelling load is on the left abutment the shear at that point is equal to the load $a\ b$, when it moves to the point 1 the reaction at the left support is equal to $c_1\ a$ acting upwards and this is constant as far as the point 1, at this point load AB shears the beam downwards and shear at this position is therefore $c_1a-ab=-c_1b$. This $-c_1b$ is constant as far as the right suppoit as shown shaded in the figure.

When the load moves to the second point the reaction at the left support $=c_2a$ and this is the shear from the left support to the point 2 and at this point load AB is to be deducted and this comes to $c_2a-ab=-c_2b$. From this point to the right support $-c_2b$ is the shear. Similarly for each point shear diagram is to be drawn as shown in the figure.

Note:—The student at first is to draw out all these diagrams step by step so as to get a clear idea as to how these diagrams are formed.

After gaining some experience he can only draw one parabola for bending moment diagram taking the greatest ordinate which measures at the centre of the beam. See figure 54.

At any position of the load on the beam bending moment may be known by dropping an ordinate to intersect the parabola and the

intersected point in the curve is to be joined to the right and left supports. For example B. M. at P=QR × H and at any other point such as U the bending moment=VW multiplied by H for the position of the load at P.

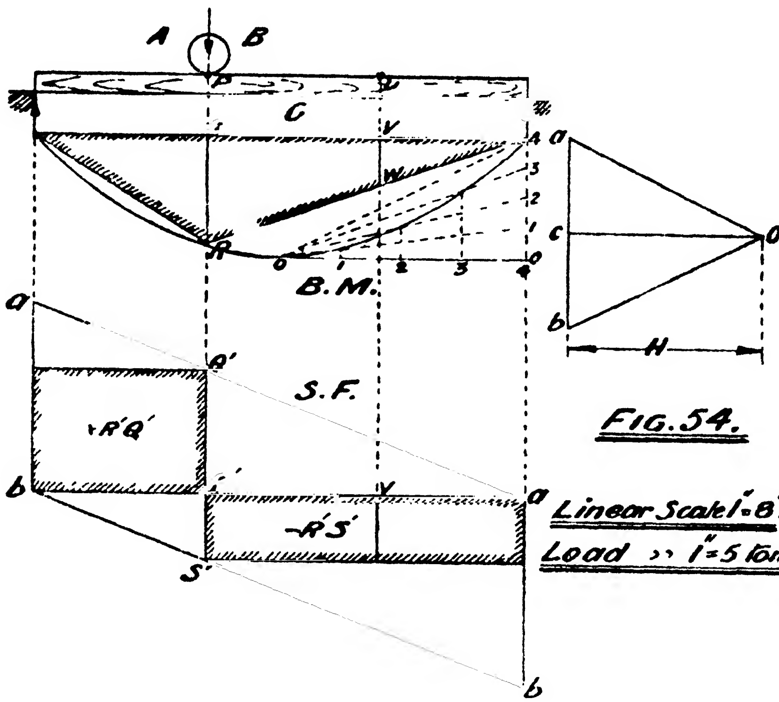


FIG. 54.

Shearing force diagram :—Draw a straight line parallel to the beam and equal to the given span. See fig. 54 From either end of this straight line draw vertical lines equal to the given load *ab*. Join *aa* and *bb*. This figure represents the shearing force diagram. Shear at P=R'Q' - Q'S'= -R'S' and shear at U for the position of the load at P is—R'S'.

SUPPORTED BEAM WITH TWO LOADS OF CONSTANT DISTANCE APART ROLLING OVER IT.

EXAMPLE 31 :—Two loads, spaced 6 feet apart roll over a girder of 40 feet span. If the leading load is 8 tons and the other 5 tons, find (a) the maximum bending moment in tons feet on the girder ; (b) the maximum shear in tons.

FIRST METHOD.

SOLUTION :—Divide the girder into 6 feet divisions as the loads are 6 feet apart. Find the centre of gravity of these two travelling loads as shown in fig. 55 (a). Locate the position of the resultant line in each division as shown. Draw two straight lines on the reaction lines each equal to sum of the travelling loads AB & BC. Draw the diagonal of the rectangle so formed as shown.

As the travelling load passes on from one division to another the ordinates shown in firm lines (Action lines of the resultant of AB and

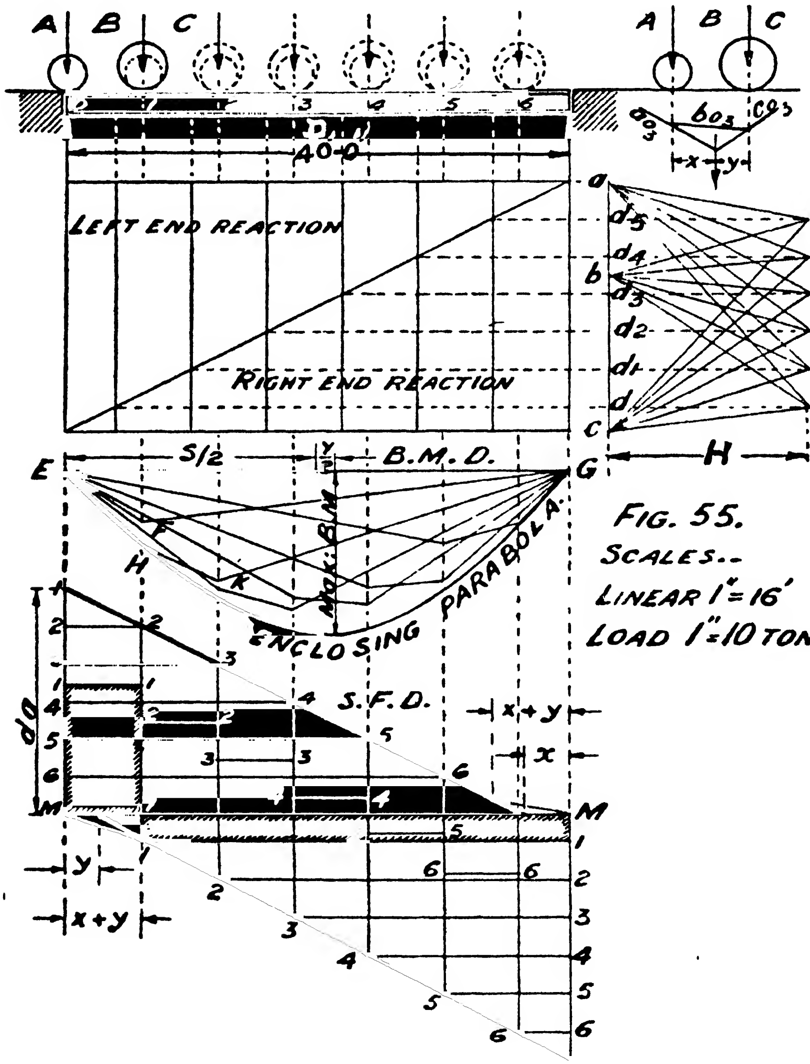


FIG. 55.
SCALES--
LINEAR 1" = 16'
LOAD 1" = 10 TONS.

BC) in the left triangle give the left end reactions and ordinates in the right triangle give the right end reactions as shown in the previous example.

These reaction points are projected on to the load line *a b c* and at right angles to these, poles o, o_1, o_2, etc. are selected with a common polar distance H as shown.

Bending moment diagram:—Assume these two loads AB and BC to occupy the first division, that is at O—1 and the bending moment diagram for this position is EFG. Suppose these two loads come to the second division, that is at 1—2 then the bending moment diagram is EHKG. Similarly draw the diagram for the rest of the positions. In the end, these external points of the bending moment diagrams are to be enclosed with one enclosing parabola as shown.

The maximum bending moment occurs under the heavy load, and picking out the ordinates under the heavy load BC in the bending moment diagram, you can draw the bending moment curve on a horizontal base as shown in fig. 56. This curve starts from a distance Y from the left end of the line and ends at the right end. The maximum bending moment will be exactly at the centre of this parabola and this point is at $\frac{QR - Y}{2}$ from the right end; and from the left end it is $\frac{QR}{2} + \frac{Y}{2}$ which is the same as $\frac{S}{2} + \frac{Y}{2}$, where S is the span and Y is the distance from the heavy load to the resultant line. For mathematical calculation see the chapter on miscellaneous examples.

Similarly by picking out the ordinates under the small load AB in the bending moment diagram, you can draw another parabola starting from the left end and ending at a distance X from the right end. This will not give you the maximum bending moment. These two curves are shown in fig. 56. The enclosing parabola gives you the maximum bending moment at the centre of the span, but this value is to be taken only when you design the girder, as this is a general practice.

Note:—In drawing separate parabolas for light and heavy loads in fig. 56, you get 6 ordinates for each load from the common bending moment diagram and if you connect the ends of these ordinates with curved lines, you observe that these two curves end in X and Y distances apart from the ends respectively, on their own accord.

Shearing force diagram:—Draw a base line MM across the span as shown, when the leading load is at the point 1, the other load AB will be at the left support. The left and right end reactions are *d a* and *c d* respectively. Plot *d a* at the left end reaction line and this is the shear acting upwards at this point, but directly at this point, load AB is acting downwards and consequently AB is to be deducted from *d a*, the balance *d b* is constant till it meets the load BC. From this upward shear the load BC is to be deducted and the balance *d c* is constant till the right end of the girder. Shearing force diagram for this position is shaded and marked 1—1—1—1. For the second position shearing force diagram is drawn and marked 2—2—2—2—2—2 and so on for the rest of the positions.

The stepped forms of the shearing force diagram are to be joined by straight lines as shown, but these two straight lines intersect the base line at X and Y distances from the ends and to this Y distance at the left end add the distance X, and at the right end add the distance Y to the distance X horizontally. At the left end this Y+X distance intersects the bottom sloped straight line at 1, and join 1 M with a straight line. Similarly at the right end Y+X distance is to be intersected with the top sloped straight line and this intersected point and the right end point M are to be joined with a straight line as shown; because at these distances, at the beginning and end of the span, one load only rolls over.

SECOND METHOD:—Bending moment diagram may also be drawn with one polar diagram as shown in figure. 56.

Draw *a b c* the load line and construct polar diagram. There are seven divisions in the girder and consequently there will be seven bending moment diagrams as shown.

The method of drawing is as follows.—Consider the loads AB BC to occupy the first division the bending moment diagram for this position is KLM and MK is the closing line; and let the loads move to the second division, then the bending moment diagram for this position is KNOP and KP is the closing line. Similarly draw the bending moment diagrams for the rest of the divisions as shown. Parallel to these closing lines of the bending moment diagrams, draw lines from pole O to intersect the load line *a b c* at d, d_1, d_2, etc. These positions will give you

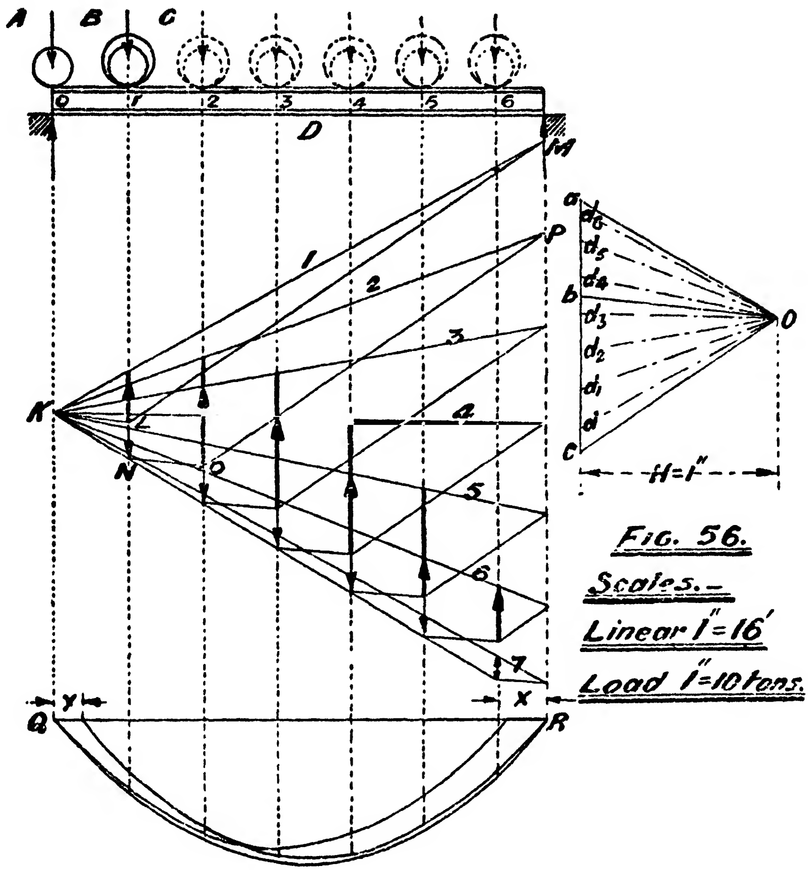


FIG. 56.
Scales.—
Linear 1" = 16'
Load 1" = 10 tons.

the magnitudes of reactions at the supports for different positions of the moving loads.

Thick lines show ordinates of the maximum bending moments for the heavier load; and ordinates between the arrow heads show, the maximum bending moments for the lighter load. Draw a base line QR and plot two parabolas as shown, one for the heavier and another for lighter load.

Now you will have to enclose these two parabolas with a circumscribing parabola as shown in the figure.

Shearing force diagram may be drawn in the same way as shown in figures 53 & 55.

THIRD METHOD:—In this third method you are to assume that the given loads are stationary and the girder is moving. Since you

assume the girder to be moving, you are to take on your drawing paper a length equal to nearly three times the length of the given span. Only one polar diagram is wanted and to start with only one funicular polygon is to be drawn, the first and the last ray may be produced to the desired length as shown in figure 57 to both the sides. Then the series of bending moment diagrams may be drawn. On inspecting these diagrams you can understand the point of the maximum bending moment.

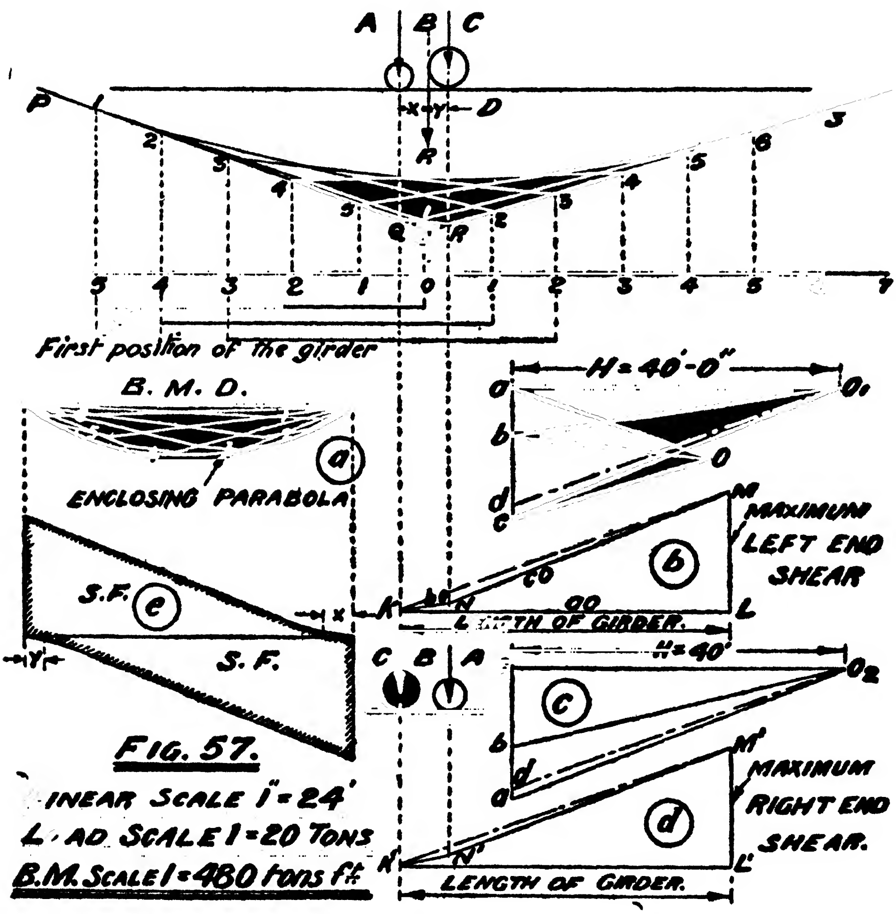


FIG. 57.
LINEAR SCALE 1" = 24'
LOAD SCALE 1 = 20 TONS
B.M. SCALE 1 = 480 tons ft.

In these diagrams linear scale is changed for want of space and consequently bending moment scale is changed. Loads AB, BC are stationary. First draw the load line *a b c* and construct polar diagram and draw the corresponding funicular polygon PQRS. Now bring the girder on the line below and divide it into as many number of equal parts as possible, say 5 divisions of 8 feet each and mark the same distances as shown along the line to the right.

Funicular polygon is already drawn. From either end of the first position of the girder project lines to intersect the funicular polygon at 1—1 join 1—1. Line 1—1 is the closing line of the funicular polygon and 1—1—Q—1 is the bending moment diagram for the first position of the girder. Shift the girder to one more division to the right ; draw similarly the closing line 2—2. In the same way you get six bending moment diagrams as shown in fig. 57.

Plot these six bending moment diagrams on a base line see fig. 57 (a) and last of all you are to draw one enclosing parabola as shown. This represents the final bending moment diagram which gives you the maximum bending moment at the centre of the span. But maximum B. M occurs at $\frac{S}{2}+\frac{Y}{2}$ distance from the left support. (see Page 45).

Shearing Force Diagram—Take a load line *a b c* and have a polar distance H equal to the span of the girder and then draw the corresponding funicular polygon as shown in fig 57 (b). Bring in the length of the girder and adjust it in such a way along the line KL that the first load AB should remain within the girder. Draw from the point L which is the right end of the girder a straight line at right angles to KL to intersect the ray co at M. Then LM represents the maximum shearing force at the left end of the girder.

Proof—KL represents the girder, AB, BC are the loads, KNM is the bending moment diagram, KM is the closing line. From pole O′ a line O′d is drawn parallel to KM. Then *cd* & *da* represent the magnitudes of reactions at the right and left end of the girder.

Two triangles *a d* O′ and MKL are similar and equal in all respects, such that *ao′* = KL, KM = *o′d* and *da* = LM. Since *da* is equal to the reaction at the left end of the girder LM is therefore equal to the reaction at the left end of the girder.

When these two travelling loads approach the right end of the girder, similar diagrams are to be drawn and these are shown in figs. (c) and (d). You will observe that the shear at the right support is the greatest as the heavier load will be nearest to that support.

On a base line equal to the span, see fig. (e) draw from left and right end of that line perpendiculars equal to LM and L′ M′ and connect them to X & Y distances as shown. Two kinks thus formed at X & Y distances may be joined with straight lines as shown. The whole figure is the shearing force diagram.

EXAMPLE 32:—*A girder of 60 feet span is traversed by a locomotive whose axle loads are given above. Draw the bending moment and shearing force diagrams graphically.* See plate I fig. 58.

SOLUTION:—To the right and left of axle loads allow a space equal to the span in your drawing paper. Draw polar diagram with a pole O for the axle loads and a corresponding funicular polygon as shown. Let the first and the last ray *ao* and *ko* be continued upwards as shown. Let the axle loads be stationary and assume that the girder is moving from left to right.

Draw a line a little bit down the funicular polygon and on it take the span length and divide it into six parts as shown. Mark this unit length along the whole length of the straight line as shown. The first position of the girder is from O to 6. Erect perpendiculars at O and 6 to intersect the funicular polygon at 1—1. Join 1—1. Similarly move one division more each time to the right and draw series of bending moment diagrams as shown. For instance the second position of the girder is between 5 & 1, then the closing line of the bending moment diagram for this position of the girder is 2—2. For the third position of the girder the closing line of the bending moment diagram is 3—3 and so on for the rest of the positions. In this way you will have to move the girder till you get the maximum ordinate between the curved line and the closing line by trial. When the length of the engine and tender is less than the span as in the present case, the maximum bending moment may be determined by bringing the centre of the girder exactly at the line of action of the resultant R of all the loads as shown in the diagram. The resultant R acts at the intersection of the first and the last ray of the funicular polygon. Transfer these bending moment diagrams on a base line MN fig. (a by means of tracing paper and draw one enclosing parabola as shown. (See example 31).

Note:—In transferring these bending moment diagrams on the base line MN, care must be taken that the closing lines 1—1, 2—2, 3—3 etc, coincide with the line MN Here MN is equal to the span length; the horizontal projections of all the closing lines are equal to the length MN and all the vertical intercepts between extreme points of each closing line must lie within the space MN.

Shearing force diagram:—From pole O draw rays O—1, O—2, O—3, O—4 etc parallel to the closing lines of bending moment diagrams 1—1, 2—2, 3—3 etc., to intersect the load line at 1, 2, 3, 4....7.

On a base line QP see fig. (b) plate I draw shearing force diagram *1—1—1—1 ; 2—2—2......2 ; 3—3—3......3 ; etc., as shown. For the first* position of the girder the reaction at the left end is $1-a$ and at the right end $b-1$, because there is only one load AB on the girder. In the second position of the girder there are three loads AB, BC and CD. Reaction at the left end of the girder is $2-a$, and this is plotted in the usual way on the base line QP. Q—2—2—2—2......2, is the shearing force diagram for the second position ; similarly for the rest. Finally a curved line is to be drawn to enclose all the stepped forms of shearing force diagram as shown.

This method of drawing the shearing force diagram is very laborious and therefore in figure (c) plate I, a modified form of drawing S. F. diagram is shown and may be studied very carefully. The method is as follows. Take a fresh pole O′ at a distance equal to the span of the girder and from this new polar diagram draw the funicular polygon as shown in figure (c). Now bring in the length TV of the girder on the line TU which is drawn parallel to the ray *ao′* and erect perpendicular line VW to intersect the funicular polygon at W. Now ordinate VW represents the shear at the left end of the girder. The proof of this will be given shortly. The span length TV includes all the axle loads, but bring the point T a few inches towards the left of the axle load BC. In this position the axle load AB is out of the girder and the base line for this is TX drawn parallel to the ray *bo′*

Next bring in the length of the girder so that the left end to be a few inches to left of the axle load BC, then point X be the right end of the girder. From X draw perpendicular XY to meet the funicular polygon at Y. Now you observe XY is less than VW, therefore VW only, gives you the maximum shear.

Proof.—Draw TW the closing line of the funicular polygon and from pole o' draw a line $o'z$ parallel to the closing line TW. Now the triangles TVW and $ao'z$ are equal in all respects, that is $ao'=TV$, $o'z=TW$ and $az=VW$; but az represents the maguitude of the reaction at the left end of the girder and therefore the ordinate VW is equal to the reaction at the left end of the girder. The reaction is nothing but the shearing force at that support, and hence the diagram TVW is the shearing force diagram. This represents only half of the diagram and a similar half is to be drawn on the bottom of the line TV drawing a line from T parallel to VW down, and equal to it with the same curve reversed. You will observe this modified shearing force diagram will be exactly equal to the shearing force diagram represented in figure (*b*).

In fig. (*d*). the curve of maximum bending moment from O to 60 feet and on, has been shown.

Note:—(*a*) The span length is to be brought perfectly horizontal and adjusted under the axle loads and not on sloping lines such as TX. The left end of the girder is to be a few inches left of the first axle load, but if that load were to be exactly over the left end of the girder the maximum shear is reduced by the magnitude of that load.

(*b*) The pole distance is always to be taken in load scale and not in linear scale, but the ordinate in the bending moment diagram is to be taken in linear scale. The reason is, that all intercepts are nothing but distances and not forces, but the pole distance is the horizontal component of the polar rays. These polar rays are forces and not intercepts.

(c) From the point Z in the load line a chain dotted line is drawn to pole O and also from points T and W dotted lines are drawn to intersect the funicular polygon at S and S. Then SS is joined by a chain dotted line and this line SS is parallel to ZO, and hence the accuracy of the diagram.

EXAMPLE 33—The live load on a Pratt Truss of 128 feet span shown in plate II fig. 59 consists of axle loads of a 195·6 ton locomotive, followed by a train weighing ·4 ton per lineal foot. The bridge is of a through type and depth is 22 feet.

Draw the bending moment and shearing force diagrams and determine the chord and diagonal stresses graphically.

SOLUTION:—Plot the axle and train loads on a load line and complete the polar diagram. Let the polar distance be a multiple of the depth of the girder. Draw the funicular polygon as shown. Keeping the loads stationary move the bridge from left to right as explained in the previous example. Draw the series of bending moment diagrams as follows—The first position of the girder is between O and 8 and the closing line for this position is ST ; the bending moment diagram is the figure enclosed between the closing line ST and the curve of the funicular polygon. The second position of the girder is between 1 and 7 and the closing line for this position is $S_1 T_1$; the bending moment diagram is the figure enclosed between the closing line $S_1 T_1$ and the curve of the funicular polygon between two points $S_1 T_1$. Similarly the closing line of the bending moment diagram for the third position is $S_2 T_2$, for the fourth $S_3 T_3$ and so on.

Determination of the chord stresses is as follows.—There are eight panels in this girder and if you can determine the maximum stresses for half of the girder, that is for four panels that will be sufficient, as the maximum stresses on the remaining four panels will be exactly the same. These four panels are marked in the frame diagram. Now for the first position of the girder, on the closing line ST mark these four points as 1, 2, 3 and 4; and for the second position of the girder mark these four points again on the closing line S_1 T_1. Similarly for all the positions of the girder and on all the closing lines mark these four points as shown in the diagram.

Then join all the points bearing number 1, number 2, number 3 and number 4 by thick firm lines separately as shown. Now you observe the greatest ordinates in the bending moment diagram of the thick lines passing through the points 1, 1, 1............1 ; 2, 2, 2............2, and 3, 3, 3............3, occur at the point T, and these ordinates are marked by arrow heads as shown. The greatest ordinate for the first panel is between 1 and T, for the second panel between 2 and T and for the third panel between 3 and T. Lastly the greatest ordinate for the thick line passing through the points 4, 4, 4............4. occurs between T and T_1 and this is marked by arrow heads. These ordinates in load scale multiplied by 4 will give you the respective chord stresses.

It was shown in pages 1 and 2 that the flange stress in any beam or girder is equal to the bending moment divided by the depth of the beam or girder. Graphically also we can prove as follows—Bending moment = ordinate of the bending moment diagram multiplied by pole distance H. Flange or chord stress $=\frac{\text{B. M.}}{\text{D}}$. Substituting the value of bending moment, we have chord stress $=\frac{\text{Ordinate}\times\text{H}}{\text{D}}$. When H is made equal to the depth of the girder, chord stress $=\frac{\text{Ordinate}\times\text{D}}{\text{D}}=$ Ordinate. In this example H is equal to 4 times the depth of the girder, therefore the chord stress = ordinate in load scale multiplied by 4.

For example take the maximum ordinate for the fourth panel and it measures to the load scale nearly 60 tons, then the chord stress is equal to $60\times4=240$ tons. Now verify this from the value of the bending moment. Bending moment at that point is equal to the ordinate in linear scale multiplied by the pole distance in load scale. Then $30'\times176$ tons $=5280$ tons feet and this divided by the depth of

the girder you get the chord stress which is equal to $\frac{5280}{22} = 240$ tons as before.

The stresses in the vertical and diagonal members can be determined from the shearing force diagram as shown in figure (*a*) plate 11. Here the shearing force diagram has been drawn in the same way as explained in example 32. plate I figure (c). The maximum shear of the left abutment is marked in the diagram and that is the shear at the supporting point of the panel number 1. This shear is not uniform throughout the length of the panel 1 and therefore you should take the average shear to determine the stress in the member AC. Therefore take half the length of one panel from the maximum shear ordinate to the left and draw a line parallel to the maximum shear ordinate, and this will represent the average shear in the first panel. Resolve this parallel to the member AC and the stress in the end post AC is determined as shown in the diagram.

Now draw the ordinates from the average shear at the intervals of each panel length as shown and these ordinates represent the average shear at the remaining three panels. From these ordinates the stresses in the diagonals and verticals are determined and these are clearly shown in the diagram (*a*). The stresses in the diagonals and verticals of the right half of the bridge are the same.

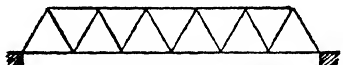
You observe here that the stress in CD is not determined in this shearing force diagram, but the tension in this member is equal to that portion of the loads between the supporting point and the second panel joint that is carried by the roadway to the floor beam. These heavy exle loads being brought on so as to cause the stress in this member a maximum.

Chapter V.

BRACED GIRDERS OR TRUSS BRIDGES.

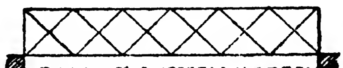
There are very many types of braced girders but only the usual types will be dealt with in this chapter. The most usual types are Warren, Howe, Pratt, Camel Back, Baltimore, Petit, K-Trusses, Bow-String Girders and Lenticular or Fish-Belly Girders.

FIG. 60.



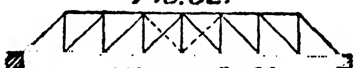
WARREN GIRDER.

FIG. 61.



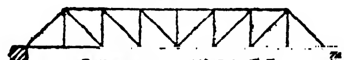
DOUBLE SYSTEM WARREN GIRDER.

FIG. 62.



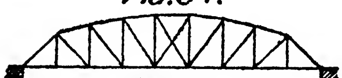
HOWE TRUSS

FIG. 63.



PRATT TRUSS OR N GIRDER.

FIG. 64.

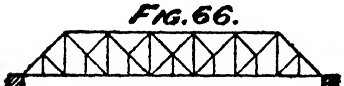


HOG OR CAMEL BACK TRUSS

FIG. 65.

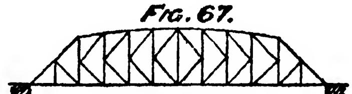
BALTIMORE TRUSS.

FIG. 66.



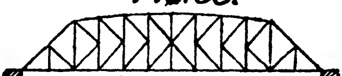
PETIT TRUSS

FIG. 67.



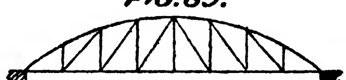
K TRUSS.

FIG. 68.



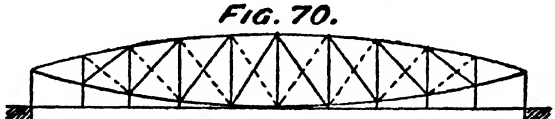
K TRUSS.

FIG. 69.



BOW STRING GIRDER.

FIG. 70.



LENTICULAR OR FISH BELLY GIRDER.

All these braced girders are used for bridges and some types of Warren and Pratt Trusses are also used for roofs and for supporting roof principles.

Bridge trusses are generally two in number and they are to carry the road ways and railway lines. Figures 60 to 70 are elevations of one

of the pairs of girders and trusses The road ways and railway lines are carried by cross girders which are usually called floor beams with lateral bracings. For railway lines rail bearers or stringers are connected longitudinally to the cross girders. These cross girders in turn, are connected to the bottom joints of trusses for through bridges and to the top joints of trusses in the case of deck bridges.

In deck type the roadway of the bridge is carried on the upper chord of the truss ; and in through type the roadway rests on the bottom chord.

The names of different parts of a bridge truss are shown in figure 71 plate III.

Note:— *(a)* In this diagram there are two Pratt Trusses and the detail connections are shown with their names. These two main trusses carry the dead load of their own, weights of lateral steel connections, road way, and live load etc.

The live load consists of locomotive axle loads in the case of Railway bridges and (for road and highway bridges) the weights of steam road rollers, tramcars or heavy motor busses.

(b) Whether the loads are concentrated or distributed they are made to act on the joints of bridge trusses and hence all the members are subjected to direct compression and tension, but floor beams are subjected to direct and bending stresses. Bottom and top laterals resist the wind pressure only.

EXAMPLE 1:—Draw the stress diagram for a Warren Girder of span 120 feet and depth 20 feet. Dead load 600 lbs. per foot per truss see fig. 72.

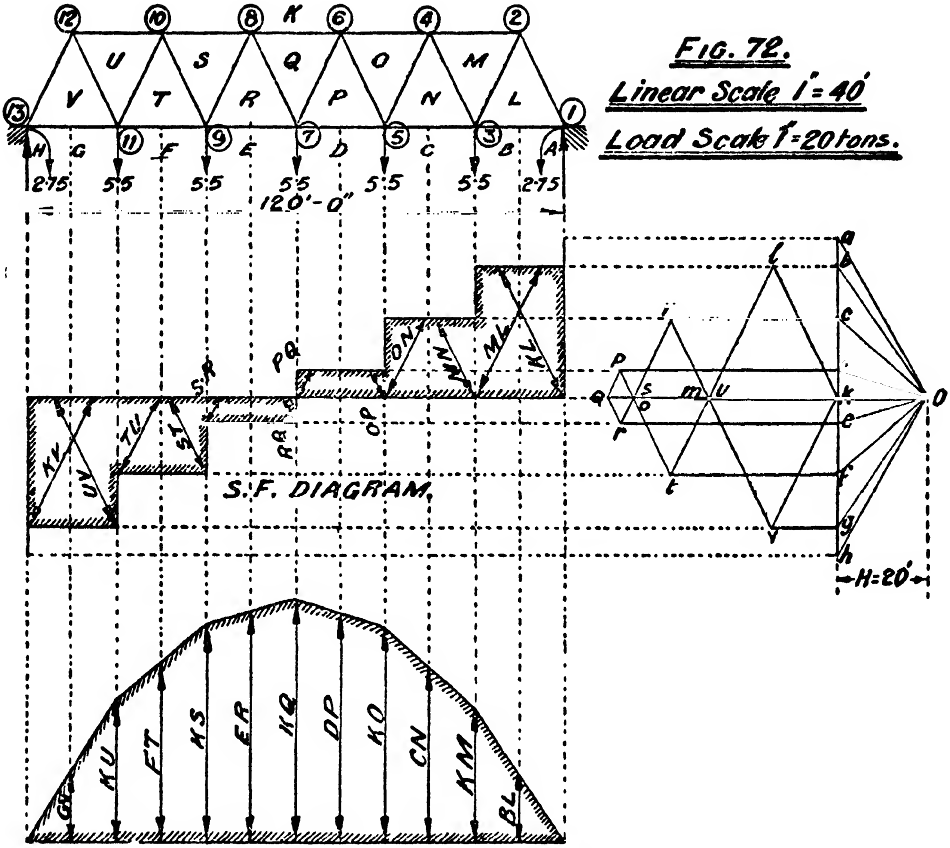
SOLUTION;—The stress diagram is drawn in the same way as you draw the stress diagram for a roof truss.

The truss in the present example is of a through type. Therefore joint loads are to be taken on the bottom chord.

The least number of trusses in a bridge is two. The load given here is per foot run of one truss. The panel load is equal to $20 \times 600 =$ 12000 lbs. or 5·35 tons or say 5·5 tons. The joints at the supports take only one half of the panel load viz. 2·75 tons.

Since the downward panel loads at the supporting joints are in lines with the upward reactions, they are separated by a little space as shown.

Take these panel loads in order, and plot them in a load line as shown. The panel loads are symmetrical, and the reaction at each support is equal to half the sum of the total load.



Commence to draw the stress diagram from the first joint at the right abutment, after solving this joint go to the second joint and then to the third Order is to be kept up.

The closing of the last line is a test for the accuracy of your diagram.

Note:—(1) stresses in the bottom and top chords can be determined from the bending moment diagram drawn with a polar diagram having a polar distance H equal to the depth of the girder as shown. See example 32.

(2) Stresses in the sloping members are to be determined from the shearing force diagram as shown.

(3) Some times Warren Girders are used to support the roof principles, in such cases loads come on the joints of top boom or chord and in certain cases loads will be on top and bottom joints as well.

EXAMPLE 2:—Given a Warren Girder, span 60′, panel 10′ and depth 10′. Loads are as shown in the diagram. Draw the stress diagram graphically.

SOLUTION:—See fig. 73. This girder is unsymmetrically loaded. You are to find the magnitudes of reactions at the supports.

There are three usual methods. First by the method of moments. Secondly by means of the equilibrium polygon and thirdly by the construction method.

Method of moments.—Take the right hand support as moment centre. $R \times 60 = (5\times5)+(3\times10)+(6\times15)+(4\times20)+(2{\cdot}5\times25)+(6\times30)+(4\times35)+(8\times40)+(8\times45)+(8\times50)+(6\times55)$.

$$\therefore\ R=\frac{25+30+90+80+62{\cdot}5+180+140+320+360+400+330}{60}$$

$$R=\frac{2017{\cdot}5}{60}=33{\cdot}62 \text{ tons.}\quad R'=60{\cdot}5-32{\cdot}62=27{\cdot}88 \text{ tons.}$$

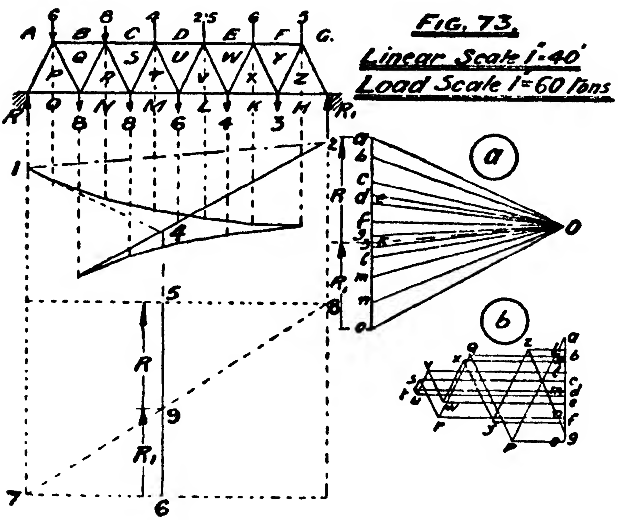


FIG. 73.
Linear Scale 1″=40′
Load Scale 1″=60 Tons

Second method.—Draw the load line as follows.—*ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gk*, *kl*, *lm*, *mn*, and *no*.

Right end reaction is named *gh* and the left end reaction is named *oa* as per Bow's Notation. These cannot come in the load line until you determine them. Consequently the bottom chord joint loads are taken in order, along with the joint loads on the upper chord as shown in fig. (*a*). Draw polar diagram and equilibrium polygon. Let the first ray *ao* touch the left end reaction line at 1 and the last ray *o o* intersect the line of action of the right end reaction at 2. Join 1—2, draw from pole *o*, a line *o*—3 parallel to 1—2. Now vertical line *o*—3 represents the reaction at the right support and 3—*a* the reaction at the left support.

Note:—Since the line of action of the resultant is nearer to the

left support, the left end reaction must be greater than the right end one. Next you are to plot the load line again as shown in fig (*b*) in the order of sequence and draw the stress diagram.

Third Method —The first and the last ray of the funicular polygon intersect at 4. The line of action of the resultant of the total load must pass through 4. Plot the magnitude of the resultant in the line 5—6 and draw horizontal lines to meet the lines of action of reactions at 8 and 7. Join 8—7. Now 6—9=reaction at the right support and 9—5=the reaction at the left support. See rolling load diagrams page 44. After determining the reactions the stress dirgram may be drawn as shown in figure (b).

DOUBLE SYSTEM WARREN GIRDER.

EXAMPLE 3:—The double system Warren Girder shown in fig. 74 is supported at the ends and carries loads as shown. Find the stresses in the members. Span 128′ panel length 20′ depth 20′. Loads are in tons.

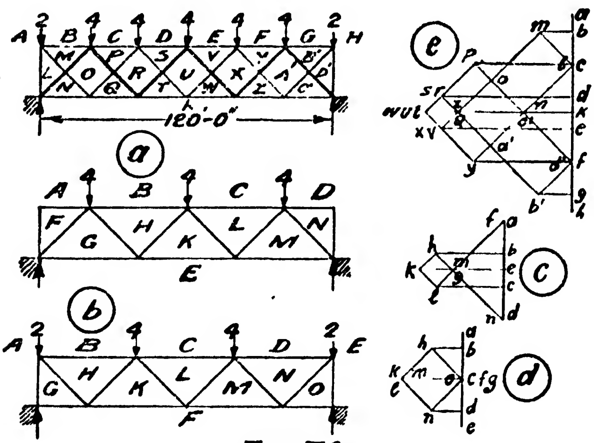


FIG. 74.
Linear Scale 1"=80'.
Load Scale 1=24 tons

SOLUTION:—In this double system girder there are three unknowns in every joint and the stress diagram cannot be started unless with some real assumption. Split these two systems into two individual systems as shown in figures (a) and (b) These two systems are shown in thick and thin lines in the frame diagram. Figs. (c) and (d) are the stress diagrams for these. From these two diagrams the stresses in each member may be summed up, or you can draw one common stress diagram as shown in fig (e) knowing that the end vertical members A L and H D′ resist only half the sum of the total load on the system shown in thin lines, and this you observe in stress diagram (d) which is equal to 6 tons. After plotting this in the

load line as shown in fig. (e) no difficulty will present in the stress diagram. Stresses on the inclined members appear twice in the stress diagram as they are named twice.

Note :—The stress diagram (e) is the summation of the stress diagrams (c) and (d). For instance the stress in the member KT or KW fig. (e) is equal to the sum of the stresses in the members EK of fig. (c) and FK or FM of fig. (d). Similarly the stresses of all the members may be summed up.

EXAMPLE 4 :—Take the same Warren Girder of example 3 and load it unsymmetrically as shown in fig. 75. Draw the stress diagram.

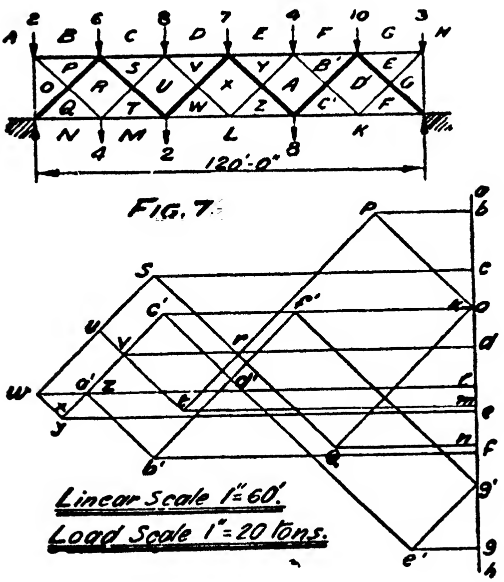


FIG. 75.

SOLUTION :—First find the reactions either by calculation or graphically for the system shown in thin lines and these work out to be 12·00 at the left support and 9·00 at the right support. The forces in the left and right end verticals are 12·00 and 9·00 tons respectively.

Magnitudes of reactions on both the supports for both the systems of loading are 26·16 and 27·84 tons respectively.

Plot all these loads on the load line and draw the stress diagram as shown.

EXAMPLE 5 :—Figure 76 represents a double system Warren Girder with end verticals at the intersection of the inclined members with even number of panels. Loading is as shown. Draw the stress diagram.

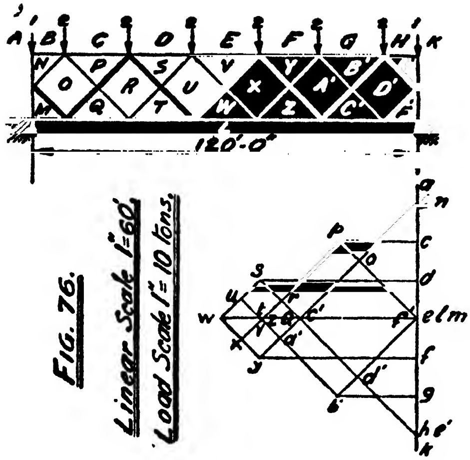


FIG. 76. Linear Scale 1"=60'. Load Scale 1"=10 tons.

SOLUTION :—Since the loads are symmetrical the reaction on either support is equal to $\frac{W}{2}$ where W = total load.

Commence from the left end supporting point, the forces at this joint are L A, AM and ML. Out of these LA is known but AM and ML are two unknowns. AM is vertical and ML is horizontal, therefore M must be with L. Hence there is no stress in the member ML. Similarly the left end top point can be solved as shown. Since the diagram closes well all the forces are balanced.

EXAMPLE 6 :—Draw the stress diagram for the double system Warren Girder with odd number of panels.

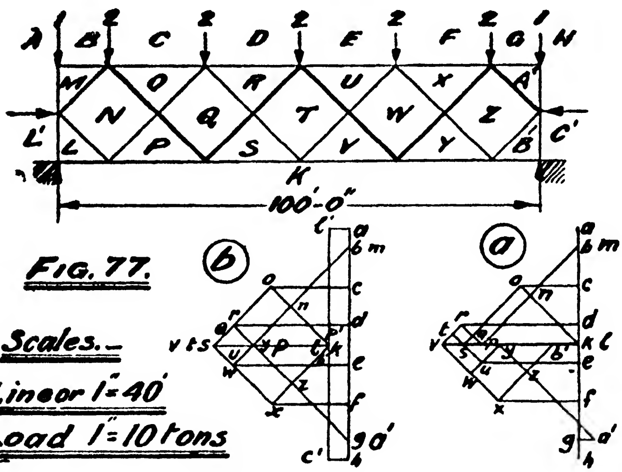


FIG. 77.

Scales.—
Linear 1"=40'
Load 1"=10 tons

SOLUTION :—

Reaction on each support is equal to half the total load.

The two systems are represented by thick and thin lines. If we draw the stress diagram by taking the loads as they are we get the stress diagram as shown in fig. 77 (a). This diagram does not close at all. Letters *b'* and *a'* must coincide with *k* and *g*, if the whole structure were to be in equilibrium.

The reason is as follows—Member LN is in tension from the load CD and member MN is in compression from the load BC and half of load DE. The other half of the load DE is not balanced. The thrust of the member MN puts a cross strain on the end vertical which

produces tension in the members BM and LK, the magnitude of which is equal to ga^1 see fig. (a). Hence no equilibrium in the structure.

Therefore the extra horizontal force which is equal to half of the load DE is to be applied at the centre of each end vertical member as shown to balance the structure.

Plot these loads as shown in fig. (*b*) and draw the stress diagram accordingly.

Note :—(1) In all odd number of panels you will have to assume the horizontal forces on either side, as is shown here to keep the structure in equilibrium.

(2) It is clear that these external forces do not exist, but the tensile stresses in top and bottom flanges at the ends act throughout the whole length of the girder to modify the stresses found in fig. (*a*). Theory requires these two forces. to keep the structure in equilibrium.

DOUBLE SYSTEM BRACED GIRDERS.

In double system braced girders the vertical members introduced in each panel, transmit the pressure to both the systems, but they are stressed to an amount equal to half of the load that directly comes on top of them. At times these vertical members are loaded at top and bottom points as shown in fig. 78 and in this case they are stressed equal to the algebraical sum of the loads.

For example CD=4 tons and HK=5 tons. The vertical member UT is stressed to the amount of $-5+4=-1$ ton. The end vertical AM and $FB^1=\frac{15\cdot5+2}{2}=8\cdot75$ tons.

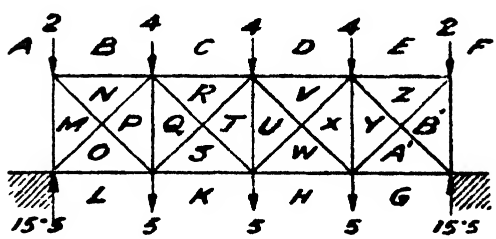


FIG. 78.

EXAMPLE 7 :—The double system braced girder is supported at the ends and carries loads as shown Find the stresses in the members, State the assumptions made.

SOLUTION :-Since every joint contains three or four unknowns you cannot proceed to draw stress diagram unless you find the stress in AL the left end vertical. Stress in AL $= \frac{12+2}{2} = 7$ tons. Now you can commence from the left end support. There are four forces acting at this joint viz. KA. AL. LN and NK. Out of these we know KA the reaction, stress in the member AL is found out. LN and NK are the only two unknowns.

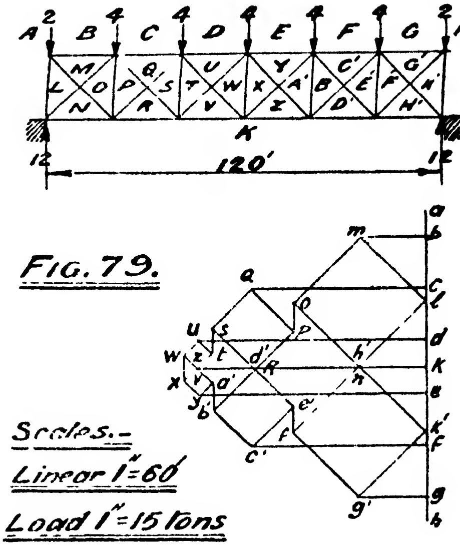


FIG. 79.

Scales.-
Linear 1"=60'
Load 1"=15 tons

This can be drawn easily as shown. As stated above, the stresses in the vertical members are equal to half of the loads that come on them. From these assumptions you can draw the stress diagram as shown.

Note :—There is a redundancy in this girder. The stresses in the members determined like this, by the above theoritical assumptions are not reliable and the stability of the structure chiefly depends on good workmanship and accuracy.

RAILWAY AND HIGHWAY BRIDGES.

The loads consist of the weight of the bridges plus the weight of the platform. In case of through bridges it is usual to take the loads all acting on the joints of bottom chord, but some engineers prefer to take $\frac{1}{3}$ of the total load to act on the joints of top and $\frac{2}{3}$ on the joints of bottom chords. Whereas in deck typed bridges all the loads are to be taken to act on the joints of top chords.

Now let us take some of the usual types of bridges and draw the stress diagrams for dead loads only.

EXAMPLE 8 :—Fig. 80 shows a Howe Truss of a through type, panel length 20 feet and depth 20 feet. Panel loads are in tons. Find the stresses in all the members. Indicate by means of arrow heads the nature of the stresses in the members.

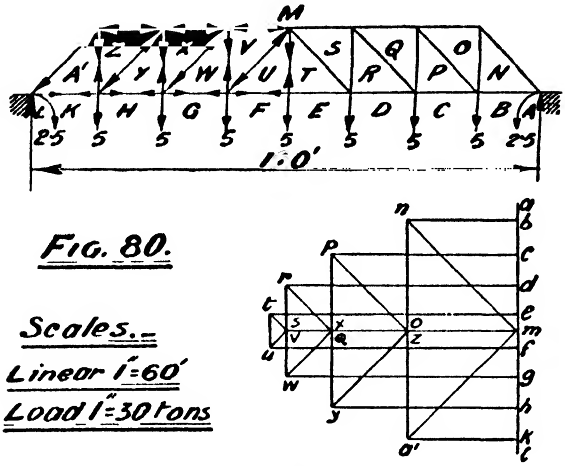


FIG. 80.

Scales.—
Linear 1″=60′
Load 1″=30 tons

SOLUTION:—First find the reaction point *m* in the load line. You can commence to draw the stress diagram either from the right or from the left support.

There will not be any difficulty in drawing the stress diagram.

Note:—In this truss all the diagonals are in compression and the vertical members are in tension. It is always safe and economical to keep short members in compression and long members in tension. This truss is therefore not an economical one as the longer members are in compression, which is objectionable This type of truss is not used for railway bridges.

PRATT TRUSS.

This type of truss is invariably used for railway bridges. This is also called "N" Girder.

EXAMPLE 9.—A Pratt Truss of 80 feet span and 12 feet high is divided into eight equal panels and is loaded uniformly of 1 ton per foot run Draw the stress diagram and show the members in compression.

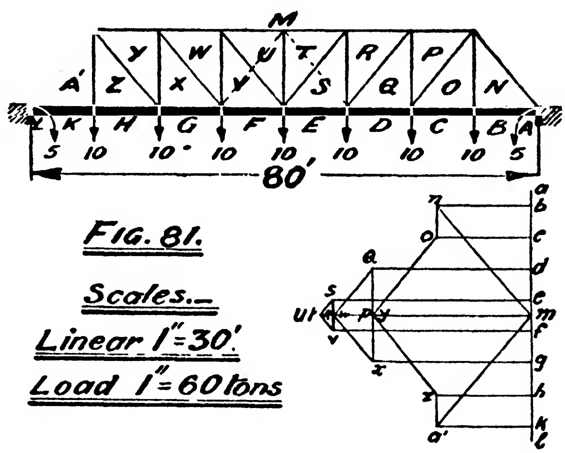


FIG. 81.

Scales.—
Linear 1″=30′.
Load 1″=60 tons

SOLUTION:—There is ambiguity in this question about the given load. It is not stated whether the load is per foot run of the bridge, or per foot run per truss. If you take the load per foot run of the bridge, then each panel load becomes 5 tons, which is exactly half of what is taken above. Note that there are two trusses in a bridge.

Note:—(1) You observe in the two central panels of the frame diagram that two more diagonals are added in dotted lines. These are called "Counter Bracings" or "Counters".

The object of counter bracing is as follows.—When the truss is subjected to rolling loads there will be a reversal of stresses in one or two panels at the centre of the truss. During the reversal of stresses the diagonals in the centre panels are subjected to tension and compression. These diagonals are meant to take only tension and not compression. Therefore by introducing two more diagonals as shown here, only one diagonal is stressed at a time and that too in tension, the other diagonal which is subjected to compression yields itself and does not act at all.

(2) Students find no difficulty if they arrange the panel loads as shown in the frame diagram and then draw the stress diagram, remembering at the same time that lines of action of the panel loads at the supports coincide with the reaction lines.

HOG OR CAMEL BACK TRUSS.

This truss is only a Pratt Truss with inclined top chords. In Pratt Truss chords are parallel. These types of bridges are used for spans from 160′ to 320′. To economise the material in steel work and also to reduce the weight of the trusses and lengths of the vertical members, inclined chords are generally used.

EXAMPLE 10:—Fig. 83 represents a Camel Back Truss Span 180 feet. Panel length 20 feet, depth at the centre 25 feet, depth at the hip 20′, dead load 4 cwts per foot run per truss. Find the stresses in all the members graphically.

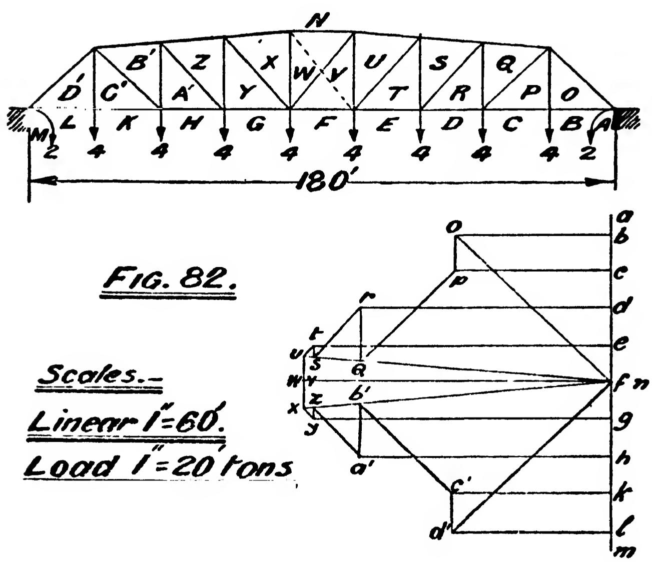


FIG. 82.

Scales.–
Linear 1″=60′.
Load 1″=20 tons

SOLUTION:—Panel load is equal to $20 \times 4 =$ 80 cwts. $= 4$ tons. Load at the supported joint $= 4/2 = 2$ tons.

Now plot the loads in the load line and fix the reaction point n exactly at the centre point of the load line. Draw the stress diagram as usual as shown.

Closing of the last line in the stress diagram ensures the accuracy of your work.

AMERICAN TYPES OF BRIDGES

BOLTIMORE TRUSS.

This truss is a modified form of N Girder or Pratt Truss, with additional members known as subverticals inserted midway along the main panels and these serving as suspenders for floor or cross beams. The advantages resulting from this arrangement are (a) reduction of panel lengths at the bottom chord, (b) shorter spacing of floor beams, (c) shorter and lighter longitudinal girders in the case of roadway, or railbearers in case of the railway.

EXAMPLE 11:—The figure below indicates a Baltimore Truss of six panels, subdivided into 12 panels by the sub verticals supporting the intermediate loads as shown. Panel length 20 feet, depth 40 feet, and dead load 5 tons per foot per truss.

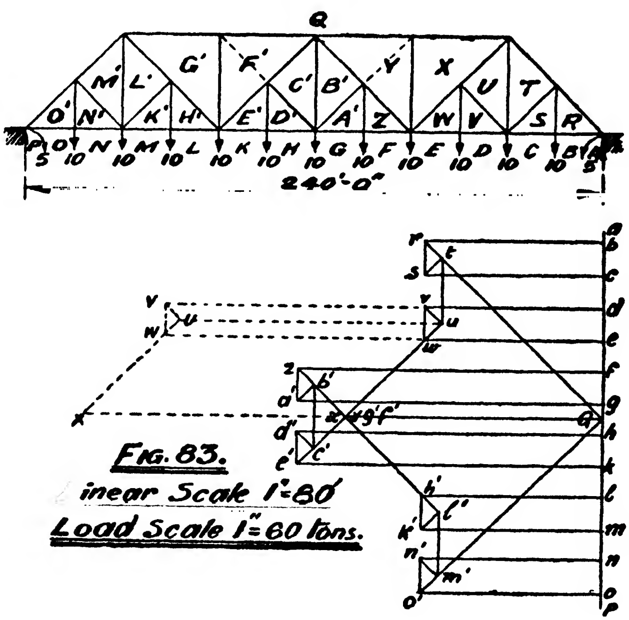


FIG. 83.
Linear Scale 1"=80'
Load Scale 1"=60 Tons.

SOLUTION:—Loads are symmetrical and therefore the supporting forces or reactions are each equal to half the sum of the total load. Plot these given loads on the load line as shown here. Commence to draw the stress diagram from the right end. There is no difficulty till

you come to the veritcal member TU; here there are three unknowns. Get the force in the vertical member TU by the following construction. *Draw two parallel lines d V and e W in dotted lines as shown and select* any point V on the line *d V* and complete the bottom joint of the member WV and similarly you can complete the top joint of the member WV. Thus you get the points VUWX. You know now, that the letter *u* must lie in the line *tu* and therefore drag the whole dotted polygon towards right till the letter *u* comes exactly in the line *tu*, as shown above. You can proceed further without any difficulty as far as the letter *b′*. Now proceed from the left end of the girder as far as the letter *c′*. Here the stress diagram automatically closes itself.

Note:—Since there is a redundancy in the two central panels, the members shown in dotted lines are omitted in drawing the stress diagram.

PETIT TRUSS.

This is one of the types of Baltimore Truss, the only difference being that this has inclined top chords. There is no difficulty in determining the stresses statically for all kinds of loading.

EXAMPLE 12:—Figure 84 shows the skeleton line diagram of Petit Truss. Panel length 25′, depth at hip 50′ and at centre 62′-6″ Dead load = ·80 tons per foot per truss. Draw the stress diagram.

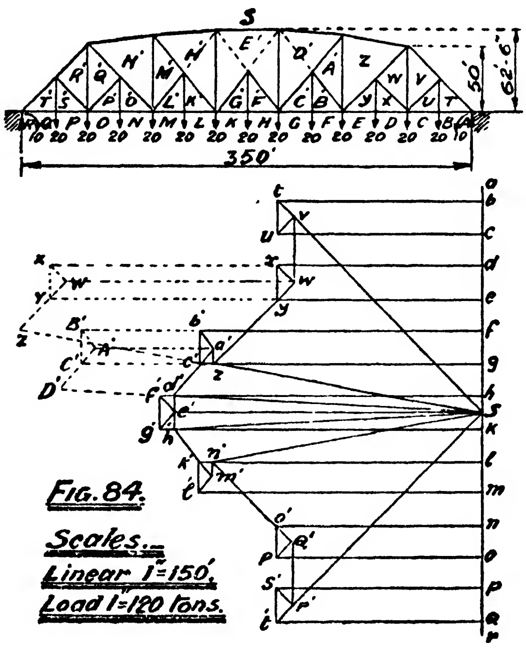


FIG. 84.

Scales.—
Linear 1″=150′.
Load 1″=120 tons.

SOLUTION:—After plotting the loads and reaction point *s* proceed to draw the stress diagram from the right support. Progress will be stopped when you come to the verticals VW and ZA′ and get through the difficulties by the construction method shewn in the diagram. Where there is a redundancy in the structure you can omit one or two members shown dotted in this diagram.

K TRUSSES.

There are two kinds of K Trusses as shown in figs. 85 & 86. Advantages of these trusses are—

(1) Economy of material (2) reduction of secondary stresses. (3) Shortness of compression members, and (4) detail connections are simple and regular.

The form of truss shown in fig. 85 is simple and the stresses can be determined easily, whereas the truss shown in fig. 86 is a difficult problem for the student as the force diagram cannot be easily drawn graphically.

EXAMPLE 13:—Draw the stress diagram for a K truss of 240 feet span depth at hip 40 feet and at centre 50 feet, panel length 20 feet. Dead load one ton per foot per truss.

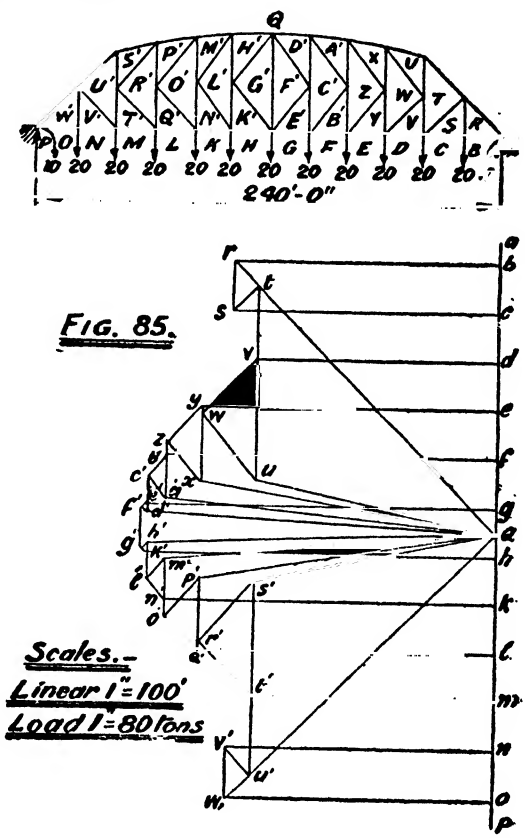


FIG. 85.

SOLUTION:—First find the reaction point Q on the load line and then you can commence to draw the stress diagram either from the right or left end of the truss.

There will be no difficulty in this diagram till you reach the central vertical F′G′. Then start the stress diagram from the other end of the support as usual and the diagram closes well as shown.

EXAMPLE 14 :—Figure 86 shows another form of K Truss of 240 feet span depth at hip 40 feet and depth at centre 56 feet. Panel length 20 feet and dead load one ton per foot per truss. Draw the stress diagram.

SOLUTION:— Commence the stress diagram from the right support and there will be no difficulty as far as the hip vertical TU. From this point there are three unknowns at every joint. Proceed as follows—From points d and Q draw lines dW and QV parallel to DW and QV of frame diagram (Points x, v and w must be in a vertical line). Select any point W on dW drop a vertical line on QV from W and select V. Draw WU and VU parallel to WU and VU of frame diagram. Now WVU is a triangle, and construct a similar triangle at some other point the same straight line as shown. Connect the two apexes of these triangles and produce it till it meets the vertical line drawn from t at u. The position of the letter u is fixed in the stress diagram. From point u draw uw and uv parallel to UW and UV of frame diagram.

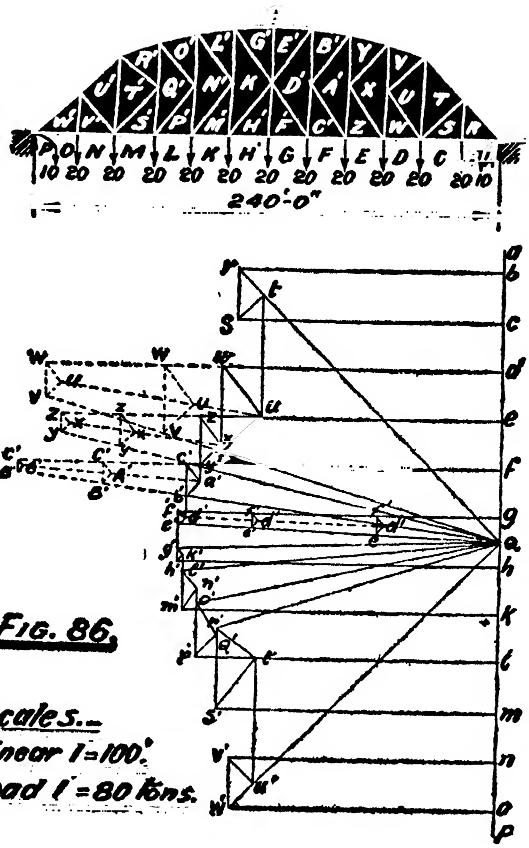


FIG. 86.

cales.—
inear 1"=100'.
oad 1"=80 tons.

To find the position of x proceed as follows.—From e and Q draw lines parallel to EZ and QY select any point Z on EZ, and construct a triangle ZYX at two points as shown. Join XX and produce it till it intersects the line wv at x. The position of the letter x is fixed.

Similarly you are to repeat this method of construction for the rest of the midjoints till you approach the centre vertical. Afterwards proceed from the left hand support and close the stress diagram as shown.

BOW STRING GIRDER

EXAMPLE 15:—Fig. 87 represents the skeleton line diagram of a Bow String Girder of 70 feet span with a height of 12 feet at the centre. Each panel load is 2 tons. It is required to draw the stress diagram.

Maximum shear is at the supports and consequently sufficient area of metal to be provided.

Note—The advantage of this sort of girder is that, we have sufficient depth at the centre to resist the maximum bending moment and no height at the supports where bending moment is nil.

SOLUTION:—Stress diagram presents no difficulty and there are counterbraces in the centre panel. Only one member will be stressed at a time and hence you are to neglect one of the diagonals in drawing stress diagram.

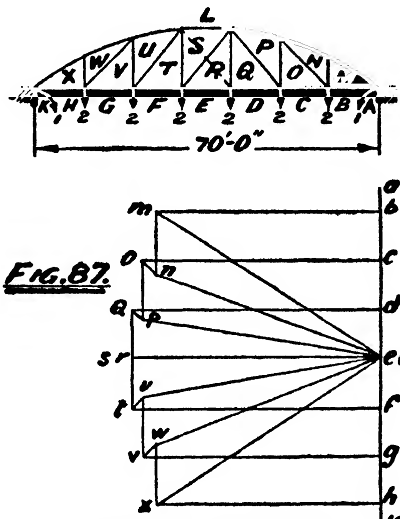


FIG. 87.

Linear Scale 1"=40'.
Load Scale 1"=8 Tons.

Note—In this girder the diagonals which are longer than the verticals are in compression. This is objectionable, and if the diagonals are reversed they take only tension and verticals which are shorter will be in compression This will be then a better design.

LENTICULAR OR FISH-BELLY GIRDER.

EXAMPLE 16:—Draw the stress diagram for the Lenticular or Fish-Belly Girder of span 170 feet, depth at the centre 30 feet and panel length 17 feet. Each panel load is 10 tons.

SOLUTION:—These types of girders carry the road way suspended through the joints of the bottom boom. Diagonal bracings in these are made to take tension only. If any one diagonal is under compressive stress, it does not act and the other diagonal must necessarily be subjected to tensile stress.

In drawing stress diagram you are to omit one of the diagonals for the reasons explained above.

As usual the stress diagram my be drawn without difficulty.

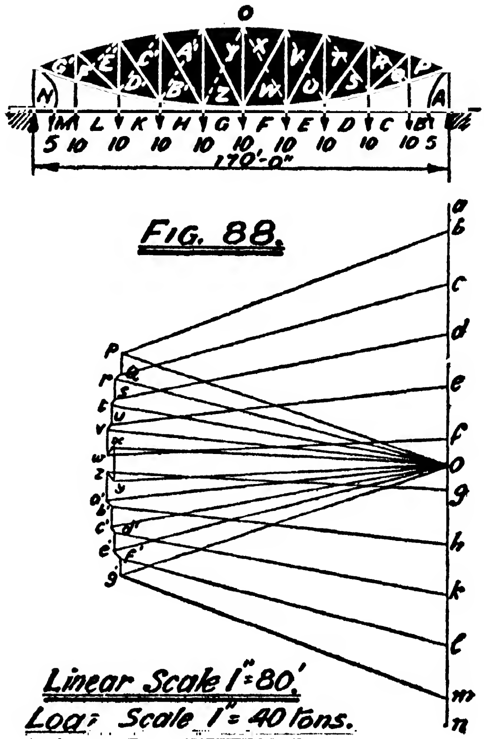


FIG. 88.

Chapter VI.

SPACE FRAMES.

In previous chapters we dealt with structures the members of which were in one plane. In this chapter we deal with members of a structure that are in different planes. Such structures are called "Space Frames" or "Framed Structures of Three Dimensions".

Gin Poles, Sheer legs, Tripods, Derricks, Jib cranes etc , are the examples.

The use of the above in actual practice—Plate girders up to about 60′ span may be swung into place by Stiff Leg Derrick or Guy Derrick and a Gin pole also may be used.

Gin poles may also be used for hoisting steel roof trusses up to about 150′ span. Two Gin poles are to be used for long trusses.

Columns and beams in office buildings may be erected with Stiff Leg or Guy Derricks.

GIN POLE.

A Gin Pole consists of only one leg of timber or steel with four guys and a block at the top through which the hoist line leads to a crab bolted near the bottom see fig. 89 and 90.

EXAMPLE—1. A Gin Pole with four guys is 60′ in height when upright and makes an angle of 15° to the vertical. Find the stresses in the guys and in gin pole when lifting a load of 5 tons.

Four guys are tied at their lower ends to logs of wood on the circumference of a circle, with a radius of 60′.

SOLUTION:—First draw the elevation and plan as shown in fig. 89. In elevation you see only two guys, but in plan you see four guys. Points 3, 4, 5 & 6 are the bottom joints of four guys. All these guys are tied at 2.

You will observe in plan points 3—1—2—5 are in one plane. 2′—3′—1′—5′ are the elevations of the same. Out of 4 ropes and one gin pole 1—2, only one guy rope 3—2 and gin pole are stressed, the rest of the guy ropes (2—4), 2—5, & 2—6 help the point 2 to remain in position. Therefore resolve the load AB parallel to BC and CA and the triangle of forces *a b c* is the stress diagram; *b c* gives the tension in the guy rope 2—3, and *c a* the compression in the gin pole.

Secondly—Arrange the guy ropes as shown in plan of fig. 90. Draw the elevation as shown. Then the pole 1—2, two guy ropes 2—3 and 2—4 are stressed. You observe here that the pole and ropes are in difleient planes, therefore the load cannot be resolved parallel to the members that are in two planes. Now join the points 3 and 4 and draw the centre line 2—7 and this is the substituted imaginary guy in plan. The guy 2—3, substituted guy 2—7 and the other guy 2—4 are in one plane, but the substituted guy 2—7 is also in the same plane with the gin pole 2—1.

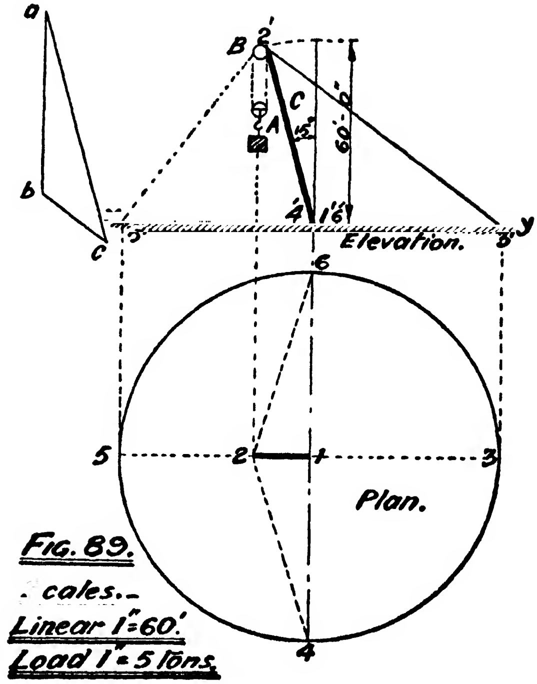


FIG. 89.

The line 2′—7′ in elevation represents the elevations of three straight lines 2—3, 2—7 and 2—4. Since 2—7 is parallel to the vertical plane, the line 2′—7′ in elevation must represent the true length of the substituted guy.

Resolve the load 5 tons parallel to gin pole 2—1 and the substituted guy 2—7.

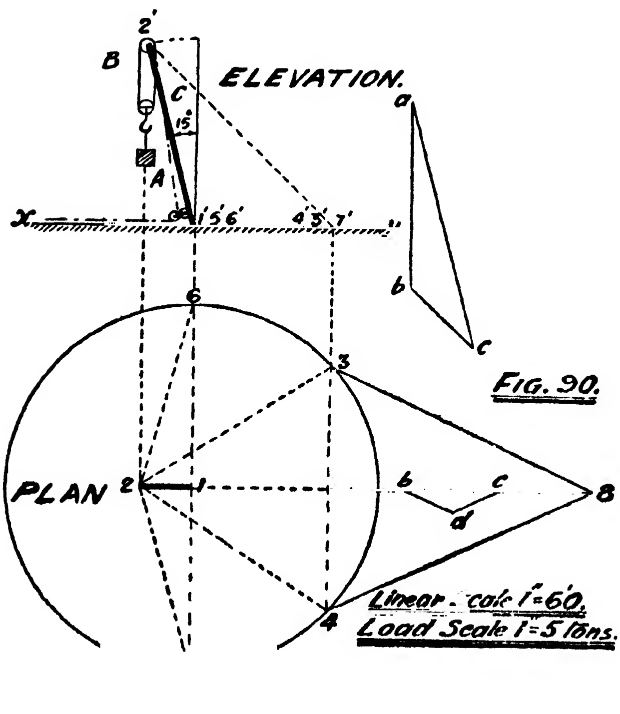


FIG. 90.

The triangle of forces $a\ b\ c$ gives you the stresses in gin pole and in the substituted guy 2—7.

The next step is to determine the stresses in two guys 2—3 and 2—4. From the point 7 draw 7—8 equal to 2′—7′ of elevation and join 8—3 and 8—4. Then 8—3 and 8—4 represent the true lengths of the guys 2—3 and 2—4 respectively. (See fig. 90). In the force triangle $a\ b\ c$ the stress in the substituted member 2—7 is represented by $b\ c$. Plot the load $b\ c$ any where on the line 7—8 and resolve it parallel to 8—3 and 8—4, then $c\ d$ is the stress in 8—4 which is equal to 2—4 and $b\ d$ is the stress in 8—3 which is equal to 2—3.

The other two guys 2—5 and 2—6 do not take any stress and are therefore useless.

Note:—In space frames it is very important to remember to substitute an imaginary member so that it may be common to both the planes concerned. Even if the guys or members are unequal the same process is to be adopted.

SHEER LEGS.

Sheer legs consist of two legs either of timber or steel with one or two guys. If the weight is to be raised to a very great height these legs are generally of built up ones, as shown in fig. 91.

EQUAL LEGGED SHEER,

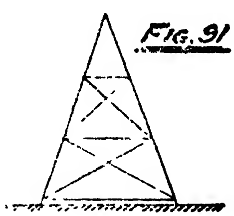


FIG. 91

EXAMPLE 2:—A pair of sheer legs is 45′ in height when upright, each leg being 60 feet long. The backstay is 90 feet long. The plane of the legs makes an angle of 30° with the vertical.

What is the stress in each leg when a weight of 50 tons is supported ? See fig. 92.

SOLUTION:—Sheer legs when upright are 45′. Take $x\ y$ as ground line and draw $v\ v$ at right angles to $x\ y$ and 45′ long, with the same radius lean it to 30° as shown. Then take 90′ the length of backstay and connect it as shown.

The distance apart of the sheer legs is shown in fig (a) and the plan of sheer legs is shown in fig. (b). The picture view of the same is roughly shown in fig. (c).

The sheer legs and backstay are not in one plane and so substitute an imaginary leg between them as shown in dotted line in fig. (c):

This dotted line is common to both the planes. In fig. 92. V V′ represents an elevation of two sheer legs as well as an imaginary leg, but V V′ *gives the true length of the imaginary leg.*

Resolve the force AB parallel to backstay and the imaginary leg as shown in fig. (*d*). Then *a* c represents the stress in imaginary leg and *b c* the stress in backstay. The distance apart of the sheer legs is

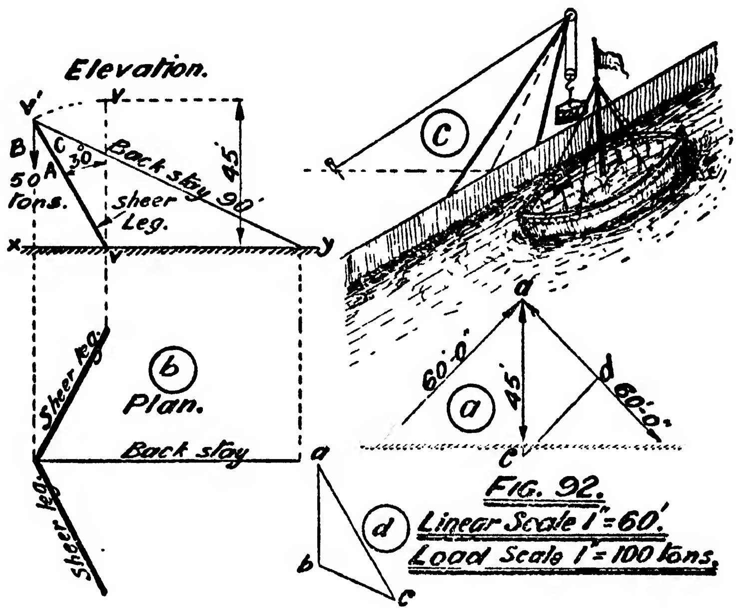


FIG. 92.

shown in fig (*a*) and the imaginary leg is midway between. Plot the force *a* c on the imaginary leg or parallel to it and resolve it parallel to the 60 feet sheer legs as shown in fig. (*a*) *a d* = c *d* = stress in each leg.

UNEQUAL LEGGED SHEER.

EXAMPLE 3:—A load of 5 tons is suspended from sheer legs. The lengths of the legs PA, PB and PC being 40′, 32′—6″ and 37′—6″, while the lengths AB, BC and CA measured along the ground are 37′—6″, 25′—0″ and 50′—0″. Determine the height of the apex P above the ground and the force acting along each leg.

(1 sc. Eng. Part II.)

SOLUTION:—Draw the plan of the base of sheer legs first as per dimensions given. Now P is the apex. The lengths of PA, PB, PC are given. In order to fix the apex point P in plan proceed as follows:—

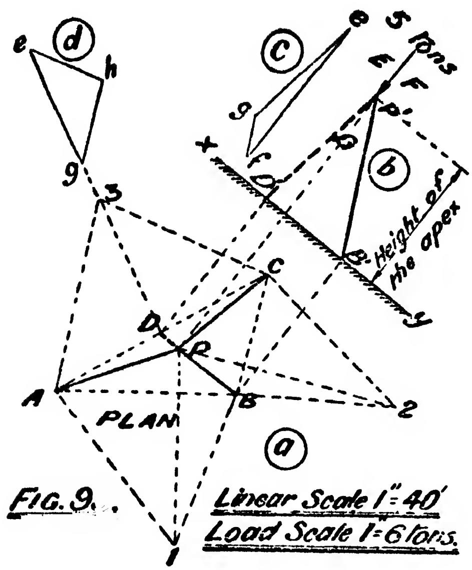


FIG. 9.

Take the length PA equal to 40 feet and strike an arc with centre A and take the length PB=32′—6″ and strike an arc similarly with centre B and let them intersect at 1 (see fig. (*a*)). Similarly take the lengths of PB and PC and get these intersected at 2.

Now the path described by the apex 1, if lifted from the ground keeping points A and B fixed, it will be at right angles to A B. So from points 1 and 2 draw straight lines at right angles to A B and B C respectively and the point of intersection of these two will be the required apex P of the sheer legs.

Now join PA, PB and PC and this represents the plan of the sheer legs, see fig. (*a*). At P the load of 5 tons is suspended. Since PA, PB and PC are not in one plane, substitute one imaginary member in a line with any one of these three legs, so that it may be common to both the planes.

Substitute a member PD in a line with PB, then PD is common to both the planes, *viz.* it is in the same plane with PB as well as in the same plane with PA and PC. Next take a ground line XY parallel to PBD. In this vertical plane you get the real length of the sheer leg PB as well as the real length of the substituted dotted member PD. Draw projections from the points PBD at right angles to X Y as usual. The projection line from P is to be continued till it is intersected at P′ with a straight line equal to the real length of PB=32′—6″. Join P′B′ and P′D′.

Resolve the load 5 tons parallel to P′B′ and P′D′. The triangle of forces *e f g* fig. (c) will determine the stresses in P′B′ and P′D′. PA, PD, PC are in one plane, bring these to a horizontal position again and let them meet at apex 3, see fig. (*a*). Now plot *g e* in a line or parallel to the substituted member PD which is equal to D3 in this new position and resolve the same parallel to the other two legs A3

and C3. See fig. (*d*) *e h* is the stress in C3 which is equal to PC and *g h* is the stress in A3 which is equal to PA.

TRIPODS.

A tripod consists of three legs made of wood or steel. Generally the base on which the legs rest is a triangle. In case of equal legs of equal inclinations to the horizontal the base will be an equilateral triangle.

EQUAL LEGGED TRIPOD.

EXAMPLE 4:—A load of 7 tons is suspended from a tripod, the legs of which are of equal lengths and inclined at 60° to the horizontal. Find the thrust on each leg.

(I Sc. Eng. Part II. 1923.)

SOLUTION:—Since the legs are of equal lengths and of equal inclinations, the base is an equilateral triangle. Describe any equilateral triangle say ABC fig. 94 (*a*). The intersection point of two medians will give you the apex point P in plan. Join PA, PB, PC. This representsl the plan of the tripod. Now substitute an imaginary leg in a line with any of these three legs. Say PD and this is in a line with the plan of the leg BP. Then you observe that this substituted leg PD is in a plane with BP and as well as in a plane with the other two legs PA and PC. Hence PD is common to both the planes.

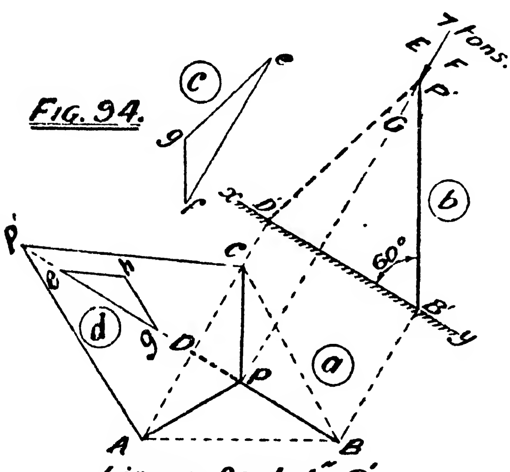


FIG. 94.

Linear Scale 1" = 8'
Load Scale 1" = 8 tons.

Next take a ground line $x\ y$ parallel to the line BPD and project the elevation B′ P′ D′ as shown in figure (*b*). In this elevation, lines B′ P′ and P′ D′ represent the true lengths of the legs BP and PD as the vertical plane is parallel to BPD. Now resolve 7 tons parallel to these two legs and you get the force triangle *e f g*. See fig. (*c*). Here *g f* is the stress in the leg PB and *g e* is the stress in the imaginary leg. Since this imaginary leg is also in the same plane with PA and

PC of fig. 94 (*a*) set out the true lengths of PA, PC and PD as shown in fig (*d*) and plot the stress *g e* on the imaginary leg or parallel to it. (This is considered as a load acting on the imaginary leg). Then resolve this force parallel to the other two legs P′ A and P′ C as shown. *gh*=*eh*=stress in PA and as well as in PC. Therefore *gf*=*gh*=*eh*.

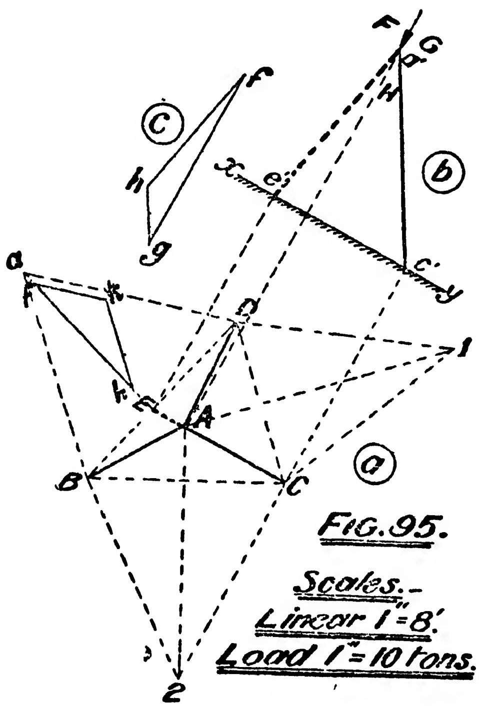


FIG. 95.

Scales.- Linear 1″=8′. Load 1″=10 tons.

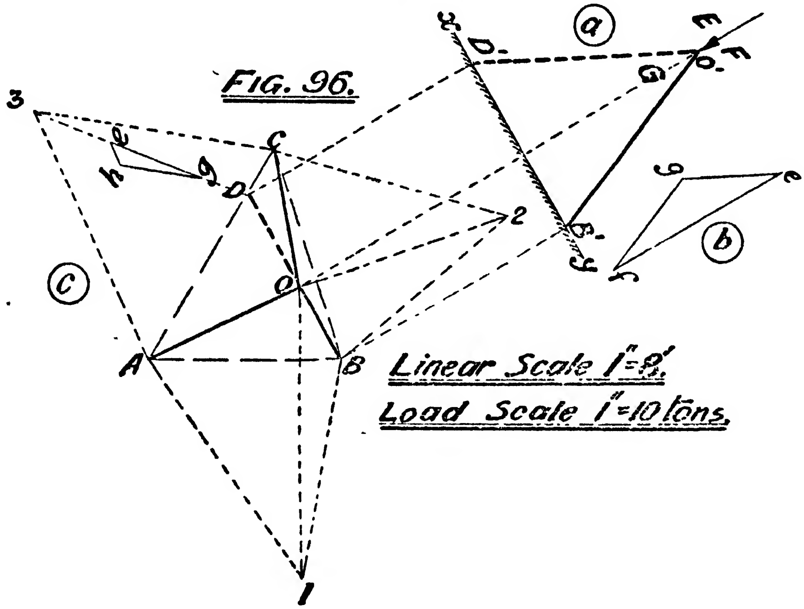
EXAMPLE 5:—A tripod is made up of poles AB, AC, AD, each 9′ long, their feet forming a triangle on horizontal ground such that BC=8′, CD=7′, BD=9′. Find graphically the forces which act down each leg of the tripod when a load of 10 tons is suspended from it. See fig. 95.

SOLUTION:—The legs are of equal lengths. Distance apart of each leg on the ground is given. First draw the plan, BC, CD and DB. To find the apex A, strike arcs of circles with a radius of 9′ from D and C and let them intersect at 1. Similarly strike arcs from B and C and let them intersect at 2. Now the point 1 of the triangle DC 1 if lifted, the path described by it will be at right angles to the base DC, similarly the path of point 2 also is at right angles to the base BC. Get these two paths intersected at A which is the apex of the tripod. Now join AB, AD, AC and this figure represents the plan of the tripod. In a line with any of these three legs, insert one imaginary leg in dotted line, say AE. Now AE is in a plane with the leg AC and as well as in a plane with AD and AB. Then take XY line parallel to the plane ACE and project an elevation of the legs AC and AE. See fig. (*b*). Now resolve the load 10 tons parallel to AC and AE as shown in fig (*c*). Here *g h* is the stress in the leg AC and *h f* is the stress in the imaginary leg AE. The stresses in the legs AB and AD are to be found as follows. See fig 95 (*a*). Strike arcs with a radius equal to the length of each leg AB and AD=9′ and let these two arcs intersect at *a*. Join

a E, then *a* E represents the imaginary leg. Plot *f h* of fig (c) which is the stres in the imaginary leg along the line *a* E or parallel to it and then resolve the same parallel to the other two legs *a* B and *a* D as shown ; *f k* and *h k* represent the stresses in the legs AD and AB respectively.

UNEQUAL LEGGED TRIPOD.

EXAMPLE 6:—The lengths of the legs of a tripod are OA = 11′, OB = 9′ and OC = 10′. The length AB = 8′, BC = 9′, CA = 10′. Find the stresses in these legs when carrying a load of 10 tons at O. See fig. 96.



SOLUTION:—OA, OB, OC are the lengths of the legs ; AB, BC, CA are the lengths of the base. Now to any convenient scale plot the base first as follows.—AB = 8′ and from B and A strike arcs with radii 9′ and 10′ respectively, the intersection point of these two arcs will give you the point C. Again from points A and B strike arcs to the lengths of the legs OA and OB, and let these two arcs intersect at 1. Join A1 and B1 in dotted lines ; A1 and B1 represent the lengths of the legs OA and OB. The path described by the point 1 when it is lifted towards the apex of the tripod will be at right angles to the base line AB. Similarly point 2 travels at right angles to the base line BC. Therefore from points 1 and 2 draw lines at right angles to AB and BC and the intersection of these two straight lines will give you the point O

the apex of the tripod. Join OA, OB and OC and this whole figure represents the plan of the tripod.

Now substitute as before an imaginary member OD in a line with OB. Then BOD is in one plane and OA, OC and OD also are in one plane. Therefore OD the substituted member is common to both the planes. Next take *x y* line parallel to BOD and project an elevation of OB and OD as shown in fig (*a*). In the elevation firm and dotted lines represent the true lengths of OB and OD respectively, as the vertical plane is parallel to OB and OD. Resolve the load 10 tons parallel to O′ B′ and O′ D′ as shown in fig (*b*), *f g* represents the stress in OB and *g e* is that of OD. To find the stress in the remaining legs OA—OC proceed as follows.—strike arcs with the radii equal to the lengths of OA—OC and let them intersect at 3. See fig. (*c*). Join A3, C3 and D3. Now plot the stress *g e* on the line D3 and resolve the same parallel to A3 and C3. Then *e h* and *g h* represent the stresses in OA and OC respectively.

DERRICKS.

Guy Derricks have vertical masts guyed with three or more guy lines and have booms which carry blocks and fall lines on the upper end. These booms are raised or lowered with rigging called "Topping Lines" or Boom lines. The load is raised by rigging called "Fall Lines" or "Falls". Guy Derricks may be swung in a full circle. Fig 97 shows the general elevation of a Guy Derrick.

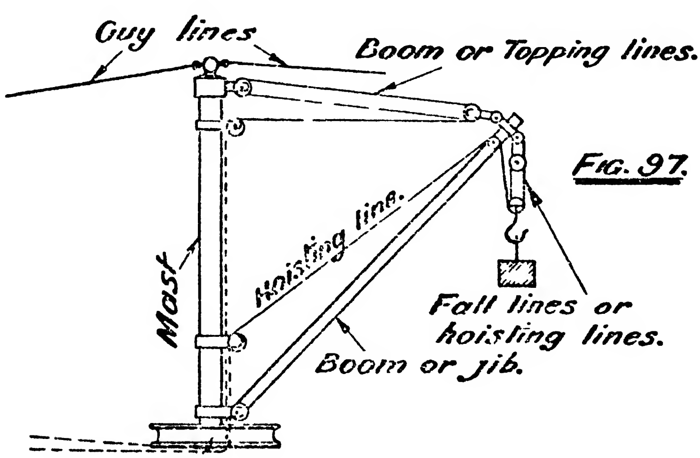


FIG. 97.

EXAMPLE 7:—The jib of a derrick is inclined at 30° to the vertical and the topping lift is attached to a point vertically over the

foot of the jib at a height equal to its length. Find the tension in the lift and the thrust in the jib when lifting 6 tons. There are three guys attached to the mast at 120° to one another and inclined at 15° to the horizontal. Find (*a*) the tension in the guys when the plane of the jib and topping lines are in the same plane with one of the guys, (*b*) when it makes an angle of 30° to one of the guys measured horizontally.

SOLUTION:—Fig. 98 shows the line diagram of the derrick. The triangle of forces *a b c* fig. 99 shows the stresses in jib and topping lift. (*a*) Since the derrick can be swung in a circle it can be stopped at any position. In the question it is stated that the derrick is to be in a plane with one of the guys. In this position the other two guys which are in different planes are only stressed and the one that is the third guy which is in the same plane with the derrick is not stressed for the reason that the guys being flexible do not take compressive stresses. Now the lengths of the guys are not given and are not necessary since the angle of inclination of each guy is given.

FIG. 98.

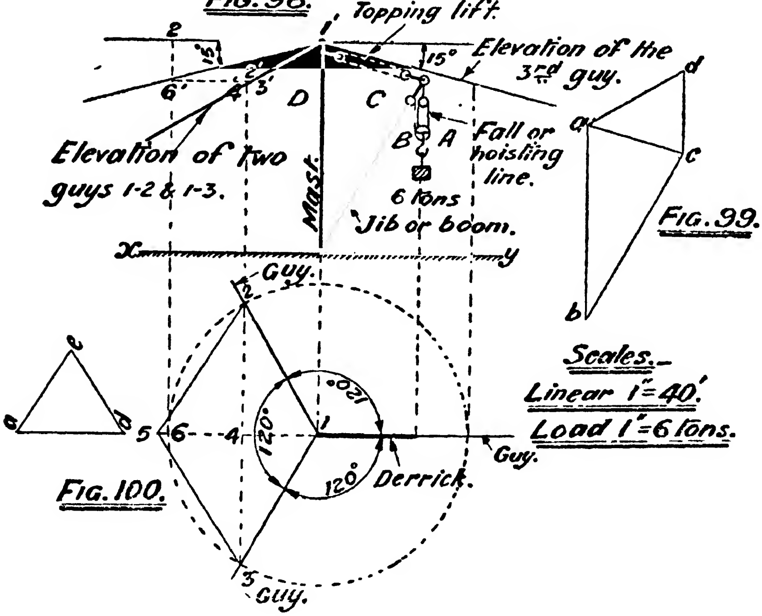


FIG. 99.

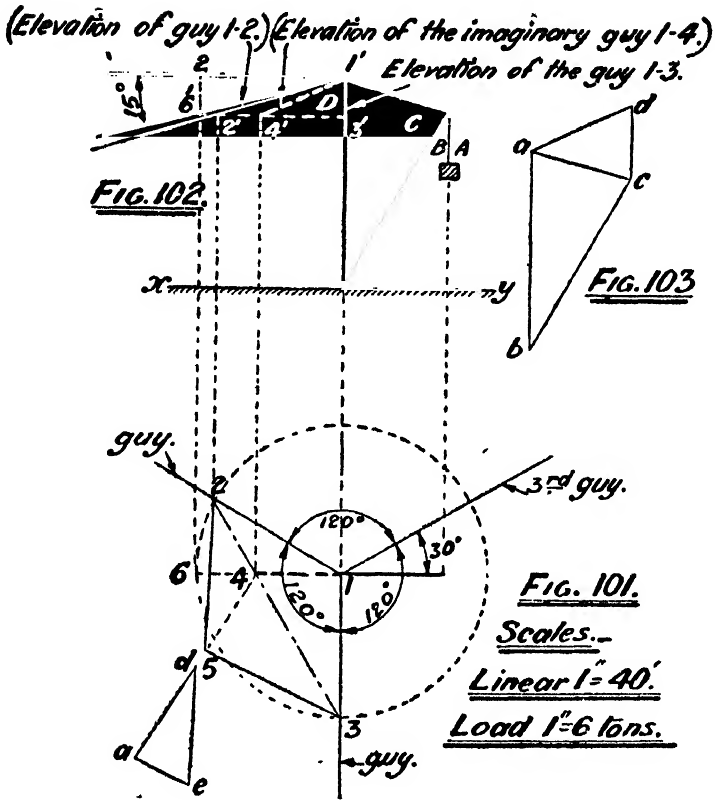
FIG. 100.

In this case you are to draw the plan first, see fig. 100. Take any equal lengths on the guys as 1—2, 1—3, and join 2—3 in dotted line. In a line with the derrick draw an imaginary guy 1—4. This imaginary guy is common to both the planes *viz.* it is in the same plane with the derrick and also in the same plane with guys 1—2 and

1—3. Next draw the elevation on $x\ y$ line taken parallel to the derrick and the imaginary guy, see fig. 98. The line 1′—2′ represents the elevation of three guys 1—2, 1—3 and 1—4. The line 1′—2′ gives the true length and inclination of the imaginary guy 1—4, as the vertical plane is parallel to 1—4 of fig. 100.

Method of getting the point 2′ in fig. 98.—(Draw from the top of the mast, one horizontal line and from the same point draw another line at an inclination of 15° as shown. This is the actual inclination of the guys. Since the guys 1—2 and 1—3 are not parallel to the $x\ y$ line, the line inclined at 15° to the horizontal does not represent the elevation of the two guys concerned. Take 1—2 in fig. 98 equal to 1—2 of fig. 100 and drop perpendicular to the inclined line at 6, and from 6 draw horizontal line till it intersects the dotted line at 2′ projected from 2 and 3 of fig. 100. Line 1′—6′ of fig. 98 is the actual length of the guys 1—2 and 1—3 of fig. 100. Join 1′—2′ and this line represents the elevation of three guys 1—2, 1—3 and 1—4; but it represents the actual length and inclination of the imaginary guy 1—4.) Now go back to fig. 99 and complete the stress diagram by circling round the top joint of the derrick as shown. $c\ d$ is the stress in the vertical mast and $d\ a$ is the stress in the imaginary guy 1′—4′. From 2 and 3 of fig. 100 strike arcs with a radius equal to 1′—6′ of fig. 98 and get them intersected at 5. Join 4—5. Plot $d\ a$ in a line with 4—5 and resolve parallel to 2—5 and 3—5. Then $a\ e$ and $d\ e$ represent the stresses in two guys 1—2 and 1—3 respectively.

(*b*) In this part of the question the method of working is the same as above. The plan fig. 101 is to be drawn first, the plane of the jib and topping lines is at 30° to the third guy. At this position of the derrick the third guy does not at all come into action and only the guys 1—2 and 1—3 are stressed. Here 2 and 3 are joined and the imaginary guy 1—4 is taken in a line with the jib and topping lines. In elevation see fig. 102 lines 1′—2′, 1′—3′ and 1′—4′ are the elevations of 1—2, 1—3 and 1—4 of fig. 101. Line 1′—6′ represents the actual lengths of guys 1—2 and 1—3, and the line 1′—4′ is the actual length of the imaginary guy 1—4. Fig. 103 represents the stress diagram and determines the stresses in the vertical mast, jib, topping lines and imaginary guy 1′—4′; the force triangle $a\ d\ e$ of fig. 100 determines the stresses in the guys 1—2 and 1—3.



STIFF LEG DERRICK.

The Stiff Leg derrick has one vertical mast braced by 2 triangular frames set at right angles to each other. This derrick has a free swing of about 240 degrees. The mast may be turned by hand or by means of a bull wheel operated by a line from the hoisting engine. Figures 104 and 105 represent the plan and elevation of a Stiff leg Derrick.

EXAMPLE 8:—A stiff leg derrick has a mast of 40 feet high and a jib or boom 50 feet long inclined at 30 degrees to the vertical. The horizontal legs of the triangular frames are 40 feet long. Find the stresses in all the members when a load of 12 tons is lifted and when the jib of the derrick is at an angle of 135 degrees with the triangular frames measured horizontally.

SOLUTION:—Figure 106 is the plan of the derrick and figure 107 is the elevation. Since the triangular frames 1—2 and 1—3 are in different planes, insert an imaginary back leg 1—4, in a line with the jib and topping lines, see figure 106. In figure 107 line 1′—2′—3′—4′

represents the elevation of three lines 1—2, 1—3 and 1—4 of fig. 106 but it gives you the real length and inclination of the imaginary leg 1—4. The real lengths and inclinations of the sloping members of the triangular frames 1—2 and 1—3 are represented by one single line $1'—x'$ in the elevation.

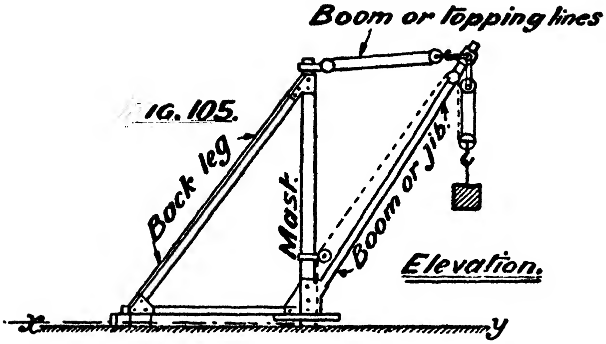


FIG. 105.

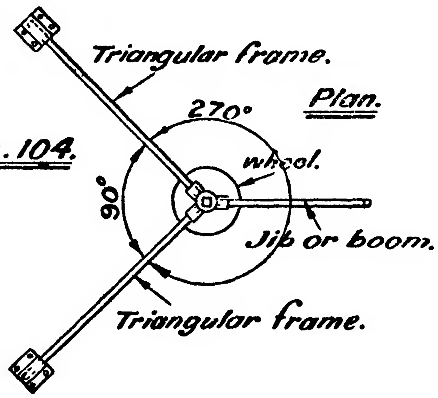


FIG. 104.

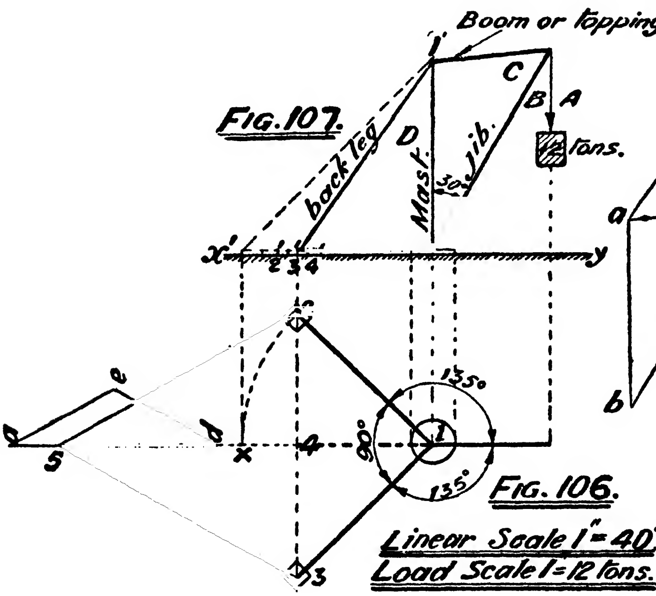


FIG. 107.

FIG. 106.

Linear Scale 1" = 40'.
Load Scale 1 = 12 tons.

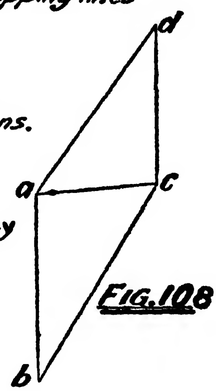


FIG. 108

Now work out the top joints of the jib and mast and draw the stress diagram as shown in fig. 108. Line *d a* is the stress in the imaginary back leg. From points 2 and 3 of fig. 106 draw 2—5 and 3—5 equal to the true lengths of the back legs and join 4—5. On 4—5 plot *d a* the stress in the imaginary leg and resolve the same parallel to 2—5 and 3—5. Then *d e* and *a e* represent the stresses in 3—5 and 2—5 and these in turn are equal to the inclined members 1—3 and 1—2 respectively. The horizontal components of these stresses are the stresses in the bottom horizontal members of the triangular frames.

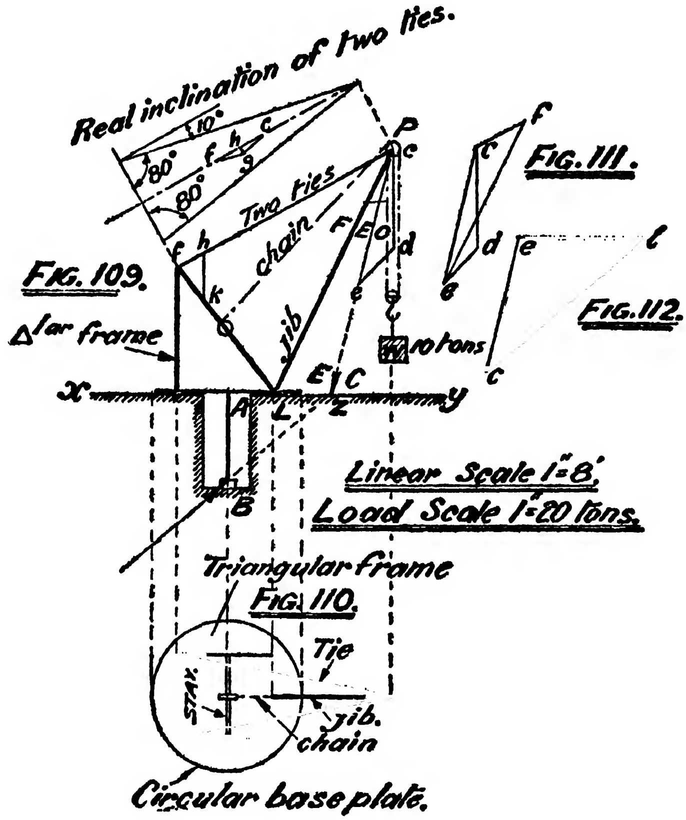
CRANES.

Cranes are generally used for lifting and shifting light and heavy machines etc. The following aro the usual types of cranes.—Wall Cranes, Jib Cranes, Foundry Cranes, Warf Cranes, and Travelling cranes and so forth according to the locality and purpose for which they are used.

EXAMPLE 9:—The elevation and plan of a Jib crane are represented in figures 109 and 110. It has two triangular frames, two ties and one jib. The angle of inclination of two lies is 80°. The foundation pin has gone right down into the ground and rests on a pivot B. The tension in the chain is by means of tackle, made half the weight. Determine the stresses in the frames, jib and tie rods, the horizontal stress at A and the magnitude of reaction at B when the load carried is 10 tons.

SOLUTION:—The effective load at P is the resultant of the load W = 10 tons and the tension in the chain = 5 tons. It is shown in fig. 109 where you can see *c d* = *w*, *d e* the tension in the chain and *c e* is the resultant. This resultant *c e* is resolved parallel to the jib and to the centre line of two ties in figure 111. Hence *f e* represents the stress in the jib and *f c* is the stress in the centre line of two tie rods. In fig. 109 the true angle of inclination of two tie rods is shown and *f c* the stress in the centre line is plotted and resolved parallel to the two tie rods; *f g* and *c g* give the stresses in two tie rods. These two tie rods are joined to two triangular frames and pull the frames at an inclination of 10° and therefore the stresses in the tie rods are to be resolved parallel to the frames; the components which are parallel to the frames are *f h* and *h c*. Hence each tie rod pulls the frame to the amount of *f h* or *h c*. Consequently *f h* is the pull at the apex of the triangular frame and is resolved parallel to the vertical and

inclined member of the frame. *f k* is the stress in the inclined member and *h k* is the stress in the vertical member.



The resultant load at the point P and the horizontal stress at A meet at Z and therefore the reaction at B must also meet at Z owing to the concurrence of three forces in equilibrium. The force triangle *c e l* fig. 112 determines the magnitudes of reactions.

Note:—The resultant load *e* c and two reactions (one at A and another at B) keep the whole crane in equilibrium, consequently these three must meet at a point as these are coplanar concurrent forces.

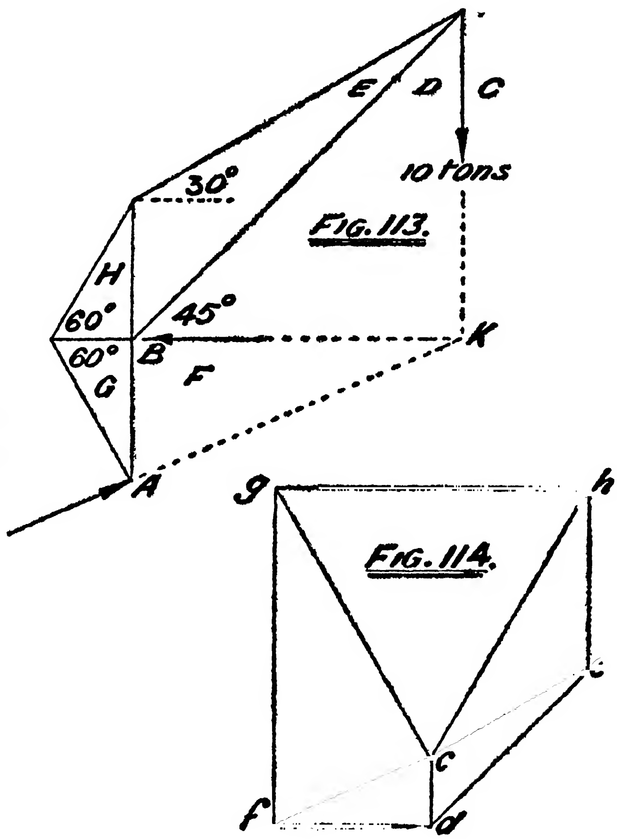
EXAMPLE 10:—Draw the stress diagram of the crane shown in figure 113 and distinguish between ties and struts. A is a footstep bearing and the reaction at B may be assumed horizontal.

(I. Sc. Eng. 1922.)

SOLUTION:—The load to be lifted is 10 tons. First letter the load and spaces as per Bow's Notation as shown. Since there is only

one load and two supporting points A and B, the forces acting at these two points and the given load must pass through a common point. The direction of reaction at B is known and the direction of the load *to be lifted is also known, these two if produced will meet at* K and the third force therefore acting at A must also pass through K.

Commencing from the top joint the stress diagram can easily be drawn as follows.—The triangle of forces at the top joint is c *d e* see fig. 114; at the junction of tie and the vertical member at 30° is *e h c,* at the third joint at 60° inclination *h g c,* at the point B the force polygon is *h e d f g h* and the force triangle at K is *f d c.* To distinguish between ties and struts the student should designate the joints of the members of the frame diagram by means of arrow heads. See roof truss chapter Part I.



EXAMPLE 11:—The plan and elevation of a hand wharf crane are shown in figures 116 and 115. The crane consists of two ties, two triangular frames and two jibs braced together. The whole crane swings through a complete circle. The weight to be lifted is 10 tons and the stress at P is assumed to be horizontal. Determine the stresses on all the members and reaction at Q.

SOLUTION:—The working method is same as shown in example 9. The true inclinations of ties and jibs are shown in figures 117 and 118 respectively. The given load W=10 tons and the tension in the

chain $=\frac{W}{2}=5$ tons are combined for the resultant at the head of the jib, see fig. 115 and this resultant *a c* is resolved parallel to the centre lines of jibs and ties in figure. 119.

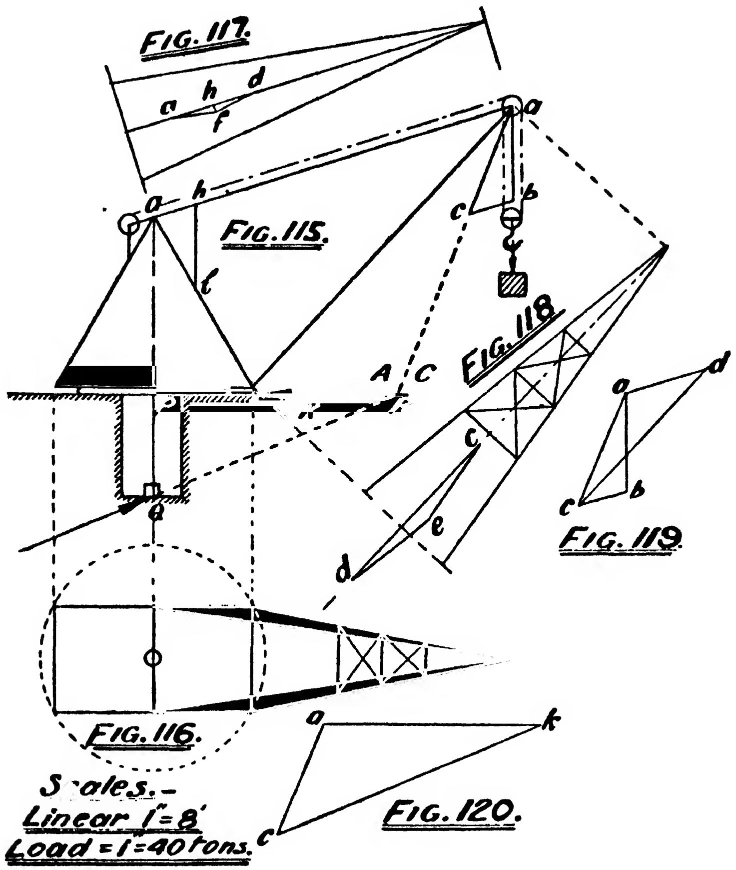


FIG. 117. FIG. 115. FIG. 118. FIG. 119. FIG. 116. FIG. 120.

Then *a d* and *c d* represent the stresses in the centre lines of ties and jibs respectively. Consequently the stress *a d* is plotted on the centre line of ties in fig. 117 and resolved parallel to two ties. Similarly the stress *d c* is plotted on the centre line of jibs in figure 118 and resolved parallel to two jibs. Now the two triangular frames are parallel to the vertical plane and the two ties which incline to both the horizontal and vertical planes of projections are attached to top apexes of these two triangles and hence the stresses determined in the ties are to be resolved parallel to the plane of triangles. Therefore *a h* and *h d* the components of *a f* and *f d* (see figure 117) are parallel to the plane of the triangles. The force *a h* is taken in the

same plane of the triangle in fig. 115 and resolved parallel to both the inclined members of the triangular frame and thus their stresses are determined.

The resultant load AC and the reaction at P intersect at R and therefore the reaction at Q must pass through the point R. Reactions at P and Q are determined in the triangle of forces in figure 120.

EXAMPLE 12:—The travelling jib crane is shown in figure 121. It carries a maximum load of 10 tons, and revolves round, on ball bearing centre pin at X. Determine the stresses in all the members and find the counterbalance weight required at Y to maintain equilibrium.

SOLUTION:—The frame of this crane is in one plane. Generally the jib cranes of heavy types are built in different planes and in such cases the working method is, as shown in example 11. The load is 10 tons and the tension in the chain is 5 tons as usual. The effective load at P is the resultant of the loads 10 and 5 tons respectively.

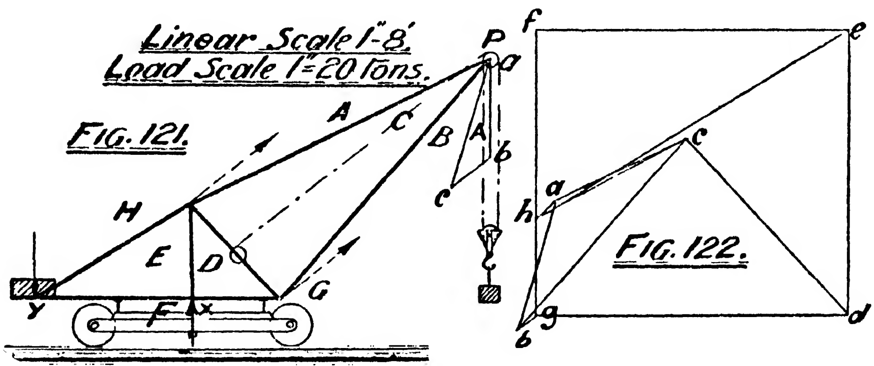


FIG. 121.

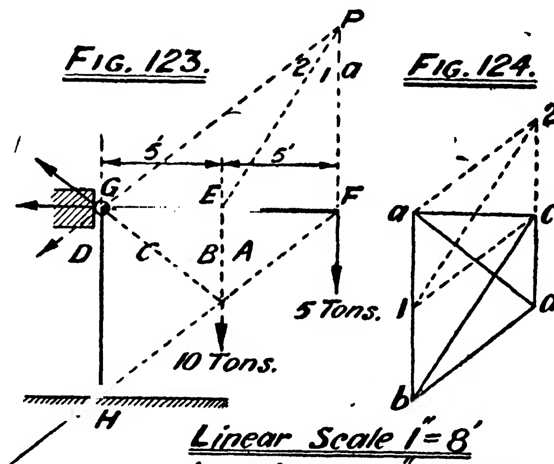
FIG. 122.

The tension of 5 tons in the chain, which is wound round the drum at the centre of the member CD, produces additional two loads of 2·5 tons at each end of the same member. Unless these two loads are plotted in the load line in figure 122, the stress diagram does not close. Hence this is a clear proof that there exist two loads which are the parallel components of 5 tons tension in the chain. By going round each joint successively the stress diagram can be drawn without difficulty as shown in figure 122. Working method is as follows—The resultant *a c* of the two loads *a b* and *b c* is named as per Bow's Notation as AB and this load AB is plotted in fig. 122 as ***a b***, and circling round the point P of the crane the force triangle ***a b c a*** is obtained in fig. 122. Then the bottom joint of the membr BC is

to be solved. Here the stress in CB is already determined and the magnitude of the force BG = 2·5 tons, therefore $b\ g$ is drawn in fig. 122 = 2·5 tons, then $g\ d$ and $d\ c$ are drawn parallel to GD and DC of the frame diagram fig. 121. Now coming to the bottom joint of the member CA, we know the force HA = 2·5 tons and taking $h\ a$ = 2·5 tons in fig. 122, we can draw the force polygon $h\ a\ c\ d\ e\ h$ in fig. 122 very easily. Next the bottom joint of the member ED, that is, at the centre pin X, the polygon $d\ g\ f\ e\ d$ is to be drawn in the order.

Lastly the counterbalance weight at Y is to be determined by circling round the joint at Y. The force triangle $h\ e\ f\ h$ is to be drawn and from this the counterbalance weight $f\ h$ may be measured to the scale of 1″ = 20 tons and determined.

EXAMPLE 13:— Fig. 123 represents a forge crane. Draw the stress diagram for this frame work and determine the directions and magnitudes of reactions at the supports for a load of 5 tons suspended at the free end of top horizontal member.



Linear Scale 1″ = 8′
Load „ 1″ = 10 Tons.

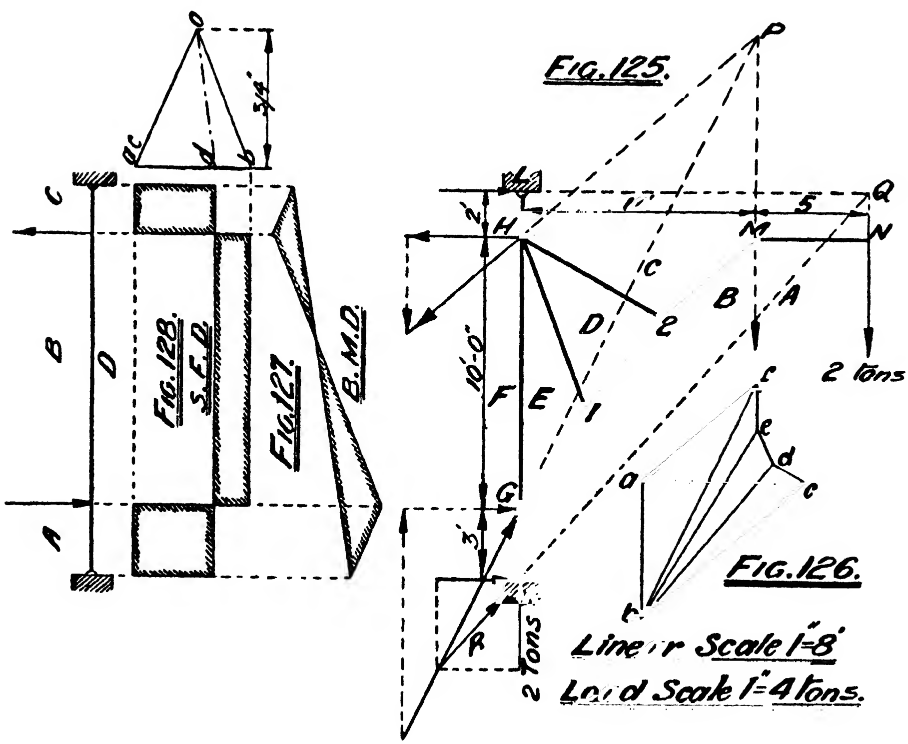
SOLUTION:—The portion EF of the top horizontal member is not braced and hence it is a cantilever fixed at E and free at F. This cantilever is subjected to bending and shear stresses *viz.* 5 × 5 = 25 tons feet (B. M. maximum at E) and 5 tons shear. The only possible and quicker method of solving such sort of problem like this, is to determine the actual vertical load at joint E, and then the triangle of forces can be drawn.

Now GF is a straight bar hinged at E and G and this may be taken as a lever with its fulcrum at E. Selecting E as moment centre we have 5 tons × 5′ = the tension in the member CD × 5. Calling T = the tension in the member CD we have $T = \frac{25}{5} = 5$ tons. The actual load at

E is therefore 5+5=10 tons. The reason is obvious as the point E is to resist the tension in the member CD which is acting at G and the pull of 5 tons at F. The stress diagram can easily be drawn as shown in the figure 124. For the joint at E the force triangle is *a b c*. Then at the joint G the stress in the member AC is known and the stress in the member CD=5 tons as determined above and the third force DA is the reaction at G. Laslty the joint H is named as DC, CB and BD and the stresses are as shown in figure. 124.

On refering to fig. 124 we see that the two reactions at G and H and the load at E all meet at one point. The dotted lines in fig. 124 show the graphical solution of this example.

EXAMPLE 14:—A more difficult sum on wall crane similar to the crane worked out above is represented in figure 125. Determine the stresses on all the members, the thrust at G and K and pull at H and L. Determine also the bending moment and shearing force in the vertical member LK. Load 2 tons. HMN is a continuous member



SOLUTION:—The equivalent load at M for a load of 2 tons at the free end is as follows.—Tension in the member $EF \times 10 = 2 \times 5$ (M as moment centre) $T = \frac{10}{10} = 1$ ton. The load at $M = 2 + 1 = 3$ tons. Now the stress diagram can be drawn by going round each joint successively as shown in figure 126. While circling round joints G and H, we know the stress in the member $EF = 1$ ton and plotting this to the scale in the stress diagram the reactions at G and H can be determined as shown. The horizontal component of reaction at G is the thrust at G and similarly the horizontal component of reaction at H is the pull at H. The horizontal components of these two reactions are equal (see figs: 125 and 126). Bending moment and shearing force diagrams are drawn in figures 127 and 128. Here you will observe the thrust at K is equal to the pull at L. Two reactions and the load at M meet at P. Also the thrust at K and the vertical reaction which is equal to the load lifted, when combined for their resultant and produced will intersect the external load line at Q The pull at L which is equal to the thrust at K if produced will also meet at Q. (See fig. 125.) This fact shows that our assumptions and method of working are correct.

Note:—HMN is one member. While calculating the moments about the fulcrum at M the tension in the member EF only is taken, but there may be some doubts to the students that members CD and DE also may help to keep the bar HMN in position and their stresses also to be taken in calculating the moments. The introduction of the members CD and DE is only to keep the points 1 and 2 in position. The vertical member EF plays an important part in resisting the tension caused by the lever HN. Suppose the member EF is cut off, the joints G and H will no longer be in position and the effect would be instantaneous. We may remove the members CD and DE and the result is, that the joints 1 and 2 may shoot in gradually or may not without causing any appreciable change in the joints H and G; because member G—1—2—M becomes one curved member and supports the horizontal member HMN. Hence the tension in the member EF is taken in calculating the moments about M.

EXAMPLE 15:—A foundry crane is arranged as in figure 129. Assuming that AE is a continuous member, and that A, B, C and D are frictionless pin-joints and that the pulleys are frictionless determine:—

(*a*) The bending moment resisted by the member AE at D (*b*) The direct thrust, or tension, in the members AD, AC, CD and CB.

(C. E, sub: 1925)

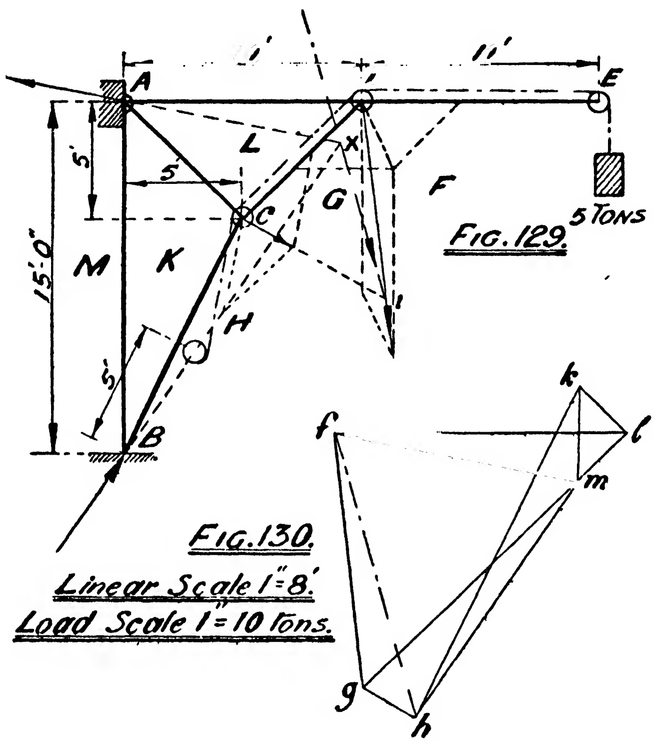


FIG. 129.

FIG. 130.

Linear Scale 1"=8'.

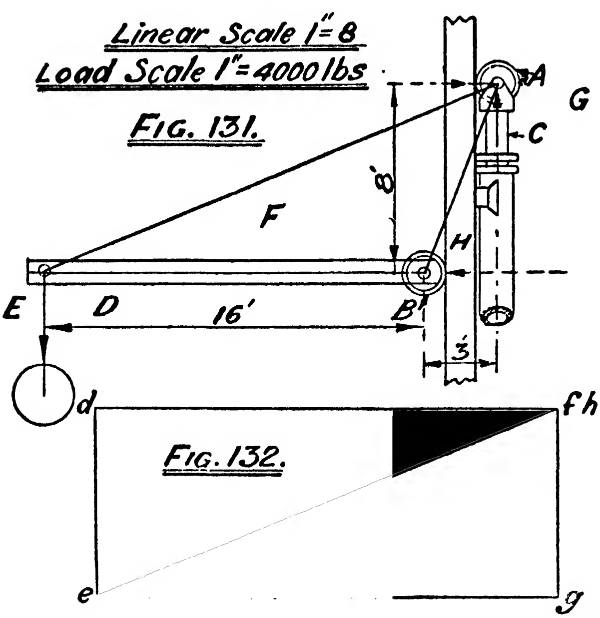
Load Scale 1"=10 Tons.

SOLUTION:— This is similar to the example 14. ADE is a continuous member and we may take this as a lever with a fulcrum at D. The equivalent load at D for a load of 5 tons at E is equal to 10 tons. (D as moment centre. Tension T in the member MK $\times$ 10 $=$ 5 $\times$ 10, $\therefore$ T $= \frac{50}{10} =$ 5 tons) The joint at D is to resist 5 tons at E plus 5 tons tension in the member MK $=$ 10 tons. In addition to this, the joint D is to resist the resultant load of 5 tons tensions on two portions of the chain that go round the pulley at D. The effective load at D is shown by the diagonal of the parallelogram of forces shown in firm line. The load at C is equal to the resultant of 5 tons tension on two portions of the same chain that go round the pulley at C and this resultant is shown in firm line at C. These loads are named as per Bow's Notation. The polygon for the joint D is *f g e f* see fig. 130 and the polygon for the joint C is *g h k l*. Determination of reactions at A and B is as follows.— The tension in the member KM$=$5 tons. From the point K in the figure 130 plot KM$=$5 tons and thus the point *m* is fixed in the stress diagram. The reactions at A and B are named as MF and HM respectively. Therefore in figure 130 join points *f* and *h* to *m*, then *m f* and *h m* give you the magnitudes and directions of reactions at

A and B respectively. When these directions of reactions are produced from points A and B, they meet at X and the resultant load of FG and GH also meets at X. The direction of the resultant load of *f g* and *g h* is shown in figure 130 in dotted line (*f h*). In figure 129 the load GH if produced intersect the load FG at 1 and from the point 1 if we draw a line parallel to the line of action of the resultant *f h* of figure 130 we see it exactly meets at X of fig. 129.

This is a clear proof to show that all our assumptions and method of procedure are correct. The bending moment resisted by the member AE at D is equal to $5 \times 10 = 50$ tons feet. See the note of example 14.

EXAMPLE 16:—A crane (figure 131) consisting of a triangular frame with rollers at A and B is lifted bodily with its load of 4000 lbs by a hydraulic ram, C. Find the stresses in the members of the frame. (City & Guilds. Exam: in Mech. Eng:).



SOLUTION:—The crane has two rollers A and B, the reactions must therefore be at right angles to the surface in contact. The hydraulic ram C lifts the whole frame and of course the direction of the push should be upwards. These are the preliminary reasonings; and the arrow heads are to be marked as shown in figure 131. Now the stress diagram can be drawn as usual commencing from the joint where the load is suspended. The force triangle for this joint is *d e f* see figure 132. Next the joint at the roller B is to be attempted, here the stress in DF is known and the stress in FH and the reaction HD are to be determined. The letter *h* is to coincide with *f* as the reaction at B is named *h d*. Hence the stress in the member FH is zero. This is quite clear as the roller B is

kept in equilibrium by two forces $d\,f$ and $h\,d$ which are equal and opposite and the third force is not necessary. Lastly the joint at the roller A is to be solved, the force polygon is $f\,e\,g\,h$ as shown in the figure 132. The member FH is introduced here, only to prevent the roller shooting up or down the pllar.

EXAMPLE 17:—Figure 133 shows a wall crane carrying a load of 10 tons. Find the reactions at the points of supports and draw the force diagram for all the members of the structure. Give the nature and magnitude of the force in each member.

SOLUTION:—Letter the given load and the members of the frame as per Bow's Notation. Start the stress diagram from the free end, $a\ b\ c$ is the force triangle for this joint. In the next joint the member CD has no stress as the letter d coincides with c. The polygon for the next joint is $c\ d\ e\ a\ c$. Similarly proceed on, till you fix the letter g in the stress diagram. Then circle round the joints at the supports; the forces acting at the bottom support are $g\,f$, $f\,b$ and $b\,g$; out of these you know the forces $g\,f$, $f\,b$ and join $b\,g$. Then $b\,g$ represents the magnitude and direction of the reaction at the bottom support. Similarly $g\,a$ represents the stress in the member GA and the magnitude and direction of the reaction at the top support. The external load AB and two reactions BG and GA meet at a common point on the load line. (See the frame diagram fig. 133.) To find the nature of the stresses in the members, read the corresponding stress in each member in the stress diagram and put the arrow heads as shown. When the arrow head points towards the joint that member is in compression and when the arrow head points away from the joint that member is in tension. Read chapter III page 41, Part I.

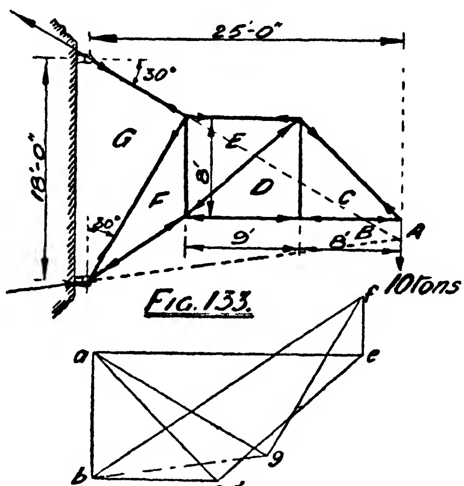


FIG. 133.

Linear Scale 1"=15'.
Load Scale 1"=15 Tons.

EXAMPLE 18:—The figure 134 shows an elevation of a jib crane with the jib swung back against the guys. If the load on the crane

in this position is 4 tons, find the load carried by the member FG and the forces in all the members

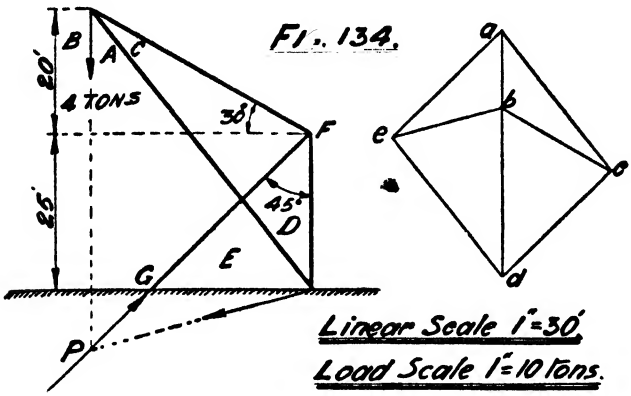


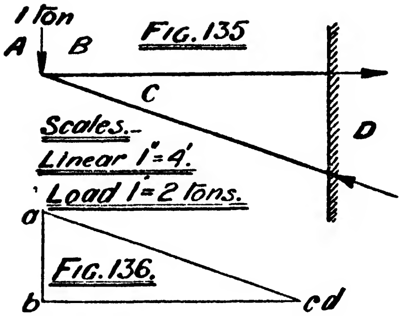
Fig. 134.

Linear Scale 1"=30'
Load Scale 1"=10 Tons.

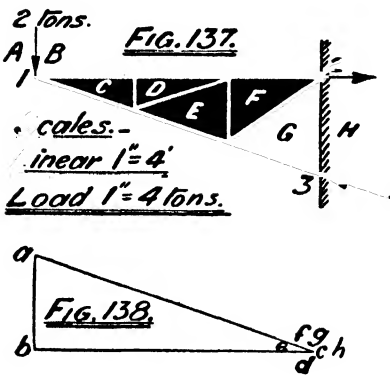
SOLUTION:—Letter the load and all the members, and draw the stress diagram commencing from the top joint of the crane. There are two supporting points one on the right and another on the left, and the supporting forces are named clockwise as BE and EA, respectively. After fixing the letter *e* in the stress diagram get the magnitude and direction of each reaction as shown. Two supporting forces and the external load meet at a common point P.

Chapter VII.

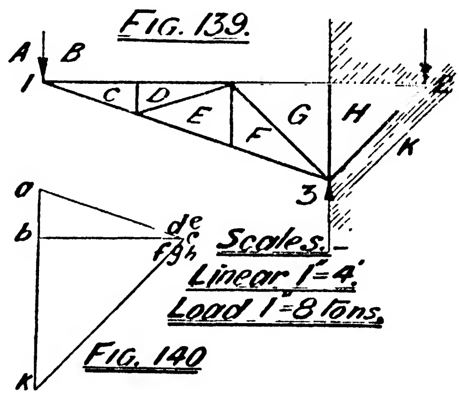
BRACED CANTILEVERS.



In braced cantilevers the determination of reactions is very instructive The cantilever fig. 135 is secured to the wall by means of bolts at the top joint and the reaction is assumed to be horizontal. Figure 136 shows the stress diagram showing the magnitudes and directions of reactions. One load and two reactions meet at a point.



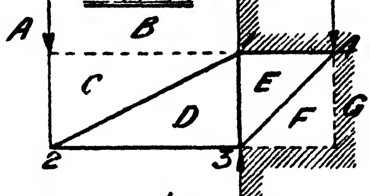
In figure 137 the same cantilever of fig. 135 is taken and braced inside as shown. Figure 138 is the stress diagram. In this diagram the internal braced members are not stressed. The reason is as follows —The triangle 1—2—3 is rigid and does not alter its shape. When this is loaded at 1 the embracing members 1—2 and 1—3 only are stressed and 2—3 also is not stressed as the reactions are in lines with these two members. These two members are therefore held in position without stressing the internally braced members.



The cantilever shown in figure 139 is anchored inside the wall. Figure 140 shows the stress diagram and magnitudes of reactions. Even in this cantilever no internal members are stressed for the reason that 1—2—3 is a rigid triangle and there are loads on three apexes of this triangle. Only the embracing members 1—2, 2—3 and 3—1 are stressed.

Now observe the frame diagram fig. 141 and the corresponding stress diagram fig. 142, you see that there is no stress in the member BC of fig. 141, bceause the load is directly taken by the vertical member CA and transmits the same to the joint 2. Similarly the reaction R acting at 4 is borne by the triangular frame 1—3—4 and consequently the dotted members inside the wall are not stressed.

FIG. 141.



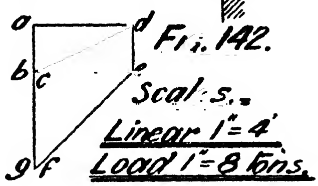
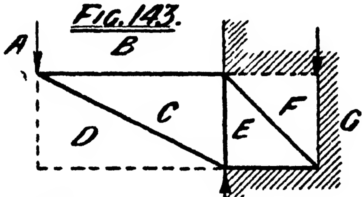


FIG. 142.

FIG. 143.



If you were to solve such sort of cantilever as shown in figure 143 you can quitely omit the members shown dotted in drawing the stress diagram for the reasons explained before. Figure 144 shows the stress diagram,

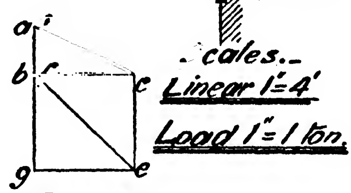


FIG. 144.

A little bit puzzling braced cantilever is shown in figure 145 and its stress diagram is shown in figure 146. There is no difficulty till we get the letter *e* in the stress diagram. There are two balancing couple AB, HA and BG, GH. Now the force *b g* must be in a line with *b e* and *h a* must be in a line with *a b* and equal to it. Consequently the letter *h* must coincide with *b*. By similar reasonings the letters *f* and *g* must coincide with *e*.

FIG. 145.

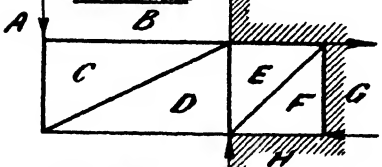
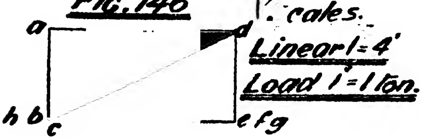
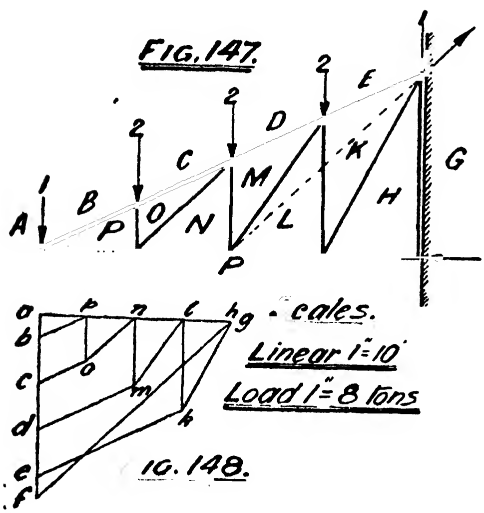


FIG. 146.



EXAMPLE 1:—A braced cantilever 20 feet span with vertical loads as shown in figure 147 is given. Draw the stress diagram and determine the directions and magnitudes of reactions.



SOLUTION:—The bottom horizontal member is in compression and therefore this member is divided into four equal parts. Care must be taken to reduce the lengths of compression members and keep them as short as possible or the truss will be very heavy and costly too. Stress diagram presents no difficulty as far as the supports. It is quite possible to assume the direction of reaction at bottom of the cantilever as horizontal. Consequently the letter g coincides with h, and the top reaction is named as FG. Therefore in the stress diagram fig. 148 the point f to be joined with h. Then $f\,g$ is the direction and magnitude of the top reaction. The magnitude of the bottom reaction is $g\,a$.

Note:—(*a*) This cantilever is in equilibrium by three forces *v i z.* the resultant of the external loads and two reactions and all must meet at one point. Consequently these three forces meet at P if produced.

(*b*) All the compression members are shorter than the tension members, and hence the design of the cantilever is good.

EXAMPLE 2:—Figure 149 shows the same diagram of example 1 loaded with the dead and wind loads and it is required to draw the stress diagram for the same.

SOLUTION:—Both the dead and wind loads are combined for their resultants by the parallelogram of forces. The loads are symmetrical and parallel to one another. The line of action of the resultant of this system of loads must pass through the mid point of the sloping rafter and the direction of reaction at the bottom fixed point of the cantilever is assumed to be horizontal. These two forces meet at P and the second reaction at the top end of the cantilever must

also pass through the point P as shown in the frame diagram 149. Figure 150 shows the stress diagram.

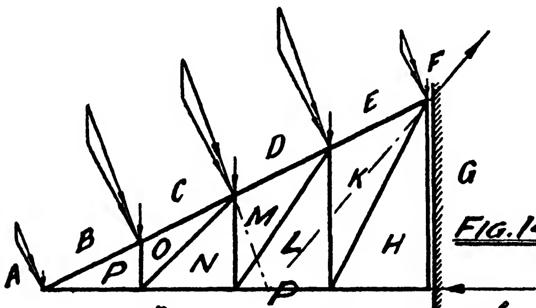


FIG. 149.

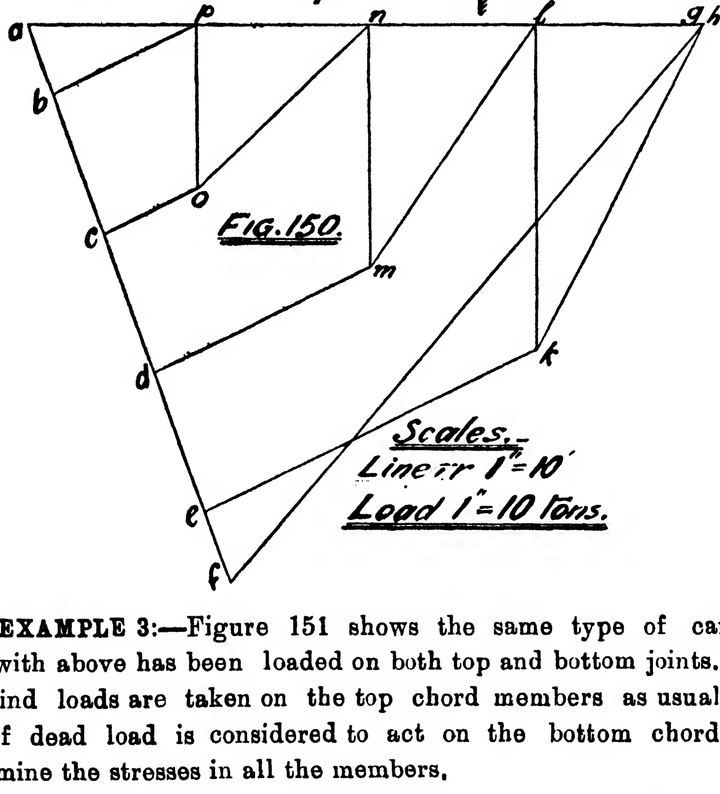


FIG. 150.

EXAMPLE 3:—Figure 151 shows the same type of cantilever dealt with above has been loaded on both top and bottom joints. Dead and wind loads are taken on the top chord members as usual and a part of dead load is considered to act on the bottom chord joints. Determine the stresses in all the members.

SOLUTION:—Plot the loads and with the help of polar diagram and the corresponding funicular polygon get the line of action of the resultant of this system of forces as shown. The horizontal reaction and this resultant intersect at x and the third force which is the reaction at the top fixed end of the cantilever must also pass through x. Figure 152 is the stress diagram.

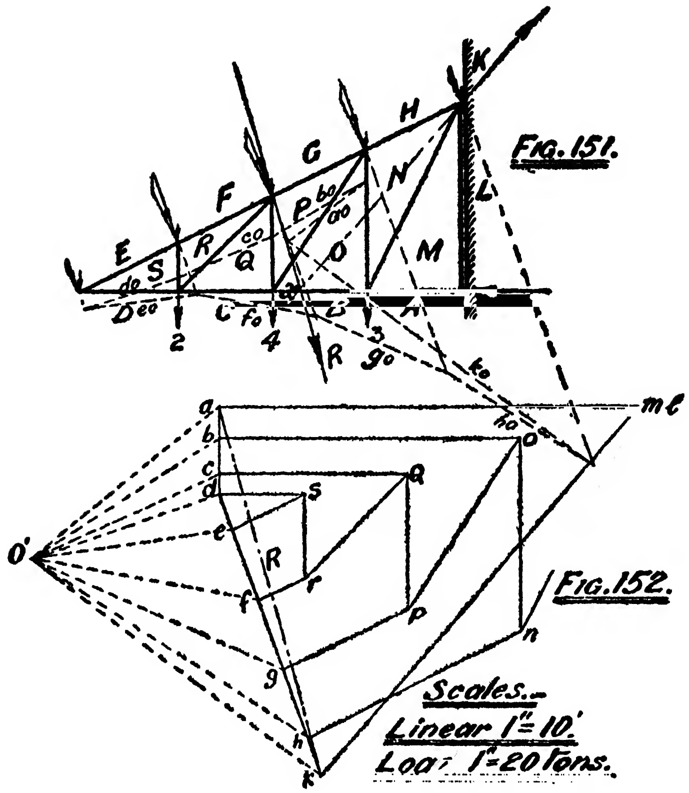


FIG. 151.

FIG. 152.

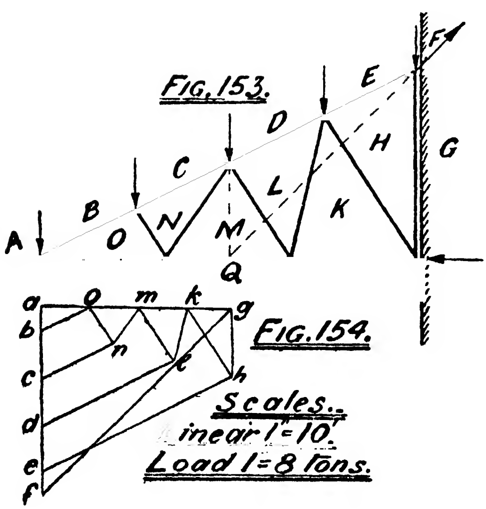
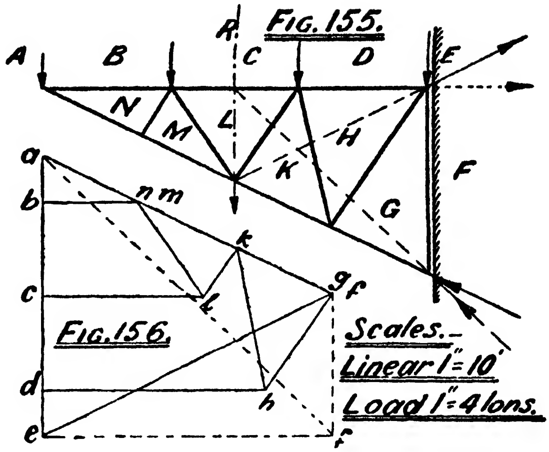


FIG. 153.

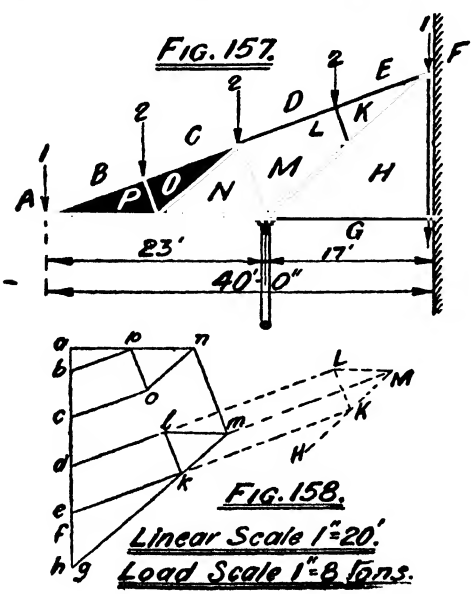
FIG. 154.

EXAMPLE 4:—Figures 153 and 154 show the frame and stress diagrams of a braced cantilever, where you can observe that the longer members are in compression and the top chord member which is in tension is divided into four equal parts and the bottom horizontal compression member is divided into three which is quite contrary to the elementary rules of structure and hence this is not a good design.

EXAMPLE 5:—The braced cantilever shown in figure 155 is not much exposed to wind pressure as the roof covering be on top horizontal member and it may be subjected to some shifting effect of wind pressure from underneath. Figure 156 is the stress diagram.



Note:—If we assume the direction of reaction at the top joint to be in a line with the top horizontal member, the direction of reaction at bottom joint will be inclined or if we assume the reaction to be in a line with the bottom inclined member, the top reaction will be inclined, but whatever may be the case, the two reactions and the resultant of the system of given loads must meet at a common point. These two assumptions above referred to, are worked out in figures 155 and 156. (To assume the direction of reaction in a line with the chord member which is not loaded is reasonable.)



EXAMPLE 6:—The braced and supported cantilever represented in figure 157 is a half French roof truss. This may be effectively used in railway platforms. The overhang is 23 feet and the space between the supports is 17 feet and hence the reaction at the wall acts downwards and the reaction on the column acts upwards In drawing stress diagram the progress is stopped at the fourth joint from the free end, where we meet three unknowns and this can be solved either by substi-

tution or by construction methods. (See Part I fig. 89, for the detail *solution of the French roof truss.*) *Fig 158 shows the stress diagram* and the construction method is used as shown.

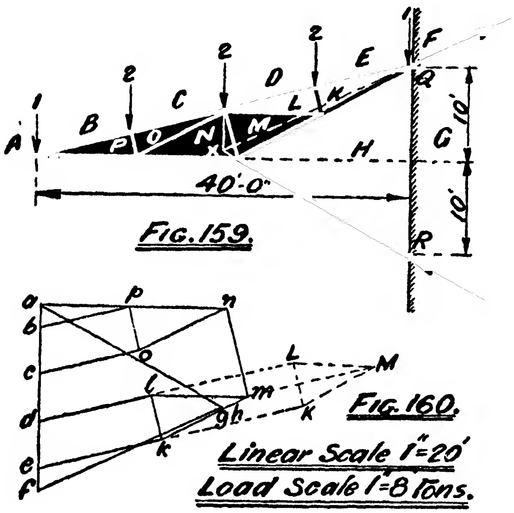


FIG. 159.

FIG. 160.

Linear Scale 1"=20'

Load Scale 1"=8 Tons.

EXAMPLE 7:—Figure 159 represents a braced cantilever similar to the figure shown in example 6 without pillar. The stsess diagram may be commenced from the free end of the cantilever. Here in this truss, directions of reactions need not be assumed and by working the fixed joints at Q and R as usual the directions and magnitudes will be determined in the stress diagram as shown. Here member MH is in compression and KH in tension although these two members are in one and the same line.

Note:—In all these cantilevers directions of reactions need not be assumed first, but they are determined in their corresponding stress diagrams.

Chapter VIII.

TRANSVERSE BENTS OR TRUSSES WITH KNEE BRACINGS.

Transverse Bents are roof trusses supported on columns and are provided with knee braces to prevent them against longitudinal movements. These kinds of trusses are generally used in steel mill buildings and are spaced from 16 to 32 feet apart. Since the lateral movements are not allowed for, the shoes of the trusses on both the ends are rigidly connected to the ends of the columns and hence the stresses in these transverse bents are statically indeterminate for wind and inclined loads. There are certain assumptions to be made on transverse bents when they are subjected to the wind and inclined loads and are dealt with in the following exam ples.

EXAMPLE 1:—A transverse bent of 40′ span, pitch of roof $\frac{1}{4}$ of the span, height of posts 20′, is represented in figure 161. Draw the dead load stress diagram. Loads are as shown.

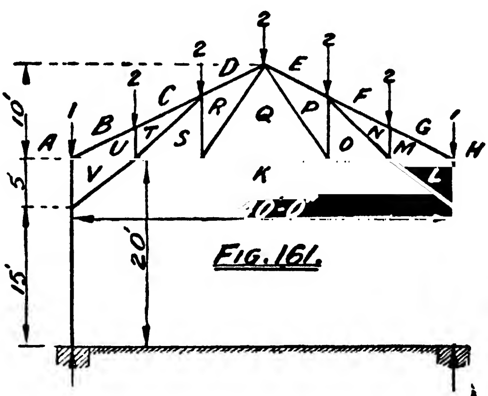
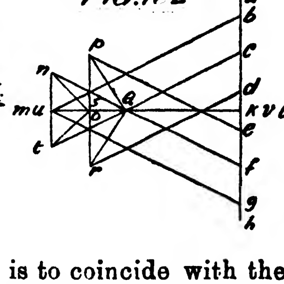


FIG. 161.

SOLUTION:—Plot the given loads to a suitable, scale, the reaction on each column is equal to $\frac{W}{2}$ where W = total load. Since the loads are vertical there will not be any stress on the knee braces. Commence to draw the stress diagram from the foot of the knee brace. This joint is named as AVK. We know KA which is the reaction on the column and it has been plotted on the load line, and the stress on the member AV is equal to the left reaction as the member AV is part of a column itself, hence the letter v is to coincide with the letter K. The knee brace is named as VK, consequently this member

Scales.-
Linear 1″ = 20′.
Load 1″ = 10 Tons.

FIG. 162



VK has no stress. Then go to the next joint on the top of the column and this joint is named AB, BU, UV, VA, clockwise, out of these AB is plotted already and from *b* draw a line parallel to BU and from *v* draw a line *u v* parallel to UV, both will intersect at *u* in the stress diagram. In the same way proceed joint by joint successively and there will be no difficulty in closing the figure. The stress diagram will be as shown in figure 162.

EXAMPLE 2:—In this example the same transverse bent is subjected to wind pressure. Columns are assumed to be hinged at bottom. Determine the stresses in all the members graphically.

SOLUTION:—Wind pressure is acting on the side and as well as on the slope of the roof truss normally, in this case the column is subjected to direct stress and as well as bending moment. Unless we truss these columns the stresses on these cannot be statically determined. The usual assumption in these cases is that the horizontal components H and H_1 of the reactions at the feet of the columns are equal to each other and each is equal to $\frac{W}{2}$ where W = the horizontal component of the resultant of the external loads as you will observe presently in fig. 164.

Now get the line of action of the resultant of the wind pressure as follows:—Plot these loads from AB to GH as shown in small letters *a* to *h* in fig. 164. The line of action of the resultant of the loads acting on the rafter of the roof truss is exactly at the centre of that rafter and this is represented by chain dotted line in fig. 163. The line of action of the resultant of the loads acting on the side of the column, determine it by the polar diagram and the corresponding link polygon as shown in the same figure. Let these two resultants meet at Q. The final resultant is *a h*, and this gives you the direction as well as its magnitude. Through the point Q draw this final resultant parallel to *a h* of fig. 164.

Then you are to determine the vertical components of the reactions at the feet of the columns. To find V take the foot of the right column as moment centre, then $R \times d' = V \times 40$ $\therefore$ $V = \frac{R \times d'}{40}$. In this example R = 10·25 tons; $d' = 2{\cdot}5'$. $V = \frac{10{\cdot}25 \times 2{\cdot}5}{40} = {\cdot}64$ tons. Again $V' = \frac{R \times d}{40} = \frac{10{\cdot}25 \times 18}{40} = 4{\cdot}61$ tons. $H = H_1$ as assumed above. Plot

these values of VH and V' H' at the feet of the columns and get the magnitudes and directions of reactions as shown in fig. 163. These two reactions and the final resultant of the external loads meet at point P'. This is a clear proof to show that our assumptions are correct.

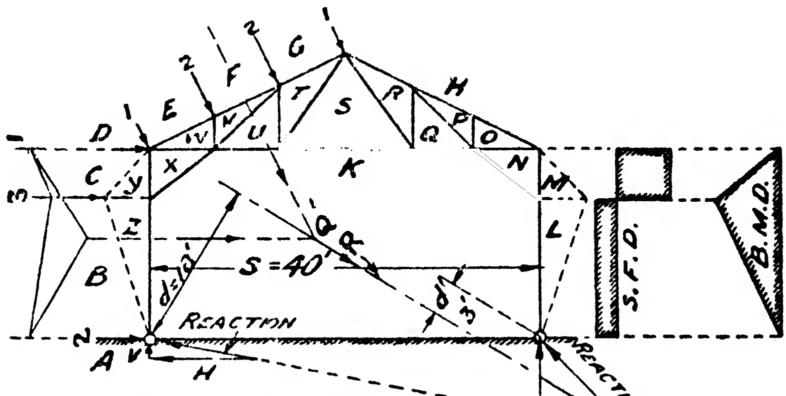


FIG. 163.
Linear Scale 1" = 20'
Load Scale 1" = 8 TONS

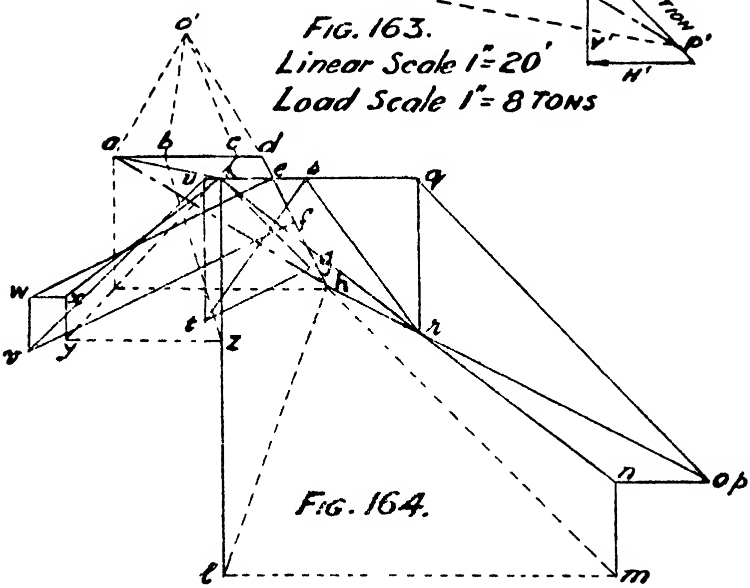


FIG. 164.

Stress diagram.—See fig. 164. Resolve the final resultant *a h* parallel to two reactions HK and KA of fig. 163. Now commence from the foot of the left column. The forces acting at this point are KA, AB, BZ and ZK out of these we know KA, AB and determine BZ and ZK by drawing parallels from points *b* and *k* of fig. 164. The next joint is the apex of the imaginary truss, the polygon

for this is $b\ c\ y\ z$. These are represented in dotted in fig. 164. The third joint is at the foot of the knee brace the polygon for this is $z\ y\ x\ k$. The next joint, that is, the fourth is at the shoe of the main truss, here there are six forces acting ; out of these we know four forces already *viz.* XY, YC, CD, DE and only EW and WX are to be determined. These you can determine by drawing lines parallel to these in figure 164. Similarly proceed on, and complete the stress diagram as shown in figure 164.

Tho maximum shear in the leeward column below the knee brace is $H_1 = 4{\cdot}4$ tons, above the knee brace is equal to the vertical component of the stress of the member $KN - H_1 = 16{\cdot}00 - 4{\cdot}40 = 16\ 6$ tons. The maximum moment occurs at the foot of the brace and is $H_1 \times 15 = 4{\cdot}4 \times 15 = 66{\cdot}00$ tons feet.

EXAMPLE 3 :—The same transverse bent is taken with the usual normal wind pressure, but columns fixed at base Determine the stresses of all the members graphically.

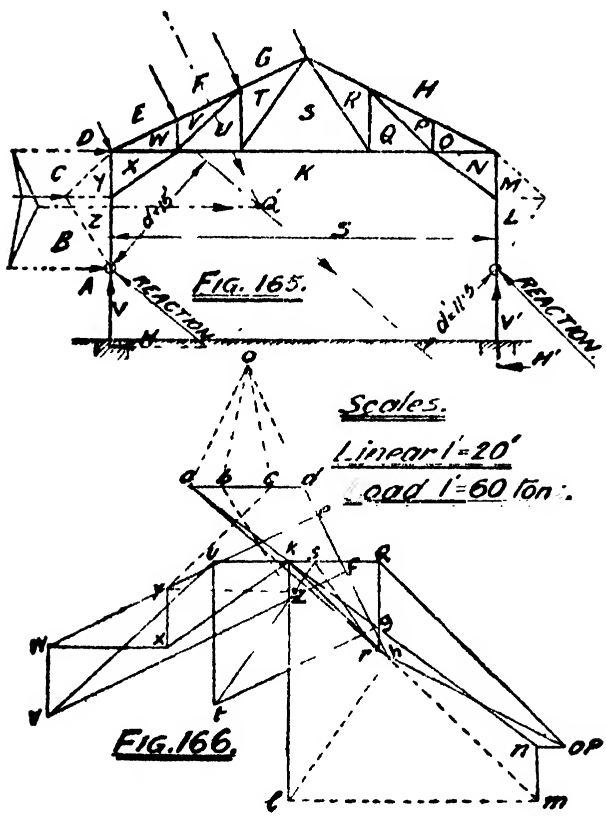


FIG. 165.

FIG. 166.

SOLUTION :—

Columns fixed at base. The usual assumption is to take the point of contraflexure midway between the foot of the knee barce and the base of the column. Considering the column as hinged at the point of contraflexure, wind pressure is to be taken into account above this point, and draw the stress diagram as done in example 2. Get the line of action of the final resultant of the external

loads and calculate V and V_1. $H=H_1$ as usual. In this example final resultant R=8·25. Taking moment centre at the points of contraflexure, $V=\frac{R\times d'}{40}=\frac{8{\cdot}25\times 11{\cdot}5}{40}=2\ 37$; $V^1=\frac{R\times d}{40}=\frac{8\ 25\times 15}{40}=3\ 18$. Plot these values of V, H and V′ H′ at the points of contraflexure then you get the magnitudes and directions of reactions as shown in fig. 165. As usual get the resultant *a h* of figure 166 resolved parallel to the directions of reactions as *h k* and *k a*. Now proceed to draw the stress diagram from the contraflexure point of the windward column. KA, AB, BZ and ZK are the mames of forces acting at that joint; out of these KA, AB are known to you, BZ and ZK are the only two unknowns and by drawing parallels to those members in figure 166, complete that joint as shown. Similarly proceed in the serial order joint by joint and complete the stress diagram as shown in figure 166.

The maximum shear in the leeward column below the knee brace is $H^1=3{\cdot}1$ tons, above the knee brace is equal to the vertical component of the stress of the member $KN-H_1=7{\cdot}8-3{\cdot}1=4{\cdot}7$ tons. The maximum positive moment occurs at the foot of the knee brace$=H_1\times 7{\cdot}5=23{\cdot}25$ tons feet; and negative moment at the foot of the column$=H_1\times 7{\cdot}5=$ 23·25 tons feet.

EXAMPLE 4:—Transverse bent, column hinged at base subjected to both wind and dead loads. Determine the stresses in all the members.

SOLUTION:—Get the line of action of the final resultant as follows:—Plot the wind loads ABCD acting on the sides first, see fig. 168 and with the help of polar diagram and link polygon get the line of action of the resultant of this system of loads; then plot the wind loads acting normally on the slope of the roof truss in continuation of the above wind loads and the line of action of the resultant acts at the centre point of the rafter; let these two resultants intersect at 1; through 1 draw a line parallel to A *h* of figure 168 which is the resultant of the total wind load. Lastly plot all the dead loads in continuation with the wind loads as shown in figure 168, the line of action of the resultant of the dead loads only must act through the ridge vertically down, draw this and get it intersected at 2 with the wind load resultant, and through this point 2 the line of action of the final resultant which is equal to A M of figure 168 must pass.

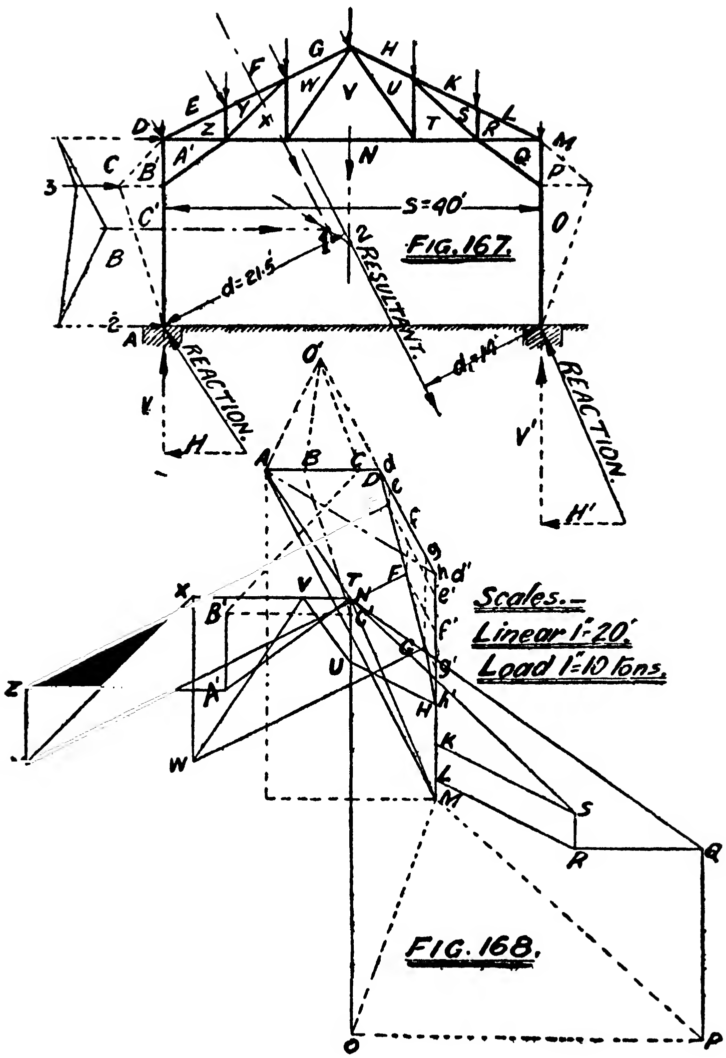


FIG. 167.

FIG. 168.

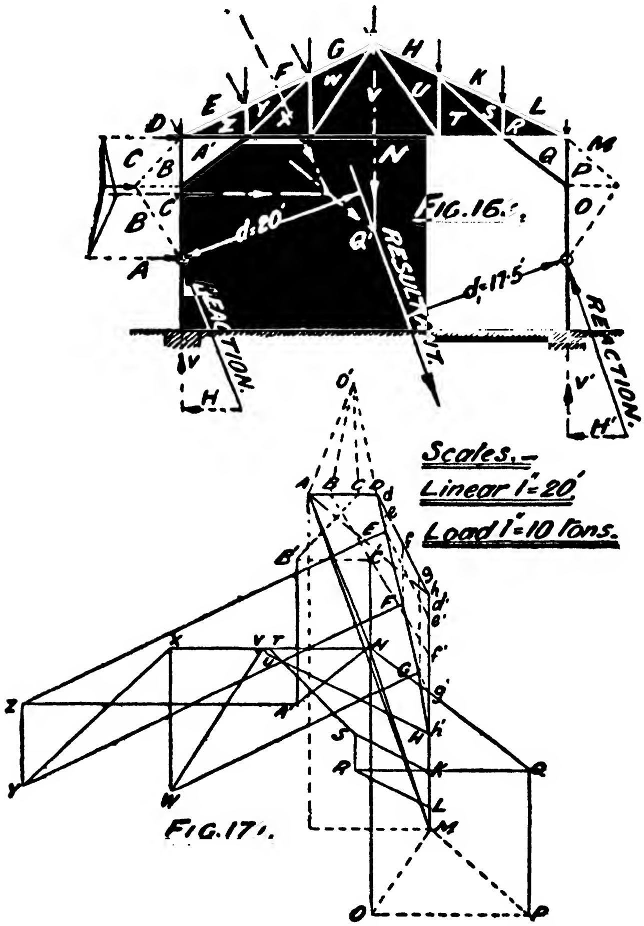
Calculate V, H and V′H′ as done previously and get the magnitudes and directions of reactions; resolve the final resultant AM of figure 168 parallel to the reactions found and proceed with the stress diagram from the base of the windward column.

Note:—In figure 168 dead and wind loads are combined for their resultants and for the detail description of this process refer figure 27. Part I.

The maximum shear below the knee brace in the leeward column

is equal to $H_1 = 4{\cdot}5$ tons, above the knee brace is $18{\cdot}5 - 4{\cdot}5 = 14$ tons; the maximum moment occurs at the foot of the knee brace and is $H^1 \times 15 = 4{\cdot}5 \times 15' = 67{\cdot}5$ tons feet.

EXAMPLE 5:—Same transverse bent with normal wind loads and vertical dead loads is illustrated in figure 169, but columns fixed at base. The procedure is same as above but the base of the column moved up to the point of contraflexture, fig. 170 shows the stress diagram.



The maximum shear in the leeward column below the knee brace is 3 tons; above the knee brace 8—3=5 tons; the maximum positive moment occurs at the foot of the knee brace and is equal to $5 \times 5 = 25$ tons feet. The negative moment occurs at the foot of the column which is equal to $3 \times 7{\cdot}5 = 22{\cdot}5$ tons feet.

Chapter IX.

PORTALS.

In through type Railway and High way steel bridges, the end posts with their top bracings are known as portals, see figure 71 Plate III. Very commonly in bridges these end posts are sloping parallel members. Portals are also frequently used on the sides of Mill Buildings for vertical columns or posts. These portals are most commonly to resist the horizontal wind pressure. The most usual forms of portals used in bridges and buildings are shown in the following pages.—

SIMPLE PORTAL (OF DIAGONAL BRACINGS) WITH COLUMNS HINGED AT BOTTOM.

Fig. 171.

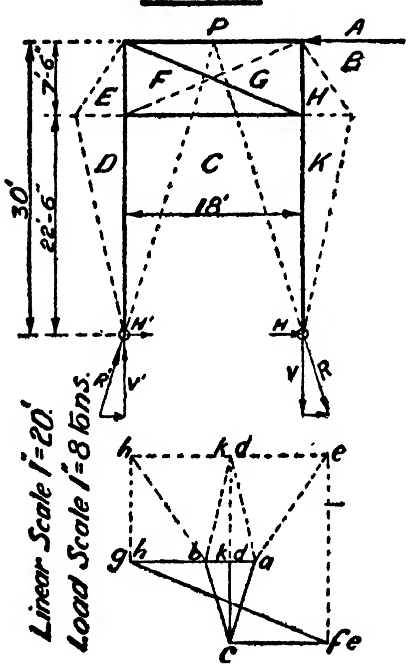


Figure 171 shows a simple portal with bracings diagonally. Columns are assumed to be hinged at bottom, and the wind pressure is taken acting horizontally on the right hand top joint. The magnitude of the wind pressure in all the following examples is taken to be 2 tons. The following reasonings are to be made in working out these portals.— The external load AB tries to pull the right column out and push in the left column and consequently there should exist, forces V and V′, one acting downwards and another upwards as shown in the diagram. Also each column is assumed to deflect towards left equally and there should exist two horizontal forces H and H′ and sum of these must be equal to the external force AB.

The columns are assumed to deflect equally for the reason that the diagonal bracings are assumed to be incompressible.

Now with equal defflections of the columns we have reason to beleive that $H = H_1 = \frac{AB}{2}$ and then follows naturally that $V = V_1$ and the

magnitude of each can be determined by selecting the moment centre at one end of the base of column. Selecting the left bottom end of the column as moment centre we have $V \times 18 - AB \times 30 = 0 \quad \therefore V = \frac{2 \times 30}{18} = \frac{10}{3} = 3{\cdot}33$ tons. Similarly $V_1 = \frac{10}{3}$ or 3·33 tons. $H = H_1 = \frac{AB}{2} = 1$ ton.

Again the structure is in equilibrium under the action of all the above forces namely AB, H, H_1 V and V_1. There are only two supporting points and one external load, the two reactions and this external load,should meet at a point. The resultant of H and V is R and of H′ and V′ is R′. If these two reactions are produced they will meet at P exactly at the mid point of the topmost member of the portal frame and the external load AB also meets this point P if produced.

These portals are all difficient frames and fall short of two members to make the frames sufficient, and moreover the columns are not braced and consequently the stresses in the members cannot be statically determined unless we add an imaginary trussed frame work to each column as shown in dotted in the figure. These trussed frames do not effect the magnitudes of the stresses of the existing members and can help us in determining the stresses graphically. columns will have to resist the bending moment and shear in addition to the direct stresses. (See, Ketchum's hand book.)

Instruction:—Plot the external force A B to some suitable scale and parallel to it. The next two forces are BC and CA which are nothing but the reactions R and R^1. Draw from the point *b* a line *b c* parallel to R and c *a* parallel to R′ and complete the force triangle *a b c* in the stress diagram. Then proceed as usual from the left base of the column and upwards in the order just as you draw stress diagrams for roof trusses or girders. For instance the forces that are acting at the left column base are CA, AD, DC and drawing parallel to these forces you get the force triangle *c a d*. Similarly proceed further and complete the stress diagram as shown.

Note:—(i) There are two diagonal braces and it will be most economical to allow them to take up only tension and in that case only one member will act. For a force AB acting at the top joint the diagonal represented in dotted line does not act at all, for the reason that it is subjected to a compressive stress for that poisition of the load AB and consequently it yields and does not resist the compressive force, then the diagonal shown in firm line comes into action and resists the

tensile stress. Therefore only the diagonal shown in firm line is lettered, and is taken into account in stress diagram.

(ii) The direct stresses in the columns are V and V' but in the stress diagram we get a little more than double of v and v' and also the stresses in the top portions of the columns EF and GH are actually zero, but we get great amount of stresses as shown in dotted lines against these members for the reason of adding imaginary trussed frames.

The direct stresses in the columns are equal to the vertical components of the reactions R and R' but the stresses in the top panel lengths of the columns (*v i z.* in EF and GH) are neutralized by the vertical component of the stress of the diagonal member FG and this is equal to V or V¹.

The maximum bending moment in the column occurs at the foot of the diagonal bracing and is equal to H multiplied by 22'—6", which is equal to 22·5 tons feet; since H = H' = 1 ton. Bending moment at bottom and top of the column is zero.

Shear from the base of the column to 22·5' is equal to H' = 1 ton and from 22·5 to 30' = the stress in FC − H' = 4 − 1 = 3 tons.

Students are advised to study carefully the above reasonings as these are applicable generally for the portals with hinged end columns.

SIMPLE PORTAL (OF DIAGONAL BRACINGS)
COLUMNS FIXED AT BASE

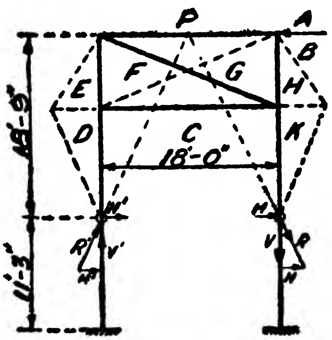
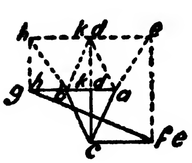


FIG. 172.



When the columns are fixed at bottom the points of contraflexure may be taken approximately, as midway between the lower edge of the portal and the base of each of the columns, that is at 11'—3" from the base. Now consider the columns hinged at the points of contraflexure, calculate the forces V, V', H and H' and apply them at these points and proceed with the stress diagram as in the previous case. To calculate V take moment centre on the left column at the point of contraflexure, then V × 18—AB × 18·75 = 0.

$\therefore V = \frac{2 \times 18.75}{18} = 2·08$ tons. V' = 2·08 tons. $H = H' = \frac{AB}{2} = 1$ ton.

Bending moment at the base of the column and at the lower edge of the portal is equal to $H \times 11{\cdot}25' = 11{\cdot}25$ tons feet; at the point of contra-flexture and at the top of the column bending moment is equal to zero.

The shear from the base of the column to the lower edge of the portal is equal to $H = 1$ ton, and from the lower edge of the portal to the top is equal to the stress in the member $FC - H = 2{\cdot}5 - 1 = 1{\cdot}5$ tons.

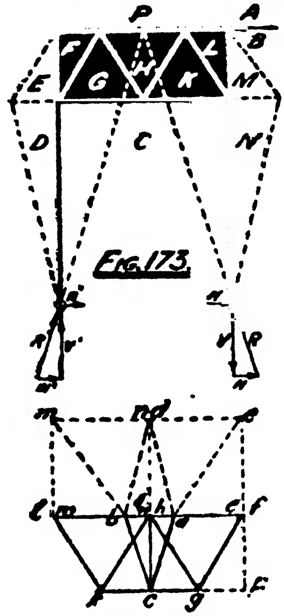


Fig. 173.

SIMPLE PORTAL
(SINGLE SYSTEM WARREN GIRDER TYPE)
COLUMNS HINGED.

As before calculate V, V′, H and H′ and draw the stress diagram as in the first example fig. 171. Bending moment for the column is the same as calculated for fig. 171 but shear is equal to $H = 1$ ton from the base of the column to the lower edge of the portal; and from the lower edge of the portal to the top (is equal to the sum of the stress in the member GC and the horizontal conponent of the stress of the member FG) $- H' = 2 + 2 - 1 = 3$ tons.

SIMPLE PORTAL (SINGLE SYSTEM WARREN GIRDER TYPE)
COLUMNS FIXED.

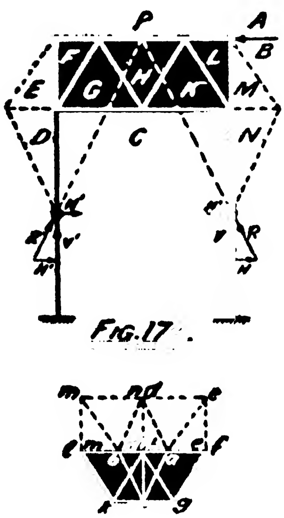


Fig. 17 .

When the columns are fixed the point of contra-flexure of each column is usually taken at midway between the base of column and lower edge of the portal. As usual $V = V' = 2{\cdot}08$ tons and $H = H' = 1$ ton. Stress diagram may be drawn similar to the figure 172.

Bending moment at the base of the column and also at the lower edge of the portal $= H' \times 11{\cdot}25' = 11{\cdot}25$ tons feet, but at the point of contra-flexure and at the top corner of the portal = zero.

The shearing force from the base of the column to the lower edge of the portal is equal to $H' = 1$ ton, and from the lower

edge of the portal to top corner of the same, the shear is equal to sum of the stress in the member GC and the horizontal component of the stress of the member FG—H′ = {(1·25 + 1·25) − 1} = 1·50 tons.

SIMPLE PORTAL (DOUBLE SYSTEM WARREN GIRDER TYPE) (COLUMNS HINGED.)

Instruction.—Calculate V, V′, H, H′, and draw the force triangle *a b c* for the three known forces AB, R and R′ as usual, the stresses in the members of the imaginary trussed frame work, and the direct stress in the leeward column can also be drawn without any further difficulty. After determining the letter *e* in the stress diagram you cannot proceed further in determining the stresses in the portal, as every joint has three unknowns. This difficulty can be overcome by two methods. (I) By separating the portals into two portals with simple bracing (as worked previously for double system Warren Girder fig. 74 page 58) taking ½ of the external load AB the stresses may be found separately, and then combined algebraically to give the stresses in the portals. (II) One combined diagram can be drawn knowing the stress of

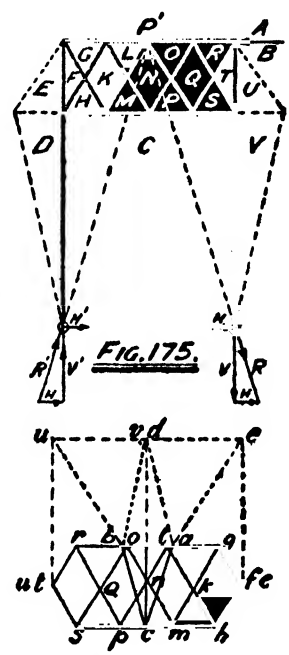


FIG. 175.

the end vertical EF or TU for both the system with ½ the load AB. This latter method is simpler and has been used in the following two examples as follows:—(See figure 173) The stress in the end vertical EF for half the load AB measures 2·17 tons, and for the reversed system of bracing, the stress in the same vertical EF is 3·85 tons. The load scale for fig. 173 is 1″ = 8 tons, by taking AB = 1 ton, the load scale then becomes 1″ = 4 tons and the same stress diagram may be used to measure the stress in the member EF. The stress in the end vertical EF for both the system is equal to 2·17 + 3·85 = 6·02 tons. Plotting *e f* = 6·02 tons in the stress diagram of figure 175 the stresses of the other members in the portal can be determined as shown without any further difficulty.

Bending moment at the base of the column is zero and at the

lower edge of the portal is maximum and is equal to $H \times 22{\cdot}5 = 22{\cdot}5$ tons feet, and at the top corner of the portal is equal to zero.

Shearing force from the foot of the column to the lower edge of the portal is equal to $H' = 1$ ton, and from this point to the top corner of the portal is equal to (sum of the stress of the member HC and the horizontal component of the stress of the member FH)$- H' = \{(3+1) - 1\} = 3$ tons. The members AG and GF are connected to the top joint and both are in tension, the sum of the horizontal component of the stress of the member FG and the tensile stress of the member AG, is the effective load on the top joint; and sum of these two is equal to $2+1=3$, which exactly coincides with the result obtained above.

SIMPLE PORTAL (DOUBLE SYSTEM WARREN GIRDER TYPE) (COLUMNS FIXED)

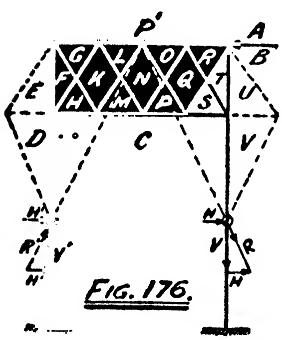
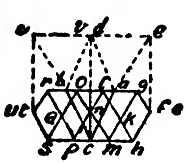


FIG. 176.



Here, columns are fixed, and take the point of contra-flexure of each column at midway between the base of column and lower edge of the portal. Then calculate V, V′, H and H′ and proceed in the same way as in the previous case. Calculate the stress in the end vertical EF of fig. 174 for both the systems of bracing by taking ½ of the given load AB. The total stress in the end vertical EF for both the system is 1·1 plus $2{\cdot}125 = 3{\cdot}225$ or say 3·23 tons. Plotting this in the diagram of figure 176 the stress diagram can be completed as shown.

The maximum bending moment occurs at two points, one at base of the column and another at the lower edge of the portal and is equal to $H' \times 11{\cdot}25 = 11{\cdot}25$ tons feet, and zero bending moment at the point of contraflexure and at top of portal corner.

Shear is equal to $H' = 1$ ton from the base of the column to the lower edge of the portal and from this point to the top is equal to the (sum of the stress of the member HC and the horizontal component of the stress of the member FH)$- H' = (1{\cdot}75 + {\cdot}75) - 1 = 1{\cdot}5$ tons.

SIMPLE PORTAL WITH KNEE BRACING. COLUMNS HINGED.

This is generally known as A Type Portal The additional members shown dotted are not stressed, and in the stress diagram all the letters connected to these members, coincide in one point. The bending moment will be a maximum at the foot of the knee brace and is equal to H′ × 20′ = 20 tons feet, and at the base of the column and at top corner of the portal, bending moment is zero. Shear from the base of the column to the foot of the knee brace is H′ = 1 ton, and from the foot of the knee brace to the top corner of the portal is equal to the horizontal component of the stress of the member of FC − H′ = 3·25 − 1 = 2·25 tons.

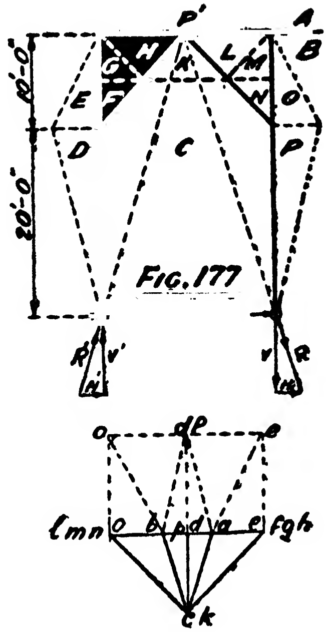


FIG. 177

SIMPLE PORTAL WITH KNEE BRACINGS. COLUMNS FIXED.

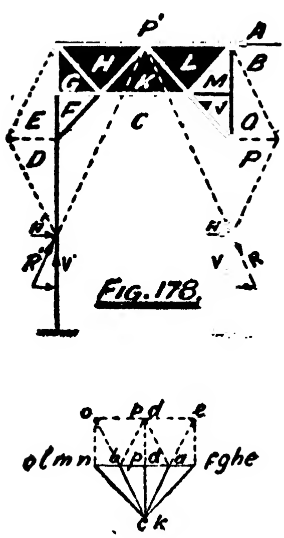


FIG. 178.

The point of contra-flexure is at midway between the base of the column and the foot of the knee brace. Calculate as usual V, V′, H and H′ and draw the stress diagram and complete it as shown. Bending moment will be maximum at the base of the column and at the foot of the knee brace, and zero at the point of contra-flexure and at the top corner of the portal. Shear is as usual from the base of the column to the foot of the knee brace equal to H′ = 1 ton, and from the foot of the knee brace to the top, is equal to the horizontal component of the stress of the knee brace − H′ = 2 − 1 = 1 ton.

FRAMED PORTAL WITH KNEE BRACINGS COLUMNS HINGED.

There is no difficulty in following this portal, as the manner of procedure is the same as adopted in previous examples. It is to be noted that end verticals EF, EG, QO and QP of the portal are stressed. EF is stressed to the amount of the vertical component of the stress of the knee brace FC—$V' = 5\frac{1}{12} - 3\frac{1}{3} = 1{\cdot}75$ tons tension. Similarly $QP = 1{\cdot}75$ tons compression. Stress in EG is equal to the vertical component of the stress of the member $GH = 1\frac{3}{4}$ tons tension, similarly the stress in the member QO is equal to the vertical component of the stress of the member $NO = 1\frac{3}{4}$ tons compression.

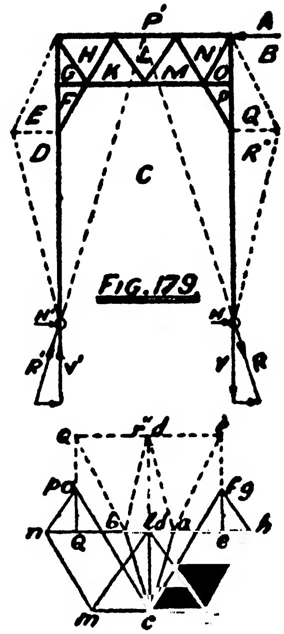


FIG. 179.

Bending moment at the foot of the knee brace is maximum and is equal to $H' \times 20 = 20$ tons feet, and at the base of the column and top corner of portal is zero. Shear at the base as usual is equal to $H' = 1$ ton from the base of the column to the foot of the knee brace; and from the knee brace to the top corner of the portal, is equal to the horizontal component of the stress of the member FC minus $H' = 3 - 1 = 2$ tons.

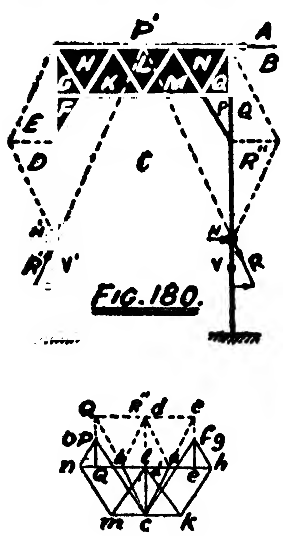


FIG. 180.

FRAMED PORTAL WITH KNEE BRACINGS COLUMNS FIXED.

Figure 180 shows the frame and stress diagrams for the framed portal with columns fixed, and needs no explanation and can be followed easily.

PORTAL (PLATE GIRDER TYPE)
(COLUMNS HINGED)

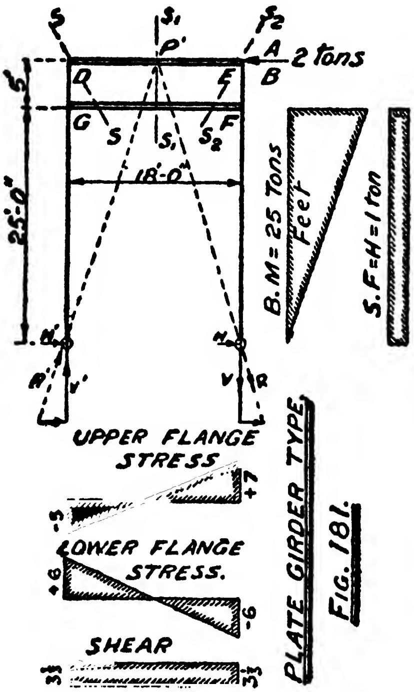


FIG. 181.

All the three members in this portal are subjected to direct stress as well as bending moment and shear. As this is not a framed portal we cannot get the stress by graphic method, but we can determine the flange stress of the girder by the method of section. See Chapter III Part I.

It is clear from the diagram that the bending moment will be the greatest at points G and F and naturally the flange stress also should be greatest. Now let us take three sections S S, S_1 S_1, S_2 S_2, at the left end, centre, and right end of the girder respectively as shown. Take section S S, let the portion to the right of the section be removed, and take G as moment centre, then upper flange stress $\times 5 - H' \times 25 = 0$. $\therefore$ Upper flange stress at this section is equal to $\frac{1 \times 25}{5} = 5$ tons. At section S_1 S_1, selecting moment centre at the centre of bottom flange we get, upper flange stress $\times 5 + V' \times 9 - H' \times 25 = 0$. $\therefore$ Upper flange stress at this section $= \frac{-\frac{10}{3} \times 9 + 1 \times 25}{5} = \frac{-5}{5} = -1$ ton. At section S_2 S_2, and removing the portion to the right of the section and selecting F as moment centre, the upper flange stress at this section $= \frac{H' \times 25 - \frac{10}{3} \times 18}{5} = \frac{-35}{5} = -7$ tons. Similarly lower flange stresses at these three sections

taking moment centres at the upper flang we have as follows:—at section $SS = \frac{H \times 30}{5} = 6$ tons; at section $S_1 S_1 = \frac{H' \times 30 - \frac{10}{3} \times 9}{5} = 0$ and at section $S_2 S_2 = \frac{H' \times 30 - \frac{10}{3} \times 18}{5} = -6$ tons. Shear in the girder is equal to V or $V' = 3\frac{1}{3}$ tons.

Bending moment is the greatest at G or F and is equal to $H \times 25 =$ 25 tons feet, and shear is as usual $= H = 1$ ton. These diagrams are drawn to the scale in the figure for verification.

PORTAL (PLATE GIRDER TYPE)
COLUMNS FIXED.

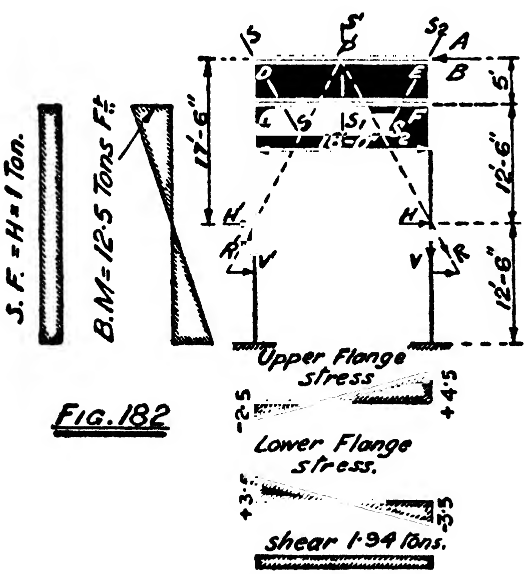


FIG. 182

Here the point of contra-flexure is at mid point between the base of the column and lower flange of the cross girder. By calculation we get $V = V' = 1{\cdot}944$ tons $H = H' = 1$ ton as usual. External force $AB = 2$ tons. Upper flange stresses at these three sections are $-2{\cdot}5, +{\cdot}99$ and 4 5 tons respectively.

Lower flange stress at these three sections are $+3{\cdot}5$, 0 and $-3{\cdot}5$ tons respectively.

Shear is as usual equal to $V = 1{\cdot}94$ tons throughout. Bending moment and shear for the columns are as shown in the diagram.

PORTAL (BEAM WITH KNEE BRACING) COLUMNS HINGED.

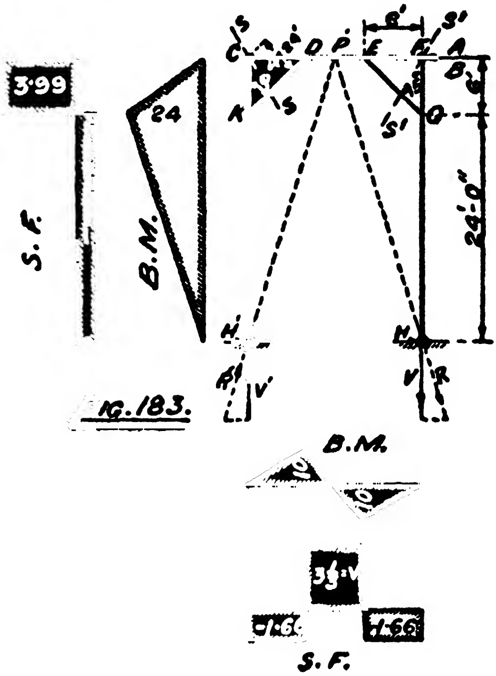


FIG. 183.

The direct stresses on the members of this pòrtal cannot be determined statically and can only be found by the method of section To know the stress in C D pass a section S S and remove the right hand portion of the structure and select K as moment centre. Then stress in CD × 6 − H′ × 24′ = 0. ∴ Stress in $CD = \frac{1 \times 24}{6} = 4$ tons tension. To know the stress in KD take moment centre at C. Then stress in K D × 4·24 − H × 30 = 0.

∴ Stress in $KD = \frac{1 \times 30}{4\ 24} = 7{\cdot}07$ tons compression.

Stress in CK can be easily determined now since we know the stress in KD. The stress in CK = vertical component of the stress in KD − V′ = 7·07 Sin $\theta - 3\frac{1}{3}$ tons $\theta = 45°$ ∴ Stress in CK = 7·07 × ·70711 − $3\frac{1}{3}$ = 4·999 − 3·333 = 1·66 tons tension.

Pass a section S¹ S¹ and remove the portion of the structure to the left of the section. Select the moment centre at G. Now the stress in EF × 6 — AB × 6 — H × 24 = 0. ∴ Stress in $EF = \frac{2 \times 6 + 1 \times 24}{6} = \frac{36}{6} = 6$ tons compression.

For the stress in EG take moment centre at F, then stress in EG × 4·24 − H × 30 = 0 ∴ Stress in $EG = \frac{1 \times 30}{4.24} = 7{\cdot}07$ tons tension. Stress in FG = vertical component of EG — V = 7·07 × ·70711 − $3\frac{1}{3}$ = 1·66 tons compression.

Compression in DE is equal to the thrust in EF − the horizontal component of the tension in EG = 6 − 7 07 × cos θ = 6 — 4·99 = 1 ton =

H. Answer. The direct stress in the right column is equal to $V = 3\frac{1}{3}$ tons tension and of the left column $= V' = 3\frac{1}{3}$ tons compression.

Columns and cross beam are subjected to bending moment and shear. Maximum bending moment for the column occurs at the foot of the knee brace and is equal to $H \times 24 = 24$ tons feet, and zero at the foot and top corner of the portal.

Shear from the base of the column to bottom of knee brace is equal to $H' = 1$ ton, and from the foot of knee brace to top of portal corner is equal to horizontal component of the stress in $KD - H' = 4{\cdot}99 - 1 = 3{\cdot}99$ or 4 tons. Cross beam is assumed to be hinged at the left and right extreme supports, and therefore the bending moment is zero at those points and maximum at the top of knee braces, that is, at E & D. Bending moment at $E = V \times 6 - H \times 30 = 3\frac{1}{3} \times 6 - 1 \times 30 = 20 - 30 = -10$ tons feet negative. Bending moment at $D = V \times 12 - H \times 30 = 3\frac{1}{3} \times 12 - 1 \times 30 = 40 - 30 = 10$ tons feet positive. Shear at $F = V -$ vertical component of the stress in $EG = 3{\cdot}33 - 4{\cdot}99 = -1{\cdot}66$ tons. Shear from D to $E = -1{\cdot}66 +$ vertical component of the stress in $EG = -1{\cdot}66 + 4{\cdot}99 = 3{\cdot}33 = V$. Ans. Shear at C is the same as at F. These are shown diagrammatically in the figure.

PORTAL (BEAM WITH KNEE BRACINGS)

COLUMNS FIXED.

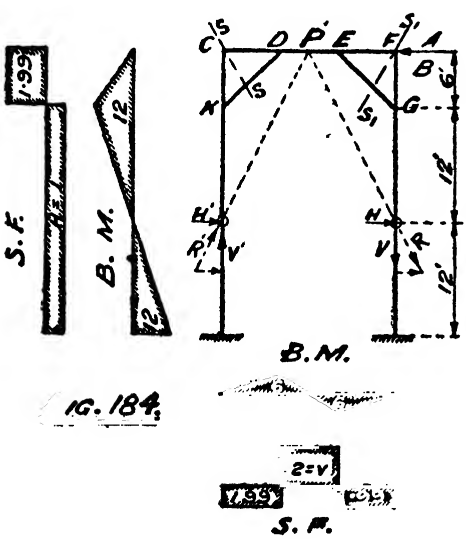


FIG. 184.

The working method of this is the same as in the previous example, and the only difference is, that H, H′ V & V′ are taken at the points of contra-flexure. Taking the point of contraflexure on the left column as moment centre, calculate V as follows:—$V \times 18 - AB \times 18 = 0$ $\therefore V = \frac{2 \times 18}{18} = 2$ tons. $\therefore V = V' = 2$ tons. $H = H' = 1$ ton.

Take section SS to know the stress in CD, and take moment centre at K. Then stress in $CD \times 6 - H \times 12 = 0$. Stress in $CD = \frac{12}{6} = 2$ tons tension.

Taking moment centre at C, stress in $KD \times 4{\cdot}24 - H \times 18 = 0$; $KD = \frac{18}{4{\cdot}24} = 4{\cdot}24$ compression. Stress in CK = vertical component of the stress in $KD - V' = 4{\cdot}24 \times {\cdot}70711 - 2 = 2{\cdot}99 - 2 = {\cdot}99$ ton tension.

Similarly stress in $EG = 4{\cdot}24$ tons tension, and $FG = {\cdot}99$ compression. At section $S_1\ S_1$ the stress in $EF \times 6 - AB \times 6 - H \times 12 = 0$. $\therefore\ EF = \frac{2 \times 6 + 12}{6} = \frac{24}{6} = 4$ tons compression. Direct stress in DE = stress in EF − horizontal component of the tension $EG = 4 - 4{\cdot}24 \times {\cdot}70711 = 4 - 2{\cdot}998 = 1{\cdot}002$ or say 1 ton, which is equal to H or H′. Bending moment, and shear for the columns and beam are shown in the figure.

Note:—Stresses in the members of portal frames can be easily determined either by graphical methods or by moments for horizontal loads, and for vertical loads on the portals, a knowledge dealing with the theory of two hinged arches is obsolutely necessary, and hence the students may refer to "The Further Problems in the Theory and Design of Stuctures by Andrews".

Chapter X.

ARCHES.

Arches are generally used for bridges and buildings ; where a large span roof and greater head room is required, such as in exhibition buildings, markets, railway platforms etc, Three Hinged Arches are used. The general classification of arches is as follows.— Masonry arches, Metal Arches, and Reinforced concrete arches. Masonry and Metal arches are dealt with in this volume. Reinforced concrete arches will be treated in the third volume later on.

MASONRY ARCHES.

Arches that are built up of stones, concrete or bricks are known as Masonry arches. Arches generally do not have vertical reactions even for vertical loads. This is obvious by looking at the figure 185. This arch is divided into 5 equal parts and the area of each, considered as a load acting at its centre of gravity. Owing to these loads the arch as a whole tries to spread itself horizontally, and the side walls or abutments are to resist this spreading action of the arch, and hence

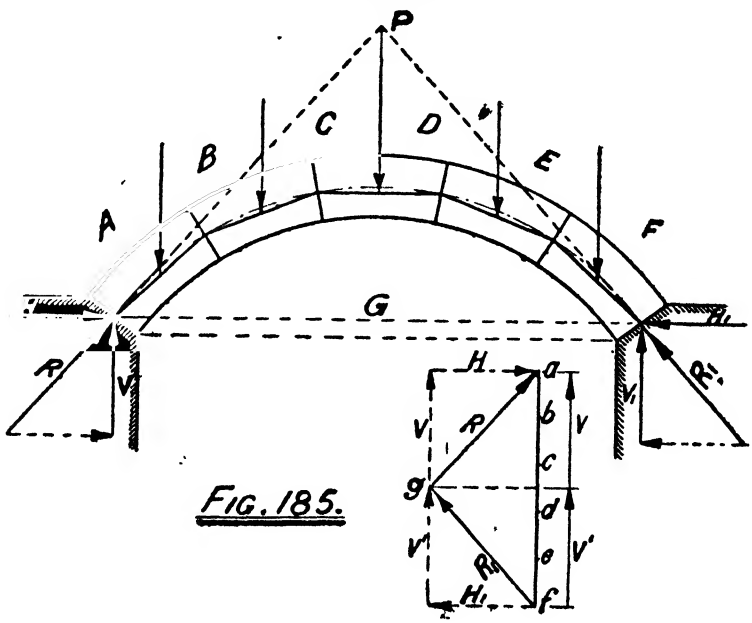


FIG. 185.

there should exist horizontal force at each end of the arch. There should exist vertical forces as well at each end, and sum of these two vertical forces must be equal to the total load of the arch. This is an established fact that, whether the loads on the arch be symmetrical or unsymmetrical, the horizontal thrust at one end must be equal to the horizontal thrust at the other end of the arch. At the resting point *viz.* at the springing of the arch therefore, there are two forces one horizontal and another vertical; the resultant of these two forces is the reaction of the arch at that point.

For equilibrium of the arch two reactions and the resultant of the external loads viz. the loads on the arch must meet at a point, and a force triangle $a\,f\,g$ may be drawn as shown in the diagram. Without determining the horizontal force we cannot find the direction and magnitude of the reaction. Hence it is obsolutely necessary to find out the horizontal force in determining the stability and also the curve of equilibrium of the arch.

The following conditions of equilibrium are to be known before proceeding to discuss the theory of arch :—

(I) Sum of all the horizontal components of forces must be equal to zero.

(II) Sum of all the vertical components of forces must be equal to zero.

(III) Sum of the moments of all the forces about any point must be equal to zero.

Now take one dimensioned masonry arch, and find out the horizontal thrust. (See figure 186.) Take the section exactly at crown and remove the right hand portion of the arch, and keep the left hand portion of the arch in equilibrium by providing a force H^2 horizontally at the centre line of the arch at crown.

(The student should understand that H^2 at crown is perfectly horizontal, because the stress in curved member of a structure always tries to travel in a straight line, and hence the stress at any point in the curved member is tangential to that point). The weight of each division of the arch is taken to be 4 cwts approximately.

The line of action of the resultant for the loads to left of the section is determined by polar and equilibrium polygon as shown, and its magnitude is 10 cwts. and call this as W.

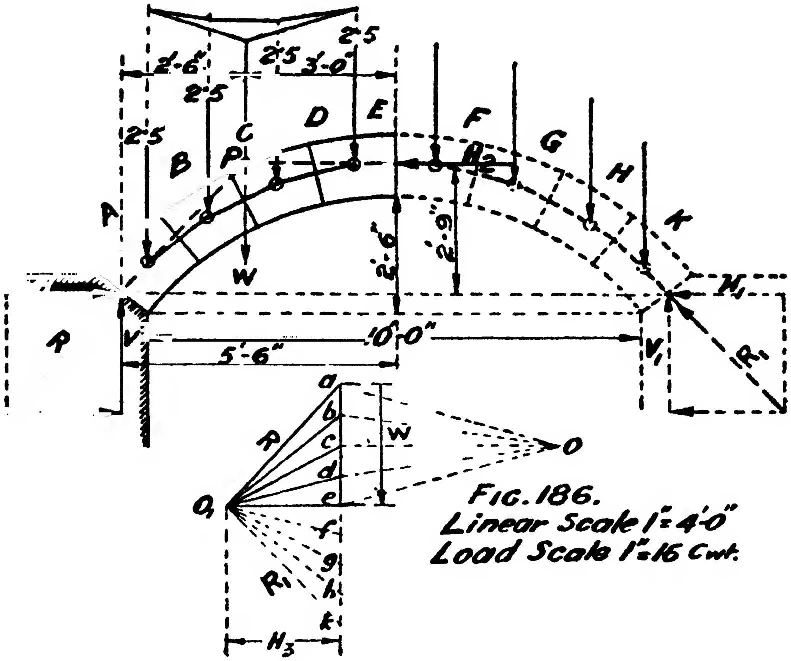


FIG. 186.
Linear Scale 1" = 4'-0"
Load Scale 1" = 16 Cwt.

$V = V' = 10$ cwts as the loads are symmetrical, and H must be equal to H' whether the loads are symmetrical or unsymmetrical; the truth of this you will see very soon.

Now take the moment centre on the centre line of the arch at crown. Then $-W \times 3 - H \times 2\frac{3}{4} + V \times 5\frac{1}{2} = 0$; $\therefore -H = \frac{10 \times 3 - 10 \times 5\frac{1}{2}}{2\frac{3}{4}} = -9{\cdot}09$ cwts. Combining this H and V for their resultant you get the magnitude and direction of the reaction of the arch as shown.

Now it can easily be proved that $H_1'' = H_2$ by taking moment centre on the centre line of the arch at springing. $-H_2 \times 2\frac{3}{4} + W \times 2\frac{1}{2} = 0$; $\therefore H_2 = -\frac{10 \times 2\frac{1}{2}}{2\frac{3}{4}} = -9{\cdot}09$, this is the same as H.

The above result may be easily obtained by graphical method as follows :—The horizontal thrust at crown and the resultant load W intersect at P and the reaction at the springing must also meet at P. Out of these three forces we know W in magnitude, and H_2 and R known only in directions, and a force triangle $a\,e\,O_1$ determines the magnitudes of these two unknowns as shown in the diagram. Since the loads are symmetrical $R = R_1$. The horizontal

components of R and R_1 are equal to H_3, therefore $H=H_1$, and H_3 is common to both R and R_1 ; hence this is an established fact that the horizontal components of any two sides of a force triangle (whether the sides equal or unequal) are equal to each other. Now $a\ O_1\ e$ is the polar diagram and a corresponding equilibrium polygon exactly coincides with the centre line of the arch. This equilibrium polygon is known as the Curve of Equilibrium or the Linear Arch.

In masonry structures tension is not at all allowed in any section, and if the resultant load falls within the middle third at any section, the whole section is subjected to compression only, and care must be taken that the maximum compression should not exceed the limit of safety Hence the curve of equilibrium or the linear arch should also be within the middle third of the arch ring. In this example the curve of equilibrium exactly coincides with the centre line of the arch ring and the arch is safe.

ARCH WITH UNSYMMETRICAL LOADING.

Fig. 187, shows the segmental arch of 10 feet span, rise $2\frac{1}{2}$ feet, thickness of arch ring 15 inches. In addi ion to the dead load a train load of 10 tons per foot run is allowed up to half of the span only. (This is considered to be the worst position of the load for arches). As usual divide the loaded area into any number of equal parts, say six parts and calculate the load taking only a foot in length of the arch. Now the loads are known and plot them on to any suitable scale as shown. The thrust at crown is not horizontal for unsymmetrical loads. By drawing the curve of equilibrium we can know the horizontal thrusts at the supports ; the curve of equilibrium should fall within the boundary of the arch ring and preferably within the middle third of the same.

Select any three points within the middle third of the arch ring, two at the supports and one at the crown, say at the centre line of the arch at KLM. Now an equilibrium polygon is to be drawn passing through these three points. (See fig. 18 part I). Select any pole O and complete the polar diagram. Then draw the corresponding equilibrium polygon commencing from the point K as shown ; K 1 is the closing line for left half of the load, and K 2 for all the loads, and from pole O draw lines O 1 and O 2 parallel to these two closing lines.

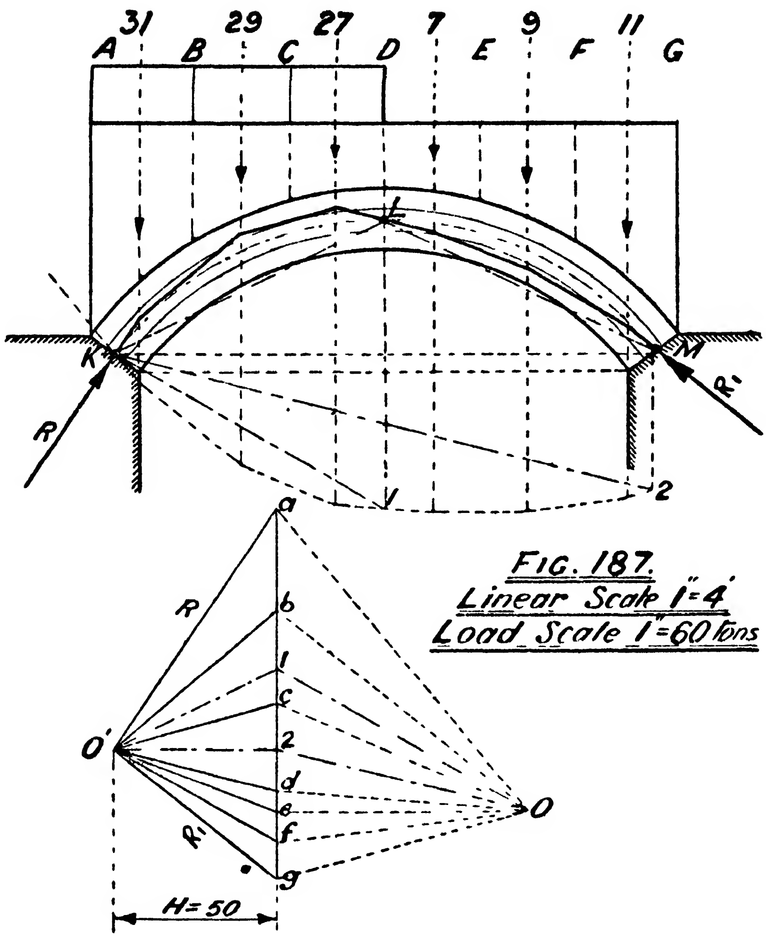


FIG. 187.
Linear Scale 1"=4'
Load Scale 1"=60 tons

(The closing line of the equilibrium polygon is familiar to the students at this stage). The points 1 and 2 on the load line do not change their positions by changing the pole distance. You want the equilibrium polygon to pass through KLM and not K 1 2, therefore join KL, KM and draw 1—O,' 2—O', from the points 1 and 2 in the load line parallel to KL and KM respectively, then O' is the real pole, and 2—O' is the magnitude of the horizontal thrust at the supports. As usual draw the equilibrium polygon from the point K and you will observe the curve of equilibrium passes through the selected points KLM. *a o'* and *g o'* represent the left and right reaction of the arch, in magnitude and direction.

The curve of equilibrium passes at more than two points beyond the middle third of the arch ring giving rise to tensional strains

at these points. In the next example we will find out bending moment shear and thrust in an arch.

BENDING, MOMENT, SHEAR AND THRUST AT ANY SECTION OF AN ARCH. (MASONRY OR STREEL)

TO DETERMINE BENDING MOMENT.

The same figure 187 is taken with its curve of equilibrium in figure 188 giving prominence to the centre line and the line of pressure. If the line of pressure coincide with the centre line of the arch, there will not be any bending moment, but in this figure the line of pressure is away from the centre line and the arch is to resist the bending moment.

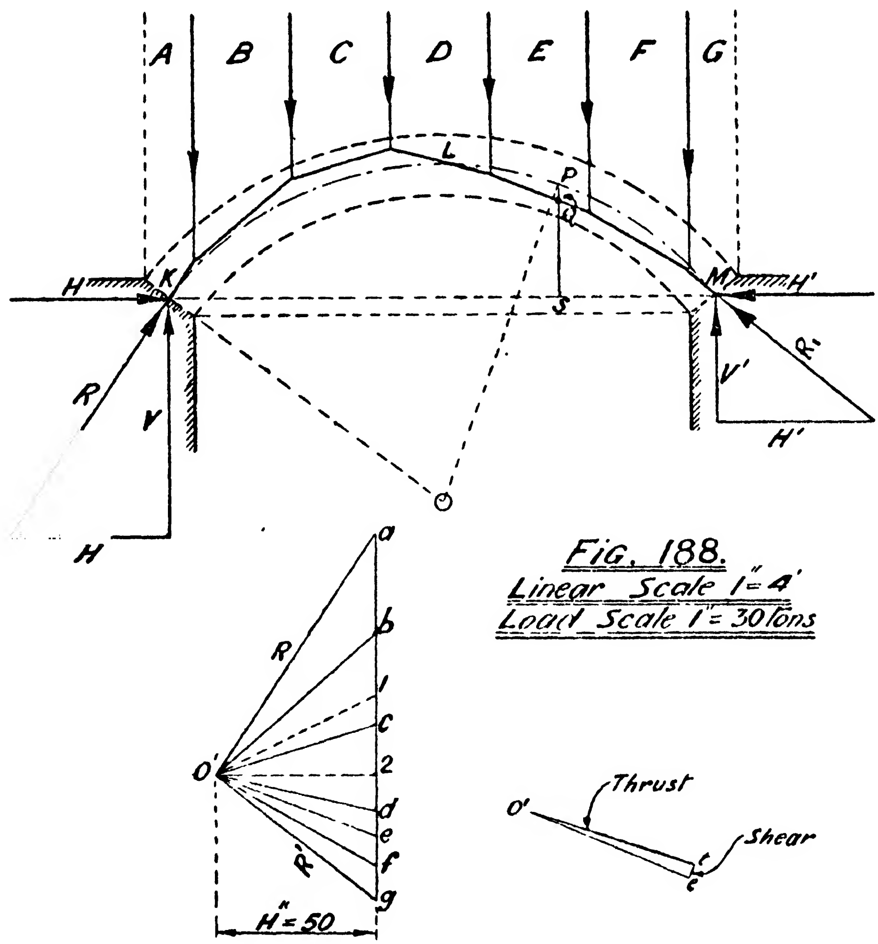


FIG. 188.
Linear Scale 1" = 4'
Load Scale 1" = 30 Tons

Now take any point P in the centre line of the arch, the bending moment at this point is equal to the intercept Q S multiplied by the pole distance H″ minus H multiplied by P S. Then B. M. at P= H″ × Q S − H × P S. = − H × P Q. Here H″ = H′ = H. Proof of this is very simple—The arch is considered to be, a bent beam with vertical loads, and for these loads the bending moment diagram is the funicular polygon K L M K drawn from polar diagram *a g o′*; but at the supporting points of this bent beam there are horizontal thrusts H and H′, which will reduce the bending moment caused by the vertical loads, as it is clear from the diagram, and this horizontal thrust is equal to pole distance H″.

Therefore the bending moment at any point in an arch is equal to the intercept between the centre line of the arch and the line of pressure multiplied by the horizontal thrust.

DETERMINATION OF THRUST AND SHEAR AT POINT P.

To determine the thrust, and shear at the point P, proceed as follows—Resolve the corresponding thrust *o′ e* (in the field E) tangential and perpendicular to the given point P. The former represents the thrust, and the latter shear at the point P, and they measure to the scale 51 and 5 cwts. respectively.

MAXIMUM COMPRESSION AND TENSION AT THE SECTION P.

The thickness of arch is 1′−3″ and a length of one foot is to be taken longitudinally for calculation as usual. Now maximum compression $= \frac{\text{Thrust}}{\text{Area}} + \frac{\text{Bending moment}}{\text{Section Modulus}}$. Here thrust=51 cwt. or 2·55 tons. Area=1·25 □′; bending moment= ·4 × 50 = 20 0 cwt. foot or 1·00 ton foot. Section modulus $= \frac{bd^2}{6} = \frac{1{\cdot}25^2}{6} = {\cdot}26$.

Maximum compression $= \frac{2{\cdot}55}{1{\cdot}25} + \frac{1{\cdot}00}{{\cdot}26} = 2{\cdot}04 + 3{\cdot}84 = 5{\cdot}88$ tons per □′. Maximum tension $= \frac{\text{Bending moment}}{\text{Section modulus}} - \frac{\text{Thrust}}{\text{Area}} = \frac{1{\cdot}00}{{\cdot}26} - \frac{2{\cdot}55}{1{\cdot}25} =$ $3{\cdot}84 - 2{\cdot}04 = 1{\cdot}80$ tons tension per □′.

The thrust has nothing to do on the cross sectional area at that point when finding maximum tensile stress, and hence we have deducted that from the equation.

Mean shear at the section $= \frac{\text{Shear}}{\text{Area}} = \frac{5}{1{\cdot}25}$ cwts. or $\frac{{\cdot}25}{1{\cdot}25}$ ton or ·2 ton per □′.

Note:—Tension is not allowed at any section in the arch ring and since we get 1·80 tons per □′, the depth of the arch ring is to be increased to 18″ which reduces the tension to less than a ton which is negligible and the arch is safe.

METAL ARCHES.

Metal arches cannot have both ends fixed at the supports and if fixed, the direction and magnitude of reaction cannot be determined statically, owing to the fact that these arches expand and contract to a very appreciable extent during hot and cold seasons. Fixed end means, that their expansion and contraction are checked, and hence we cannot correctly determine the stresses or the magnitudes and directions of reactions, unless we go into the theory of elasticity of the material of which the arch is composed. An arch of stone or brick is an unelastic one, and is subjected to the direct compression only at any section of it, and the line of pressure to be within the middle third of the arch ring. The theory therefore applied to the masonry arches is not applicable to metal arches which are capable of resisting bending, direct compression or tension, shear, and therefore the line of pressure may go outside the section as well.

THREE HINGED ARCHES.

The directions and magnitudes of reactions of metal arches can be very easily determined accurately by providing hinges at the supports and crowns. These are known as Three Hinged Arches. There are however Two Hinged and One Hinged Arches also, but for the present, three Hinged Arches are dealt with here. In Three Hinged Arches the line of pressure or the curve of equilibrium must pass through these hinged joints, you know that they cannot resist any bending moment and you will observe this in figure 189.

The span of this arch is 50 feet and rise 12′ – 6″, hinged at the supports and at crown. The load of 20 tons is carried at the crown. The arch as a whole is in equilibrium under the action of three forces namely, the external load at crown and the reactions at the two supports. Since the three forces keep a structure in equilibrium they all must meet at a point, therefore the two reactions R and R′ should meet any-

where on the load line AB. The determination of the meeting point is as follows.

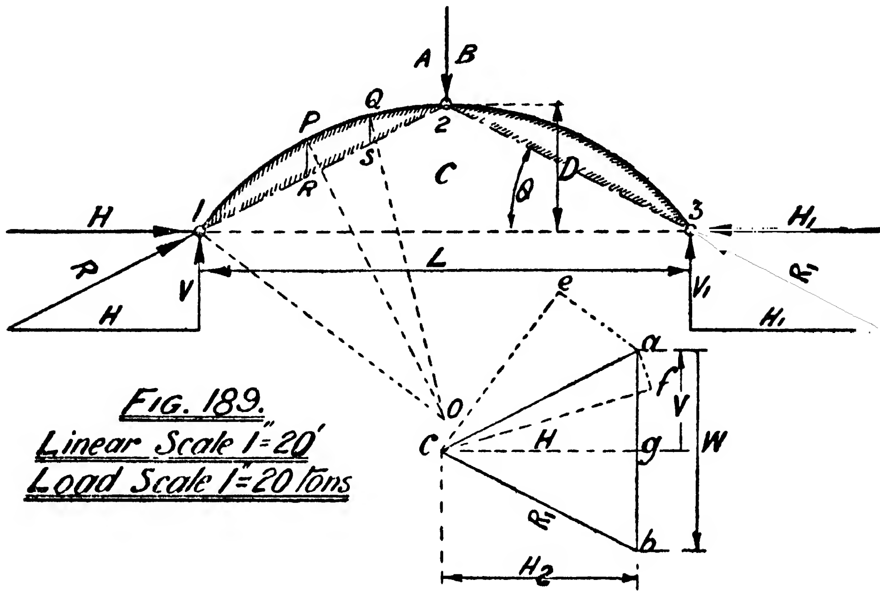


FIG. 189.
Linear Scale 1" = 20'
Load Scale 1" = 20 tons

In this arch, there are two curved members left and right, hinged at bottom and at crown, and you know the stresses do not travel through the centre lines of the curved members, but pass tangential to the curvature The lines shown dotted drawn from the hinges at the supports to the crown hinge, are parallel to the tangential lines from mid points of these two curved members, and hence the direction of reaction at the supports must be along with these.

(2) The two reactions at the supports have their corresponding components H, V and H′ V′. Taking the crown hinge as moment centre we have $-V_1 \times \frac{L}{2} = - H_1 \times D$. $\therefore V_1 = \dfrac{H_1 \times D}{\frac{L}{2}}$ but $\tan \theta = \dfrac{V_1}{H_1}$, substituting the value of V_1 in the above equation ($\tan \theta = \dfrac{V'}{H'}$) we have,

$\tan \theta = \dfrac{H_1 \times D}{\frac{L}{2}} \div H_1 = \dfrac{D}{\frac{L}{2}}$. Therefore R_1 which is the resultant of H_1 and V_1 must pass through the crown hinge. Similar reasonings for the left reaction.

Linear Arch or the Curve of Equilibrium:—Plot the load AB at the crown to some suitabe scale and draw the force triangle *a b c*. Now 1—2—3 is the Linear Arch or Curve of Equilibrium.

Determination of Bending Moment, Shear, and Thrust at any point in the arch:—For example select some three points on this arch, say one at the left supporting hinge and two others at points P and Q. The thrust at the hinge is equal to $c\ e$ which is drawn tangential to that point from the polar diagram $a\ b\ c$, and $e\ a$ is the shear which is drawn at right angle to $c\ e$ from the point a. Since there is no intercept from the centre line of the arch to the linear arch at this point, the bending moment is zero For proof see fig. 188 and page 129.

Bending moment at P is equal to the intercept PR multiplied by the pole distance H_2. The thrust is equal to $c\ a$ which is drawn tangential to the point P· There is no shearing force at this point,because the tangential line coincides with the ray of the linear arch. Again the bending moment at Q is equal to the intercept QS multiplied by H_2; the thrust is equal to $c\ f$ drawn from c tangential to Q and shear is equal to $a\ f$ which is drawn perpendicular to $c\ f$ from a.

Note:—The reason of taking the ray $c\ a$ of the polar diagram for knowing the thrust, shear and bending moment of the points 1, P and Q is as follows —The polar diagram contains only two rays $c\ a$ and $c\ b$,* and naturally the linear arch or the curve of equilibrium must contain also two rays 1—2 and 2—3 drawn parallel to two rays $c\ a$, and $c\ b$. Therefore $c\ a$ represents the greatest thrust in the left half of the arch and $c\ b$ to the right half of the arch. If the points are selected on the right half of the arch, the ray $c\ b$ is to be considered to calculate thrust and shear. Polar distance H is common to both. Here $V=V'$ and $V-W=g\ b$ and combinining this with the horizontal thrust H at crown we get the final direction of the thrust at crown parallel to $b\ c$.

THREE HINGED ARCH WITH SINGLE CONCENTRATED LOAD ANYWHERE ON THE ARCH.

Fig. 190 shows a load anywhere on the left half of the arch. Now this load is to be transferred to the nearest joints of the left half of the arch, just as we transfer the load to the joints of the member of a roof truss,if the load were to be anywhere between the joints of that member (See figure 102 part I.)

From the diagram shown below the arch, we know that $a\ b$ is to be taken at the left supporting hinge and $b\ c$ at the crown hinge. (See figure. 53 page 41). Points 1—2—3 are joined by dotted lines and

these two lines represent tangential lines to the left and right half of the curved members.

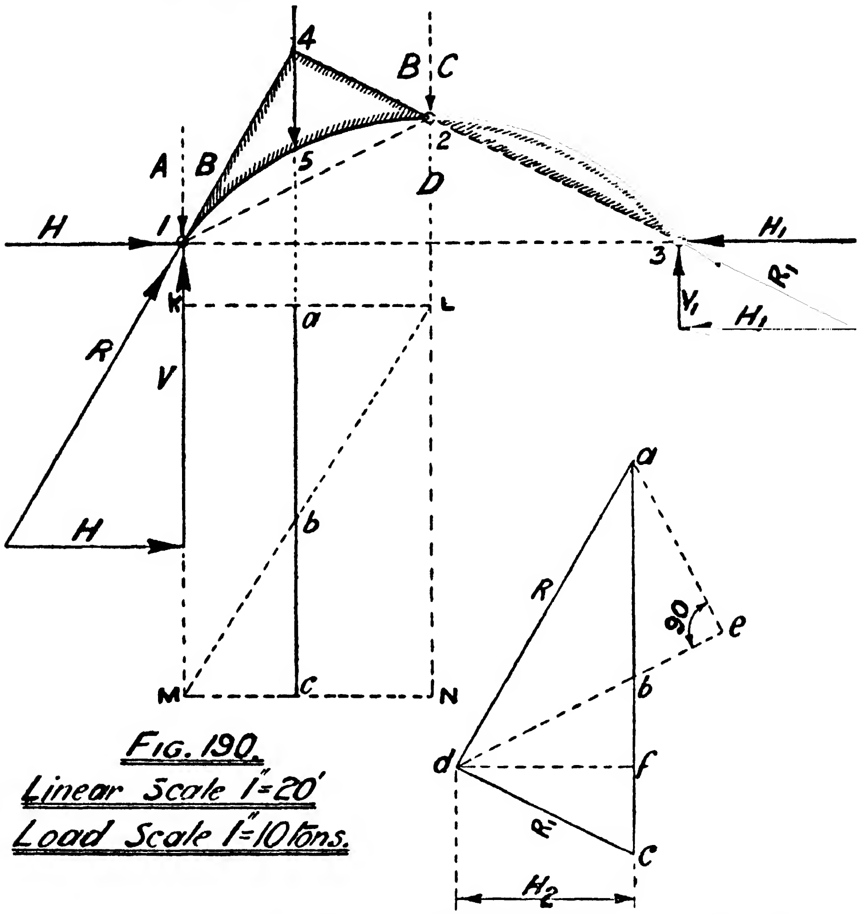


FIG. 190.
Linear Scale 1″=20′
Load Scale 1″=10 tons.

Now the stress diagram is drawn as usual. Commencing from the crown hinge we get *b c d* the traingle of forces for that joint and by going to the left and right supported joints we get the disired figure *a b c d a* as shown. Here *d a* is the left reaction and *c d* is the right reaction and 1—4—2—3 is the linear arch or the curve of equilibrium. This is the graphical way of getting the magnitudes and directions of reactions and the curve of equilibrium.

Theoritically, we know that the reaction at the right supporting hinge must pass through the crown hinge and we have proved this fact in the fig. 189. By producing the reaction at the right support through the crown hinge and getting it intersected the given load at 4 the direction of the reaction at the left supporting hinge can be determined, as these three loads must meet at a point,

BENDING MOMENT, SHEARING FORCE AND THRUST AT ANY POINT.

Now let us take any point or say at the point where the load acts at 5. The bending moment is maximum here, which is equal to the ordinate 4—5 multiplied by H_2 the polar distance. The thrust is equal to *d e* which is drawn tangential to the point 5, and shear is equal to *a e* which is drawn porpendicular to *d e*. Here V is not equal to V′ and vertical shear at the crown is equal to $V-W=f\,c$. Combining this with the horizontal thrust at crown we get the direction of the thrust at crown which is the same as *c d* .

Note:—(1) There will be naturally some doubt in the minds of the students for having selected the ray *d a* in the polar diagram to determine the thrust and shear at the point 5 in this arch, point 5 is exactly down at the intersection of the two rays *a d* and *d c* and the thrust and shear at that point may as well be determined from the ray *d c* instead of the ray *a d*, but the tangential and perpendicular components if were to be drawn to the point 5 from the ray *d c*, the results fall very much short of the values got from the ray *a d*, hence the ray *a d* has been selected to determine the maximum thrust and shear at the point 5 in the arch

(2) The determination of the Bending Moment, Shear and Thrust for any point between the left supporting hinge ond point 5, the ray *a d* is to be selected and resolved, and similarly the ray *c d* is to be taken for resolution to determine the bending moment, shearing force and thrust for any point or points between the point 5 and the right supporting hinge.

THREE HINGED ARCH WITH SYMMETRICAL LOADING.

In three hinged arches, the linear arch or the curve of equilibrium must pass through three hinges. By somehow or other if we succeed to draw the equilibrium polygon passing through these three given points or hinges we will be able to determine, the magnitudes and directions of reactions, horizontal and vertical thrusts at the supports. Bending moment, shearing force and thrust at any point in the arch naturally follow as shown in previous two examples.

Fig. 191 shows a three hinged arch of 100 feet span and a rise of 25′ with uniformly distributed loading of one ton per foot run. Divide

the load into any number of equal parts and plot the loads to a suitable scale as shown. Take any pole O′ and draw the equilibrium polygon as shown from the left supporting hinge, P Q′ R′ is the polygon, but polygon is to pass through three hinges named P Q R. Refer example 5, figure 21 Part I and get the desired pole distance H graphically as shown in figure (*b*) here, or by calculati n as follows:—H: H′ : : y': y and from this the required polar distance can be determined thus:—$H \times y = H' \times y'$. $\therefore H = \frac{H' \times y'}{y}$ or by any of the three methods shown in part I, may be adopted to find H.

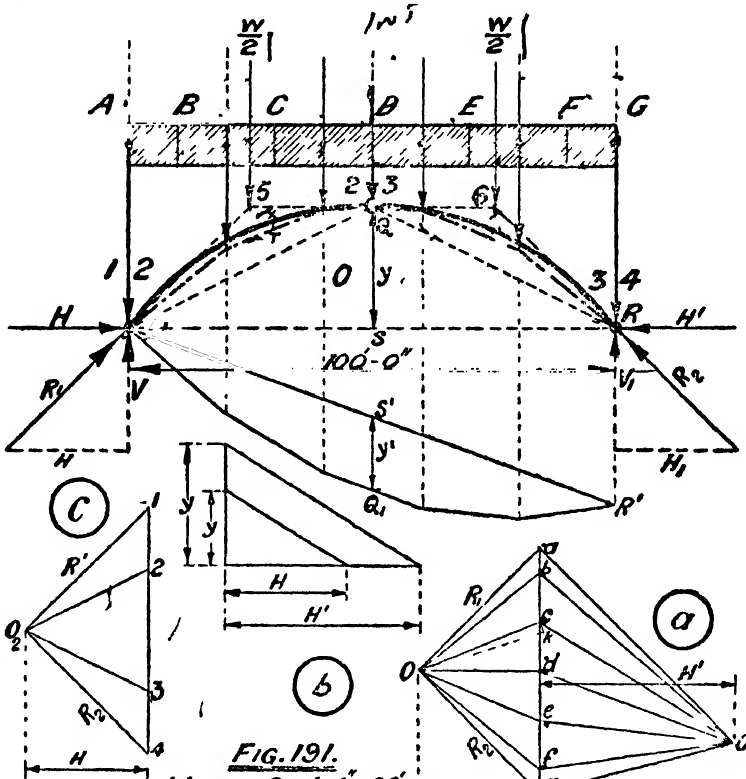


FIG. 191.
Linear Scale 1″=40′
Load Scale 1″=80 Tons.

Note:—Instead of going into laborious process for determining the real pole distance, students are advised to adopt the methods shown in figures 189 and 190. Here in this figure the load is distributed of 1 ton per foot run, and the total load is therefore 100 tons. This load of 100 tons is to be divided proportionately over the three hinged joints P, Q, R. Then the joint P receives 25 tons. Q 50 tons and R receives 25 tons.

Join P Q and Q R in dotted lines as shown, and these two dotted lines are tangential to the curved members as pointed out previously. These three loads are plotted to the load scale by naming them $1-2$, $2-3$, and $3-4$, as shown in figure (c). Now going round the ridge joint we get the triangle of forces $2-3-0_2$ which fixes the pole distance H and then $4-0_2$ and 0_2-1 give us the magnitudes and directions of right and left reactions. Finally we should fill up the loads *a b, b c, c d, d e, e f* and *f g* whithin the space $1-4$ and draw the curve of equilibrium. See how simple is this. After determining the pole distance, the linear arch or the curve of equilibrium may be drawn as shown. Here the first and the last ray of the polar diagram represent the magnitude and direcion of the left and right reaction, the polar distance H is the horizontal thrust at each support, and half of the total load is equal to $V = V'$ as shown in the diagram.

Bending moment at any point X is equal to the ordinate XT multi plied by H, the thrust is equal to OK drawn tangential to the point X from the ray *o c*, and the shear is equal to *c* K drawn perp: to OK.

Since $V = V'$ and $V = \frac{W}{2}$, therefore the vertical shear at the crown is equal to zero, because $V - \frac{W}{2} = 0$. Hence for symmetrical loading the direction of the thrust at crown is horizontal. This horizontal thrust meets the two reactions at 5 and 6 which are the points in the lines of the resultants $\frac{W}{2}$ and $\frac{W}{2}$.

THREE HINGED ARCH WITH UNSYMMETRICAL LOADING.

The arch shown in figure 192 is of 120 feet span with a rise of 45 feet. Half of the span is loaded with a distributed load of 1 ton per foot run, and the right half of the span with 2 tons per running foot. Determine the magnitudes and directions of reactions; draw the curve of equilibrium and the direction of the thrust at crown.

SOLUTION:—Divide the distributed load into some convenient divisions as shown (the more the better). Plot the loads in the load line as shown and by selecting any pole O′ draw the polar diagram and the corresponding equilibrium polygon P Q′ R′ as shown. Mark *y*′ in this polygon, but the actual depth you require is *y* and the curve of equilibrium is to pass through three hinges, and therefore obtain the pole distance graphically as shown in fig (*b*) see also fig. 21 part I. Draw from

pole O′ a line O′ K in fig. (a) parallel to the closing line PR_1 and from K draw a line KO parallel to PR and equal to H the real pole distance. Then from this new polar diagram you can draw the curve of equilibrium as shown in chain dotted line. Here the first and the last ray *o a* and *o h* represent the magnitudes and directions of reactions at the left and the right supports respectively. (Load scale for fig. 192 is 1″ = 160 tons, and 1″ = 80 tons written by mistake.)

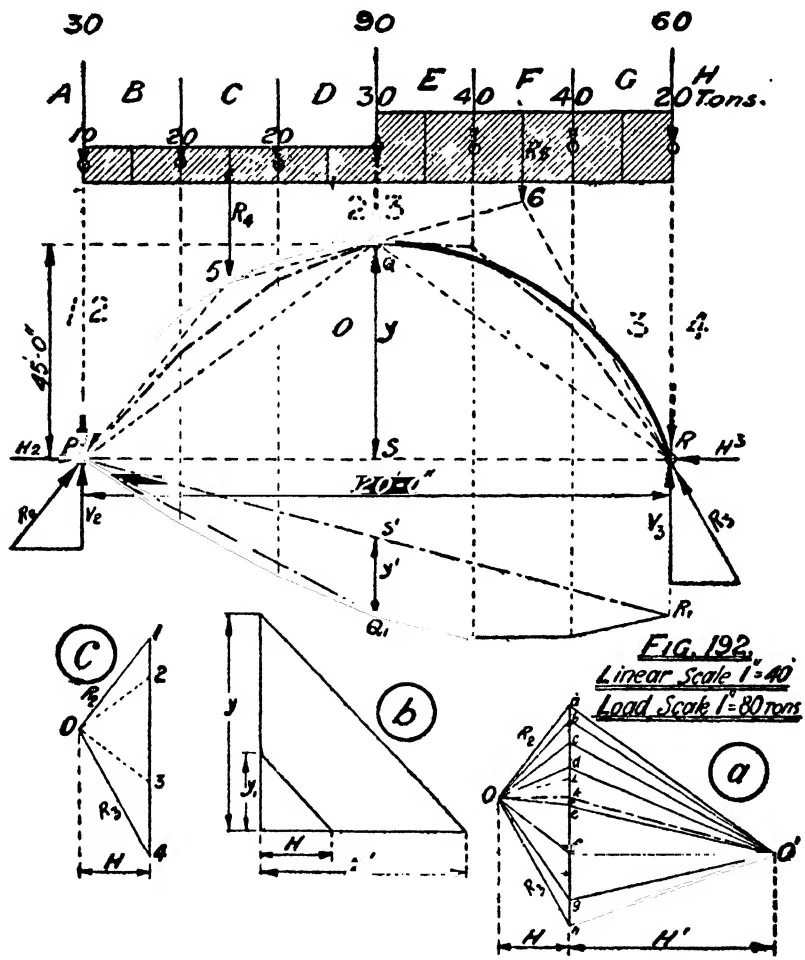


FIG. 192.
Linear Scale 1″=40′
Load Scale 1″=80 tons

Here you observe V^2 is not equal to V^3 but $V^2 - W = KL$ where W is the distributed load over the left half of the span which is equal to 60 tons; to this KL if you combine the horizontal thrust KO, the resultant would be OL, and this is the actual direction of the thrust at crown. Now L is the dividing point of left and right half of the

distributed loads. Finding the point L and joining this with the pole O you get the direction of the thrust at crown. This inclined thrust at crown intersects the reaction lines at 5 and 6 at the action lines of the resultants of the left and right distributed loads. As suggested in the previous example you can easily get the real pole distance H without going into the laborious methods as described above. Divide the total loads proportionately over the three hinged joints; join PQ and QR in dotted lines. Now left hinged joint takes only 30 tons, crown takes 90 tons and the right supporting hinge will be loaded by 60 tons. Name these loads as 1—2, 2—3, and 3—4 and work out the crown hinge and the real pole O will be fixed; O 4 and O 1 represent the right and left reactions respectively. [See fig. (c).] Fill up the loads *a b*, *b c* etc. between 1 and 4 and draw the curve of equilibrium as shown.

THREE-PINNED SPANDRIL BRACED ARCH.

A truss with two portions pin-jointed together is of 80 foot span and is assumed to weigh 9600 lbs. The load in pounds per linear foot of top chord, uniformly distributed is 1000 lbs. Draw the stress diagram. (Proposed to set for I. sc. part II 1926).

SOLUTION—The total load on the arch is equal to the weight of the arch plus the load per running foot over the length of the top chord $=9600+1000\times 80=89600$ lbs. Since there are eight panels, each panel load $=\frac{89600}{8}=11{,}200$ lbs. $=5$ tons. The magnitudes and directions of reactions may be determined by the modified method suggested in previous examples; that is, loading the hinged joints from the total load proportionately; in this case supporting hinges will receive each 10 tons and the crown hinge will be loaded with 20 tons. Three hinges are connected by dotted straight lines. Due consideration is to be given only for the curved portion of the arch where the hinges are directly connected to it ignoring the braced portion of the structure. Working round the crown hinge we can fix the point M which is the required pole, and 1—M, and 4—M will give us the magnitudes and directions of the reactions at the supports. Then filling in all the loads in detail, the stress diagram can easily be drawn as shown in the figure.

Vertical shear at crown is zero; because $V=V'=\frac{W}{2}$ then $V-\frac{W}{2}=0$. Hence the thrust at crown is horizontal and the two reactions if produced they will intersect at points 5 and 6 which are on the action

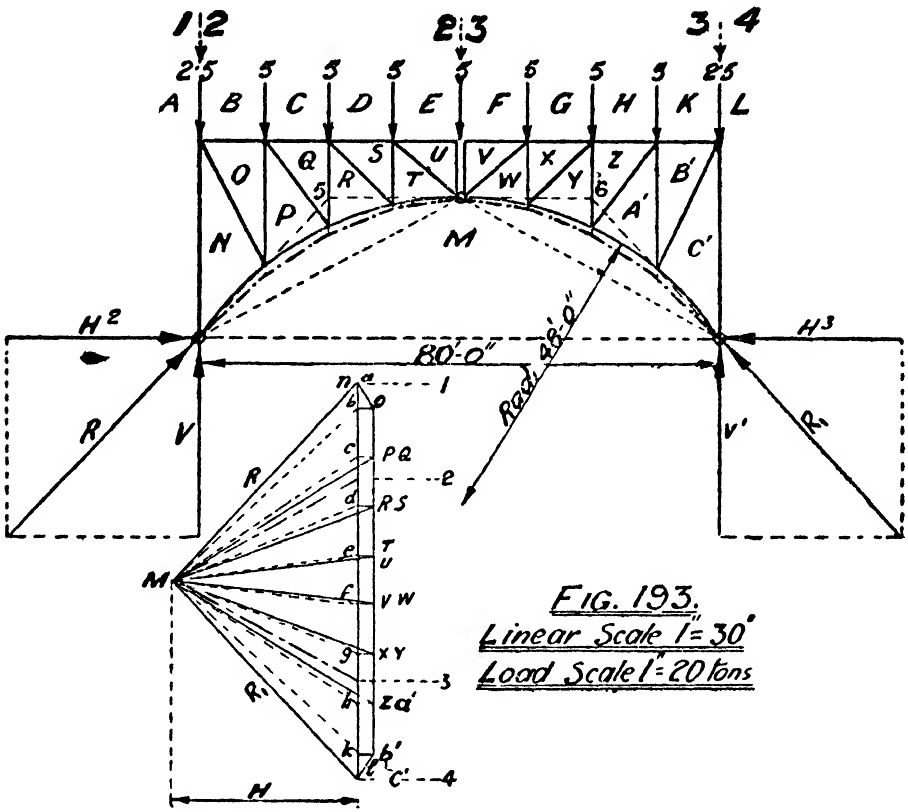


FIG. 193.
Linear Scale 1" = 30'
Load Scale 1" = 20 tons

lines of the resultants of the left and right half of the loads. Linear arch also has been drawn in thick chain dotted line in the figure.

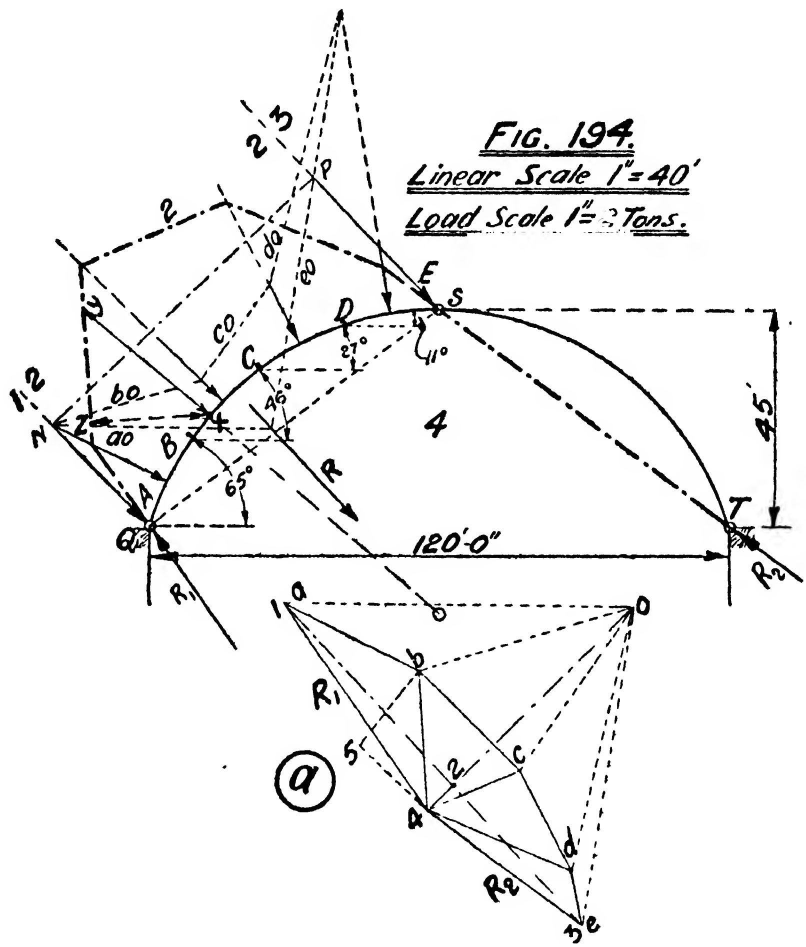
WIND AND DEAD LOADS ON THREE HINGED ARCHES.

CASE I.

WIND LOAD:—Fig. 194 shows a three hinged arch of span 120 feet with a rise of 45 feet. Spacing is 14 feet centre to centre; wind is taken to blow from the left side. It is required to determine (I) the magnitudes and directions of reactions, (II) to draw the linear arch, and (III) to determine the bending moment, shearing force and thrust at any point in the arch.

SOLUTION:—The left half of the arch only is exposed to the wind pressure. Divide the left half of the arch into any number of equal parts say 4 as shown here. Draw the tangential lines from the mid points of the divisions to intersect the horizontal lines and measure the angles of inclination as shown with the horizontal. Then calculate the

normal component of the wind pressure per square foot of roof surface, as per Duchemin's formula as follows:—$N = H \times \frac{2 \operatorname{Sin} \theta}{1 + \operatorname{Sin}^2 \theta}$, (See also part I chap: 3) where N = the normal component of wind pressure per square foot of roof surface in lbs., θ = angle of inclination of the roof surface to the horizontal, H = horizontal wind pressure per square foot of vertical surface.



Now for an angle of 65° the normal wind pressure per square foot of roof surface is 49·75 lbs. when H = 50 lbs. per □' of vertical surface. The calculation is as follows:—

$$N = H \times \frac{2 \operatorname{Sin} \theta}{1 + \operatorname{Sin}^2 \theta.} \quad (\theta = 65^\circ).$$

$$= 50 \times \frac{2 \times \cdot 9063}{1 + \cdot 9063^2} = 50 \times \frac{1\ 8126}{1\ 8213} = 49{\cdot}75 \text{ lbs.}$$

The spacing of these arches is 14 feet, and the panel length (that is each division) measures 20 feet, then the total load on this panel is equal to $49 \cdot 75 \times 14 \times 20 = 13930$ lbs. or 6·21 tons. Take this load at the centre of the first division and name this as AB. Calculate similarly for the rest of the divisions. Take all the calculated loads at mid points of the respective divisions as shown. Plot loads to some suitable scale as shown in fig. (*a*), connect these to any pole O and draw the corresponding funicular polygon and at the intersection of the first and the last ray draw the resultant R parallel to *a e*. Draw QN and SP parallel to the resultant and let these two lines meet the first and the last ray of the funicular polygon at N and P. Then NP is the closing line, and from pole O draw a line parallel to this closing line, and let it intersect the resultant *a e* at 2.

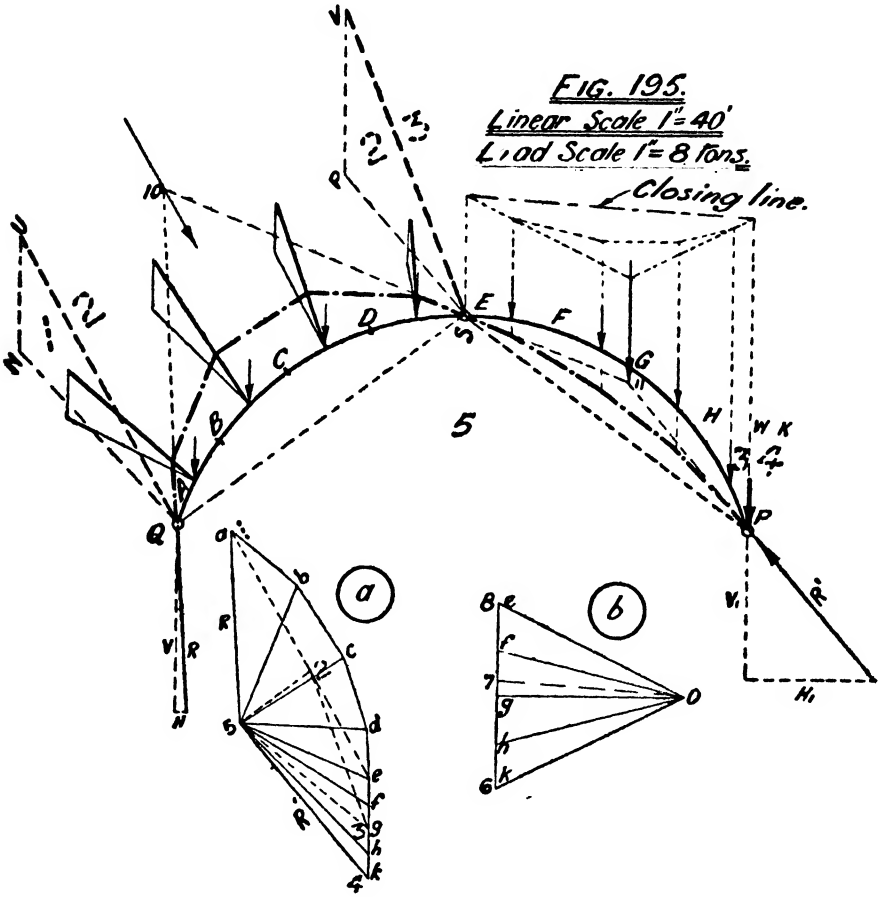
Now 2 *a* is the load at the left supporting hinge and 2 *e* the load at the crown hinge. You can neglect the rest of the loads *a b*, *b c*, *c d* and *d e*. Consider Q S T as a Vee shaped roof truss hinged at the supports and at crown with loads. 1—2 at the left supporting hinge and 2—3 at crown hinge, and work out the crown hinge in the usual way, 2—3 is the load and resolve this into two directions 3—4 and 4—2, then 2—3—4 is the force triangle for that joint. Come to the left supporting joint, the triangle of forces for that joint is 1—2—4.

3—4 is the direction and magnitude of reaction at the right support and 4—1 at the left support. The point 4 is the real pole and connect all the loads to this and draw the curve of equilibrium as shown in thick chain dotted line and you will observe that this will pass through the crown hinge.

BENDING MOMENT SHEAR AND THRUST AT ANY POINT.

The loads are not vertical, and there is no common pole distance to the loads. (See fig. 41 part I). Ordinate multiplied by pole distance method does not hold good here. Therefore select a point any where in the centre line of the arch, say at x and draw a radial section line through x to intersect the linear arch at y To the left of this section there are two forces R_1 and AB. In fig (*a*) 4—*b* is the resultant of these two forces, and therefore resolve this force 4—*b*, parallel and perpendicular to $x\ y$ as 4—5 and 5—*b*. Then bending moment at x is equal to $x\ y$ multiplied by 5—*b*; shear is equal to 4—5, and 5—*b* is the thrust, $x\ y$ to be taken in linear scale, 4—5 and 5—6 in load scale.

Another method of getting the bending moment at x is as follows:—Draw from the point x a line $z\,x$ at right angles to the line of the linear arch in the field B. The corresponding ray in the polar diagram representing this force is $4-b$. Then bending moment at x is equal to the force $4-b$, multiplied by the distance $x\,z$. The shear and thrust as before. Remember these well.



CASE II.

WIND AND DEAD LOADS COMBINED:—Wind loads on the divisions as calculated in the last example are 6·21, 5·92, 4·70 and 2·30 tons respectively, and dead loads if calculated will be as follows.—Spacing of arches is 14 feet centre to centre. Weight of arch truss roughly per square foot of ground area is (·08 × Span in feet) = ·08 × 120 = 9·6 lbs. Total load = 120 × 14 × 9·6 = 13208 lbs. and on each

division $= \frac{13208}{8} =$ 16510 lbs. or ·73 ton. Weight of corrugated sheeting on each division $= 20 \times 14 \times 4 = 1120$ lbs. and weight of purlins $= 14 \times 20 \times 4 = 1120$ lbs. Total 2240 lbs. or 1 ton. Grand total weight on each division $= \cdot73 + 1 = 1\cdot73$ tons or say 2 tons. Plot the wind and dead loads on the left portion of the arch and combine them for their resultants and draw the dead loads on the right side. Plot these loads as shown in fig. (*a*).

Fig. (*a*) has been drawn to a scale of $1'' = 16$ tons for want of space. The modified method of determining the reactions and a real pole for the curve of equilibrium, is to sum up all the loads and concentrate them only on three hinges. Consider the arch to be a Vee shaped roof truss. First take only the wind load acting on the arch and for this the loads on the left supporting and crown hinges are Q N and S P as found out in the previous example Then determine the dead loads on three hinges as follows:—Take the dead loads on one half of the arch, that is, on the right half, draw the load line, polar diagram, and a corresponding equilibrium polygon as in fig. (*b*), and a line drawn from pole O parallel to the closing line will divide the line *e k* in 7, thus 7—6 is the load on the supporting right hinge and 8—7 on the crown hinge. The crown hinge in subjected to a further load equal to 8—7 from the left half of the arch.

Next combine the wind and dead loads from the left bottom hinge as shown.

Note:—N U the dead load, is combined with the wind load Q N and the resultant is U Q. Similarly at crown V S is the resultant and at the right bottom hinge W P is the load. Now join Q S and S P with dotted lines. Plot these loads in fig (*a*) commencing from the point *a* as 1—2, 2—3, 3—4, shown dotted, ignoring the rest of the loads. Now work round the crown hinge and draw the triangle of forces 2—3—5, here the point 5 is fixed up, then 4—5 and 5—1 will give you the directions and magnitudes of reactions at the right and left supports respectively. The point 5 is the real pole and to which you can join the points of the other loads *a b*, *b c*, *c d*, *d e*......... *h k* with the rays. You can then draw the curve of equilibrium as shown.

The thrust at crown is not horizontal as the loads are not symmetrical and it will be parallel to the ray 5—*e* of fig (*a*), because *e* is the dividing point from the loads of left and right half of the arch.

When the directions of reactions are produced you will observe that these will intersect at 10 and 11 which are the points on the lines of action of the resultants of left and right half of the loads. 10—11 shows the direction of the thrust at crown. Bending moment, shear and thrust at any point in this arch may be determined as shown in the previous example, and for the right half of the arch vertical ordinate may be taken to determine the bending moment, but for the left half, radial ordinate is to be taken similar to the methods shown in figure 194.

DEAD LOAD DIAGRAM FOR THREE HINGED FRAMED ARCH.

Fig. 196 exhibits a usual three hinged framed arch with a span of 150′ and a rise of 70′. Spaced 30′ centre to centre. Roof covering consists of galvanized corrugated iron sheeting on braced steel purlins. Determine the reactions, and draw the curve of equilibrium and stress diagram. With the help of the curve of equilibrium determine the stress in the member CX.

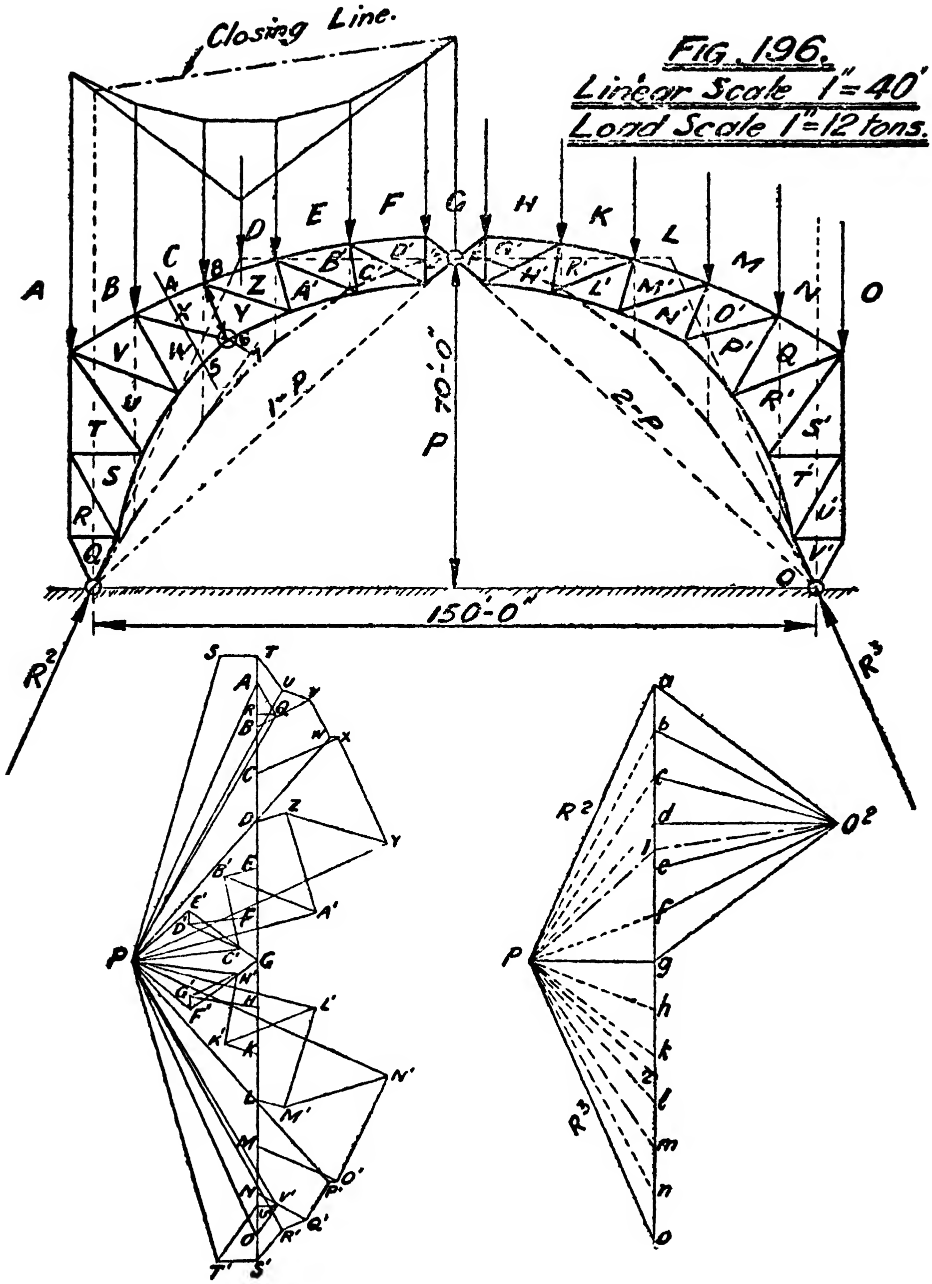
SOLUTION:—Spacing of arches is 30′. The dead load on the arch is composed of the weight of steel work in the arch, roof covering and the weights of purlins. The weight of the steel work in the arch per square foot of ground area covered in lbs. is equal to ·07 multiplied by the span in feet $= {\cdot}07 \times 150 = 10{\cdot}5$ lbs. The area covered by one arch is equal to the span multiplied by the spacing of the arch $= 150 \times 30 = 4500$ □′. Therefore the weight of the arch $= 4500 \times 10{\cdot}5 = 47250$ lbs. The weight of G. C. I. sheeting $= 4500 \times 3{\cdot}5 = 15{,}750$ lbs. where 3 5 lbs. is taken as the weight of G. C. I. sheeting per □′ of ground area covered.

The weight of braced purlins $= 4500 \times 4$ lbs $= 18{,}000$ lbs. 4 lbs is the weight per □′ of ground area covered.

Total load $= 47250 + 15750 + 18000 = 81{,}000$ lbs. Each panel load $=$ $\frac{81000}{12} = 6775$ lbs. or say 3 tons on each joint.

With dotted lines join the supporting hinges to the crown hinge as usual. Divide the total load into three loads to act on 3 hinges as explained in previons examples. Take only half of the arch with its load and connect them with a pole O^2 and draw the corresponding equilibrium polygon, and a closing line to touch the lines drawn parallel to the loads from the left and crown hinges. From pole O^2 draw a line parallel to this closing line and it gives you the point 1 in the load line. Then $1-g$ is the load on the crown hinge and $a-1$ is the load on the

left hinge. Similarly you get the same amount of loads on the crown and right hinge from the loads of right half of the arch, as the loads are symmetrical.



The total load on the crown hinge is equal to twice the load of $1-g$ and the load on the left and right hinges are each equal to $a-1$. In the diagram $a-1$, $1-2$, and $2-0$ represent the loads on left, crown and right hinges. From 1 and 2 in the load line draw lines $1-p$ and $2-p$ parallel to the dotted lines which join crown, left and right hinges.

The point p is then the real pole from which you can draw the curve of equilibrium as shown. $p-a$ and $o-p$ represent the directions and magnitudes of the reactions at left and right hinges respectively, and the stress diagram can be drawn as usual commencing from the left or right support as shown.

Note:—(I) In drawing stress diagram great difficulty is experienced in getting the last line to close well, a little error in parallelism at the beginning magnifies itself in the end to a very great extent specially in large arches like this. If the students desire to draw the stress diagram correctly, they are advised to draw the frame diagram to a very big linear scale and load scale may be selected at will.

(II) The curve of equilibrium may advantageously be made use of, in getting the stress of any individual member of this framed arch as follows:—Suppose you require to determine the stress in the member CX. Take a section 4—5 and remove the portion of the strutcture to the right of the section, take the moment centre at 6 the intersection point of the other two cut members. There are three forces to the left of the section the reaction PA, loads AB, BC; and the resultant of these three loads is PC. The line of the curve of equilibrium which is parallel to the resultant $c\ p$ is in the field C. Produce this line of the curve of equilibrium as shown to get the perpendicular distance $7-6$ about the moment centre, and the perpendicular distance of the member CX about the moment centre is $6-8$.

Hence the stress in $C\,X \times 6-8 = \text{force } c-p \times 7-6$.

$$\therefore\ C\,X \times 13 = 14{\cdot}6 \times 5.$$

$$\text{Stress in } C\,X = \frac{14{\cdot}6 \times 5}{13} = 5{\cdot}61 \text{ tons.}$$

This result exactly tallies with the result obtained from the stress diagram.

WIND LOAD DIAGRAM FOR THREE HINGED FRAMED ARCH.

Take the same framed arch of the previous example, calculate the wind pressure, draw the curve of equilibrium and stress diagram.

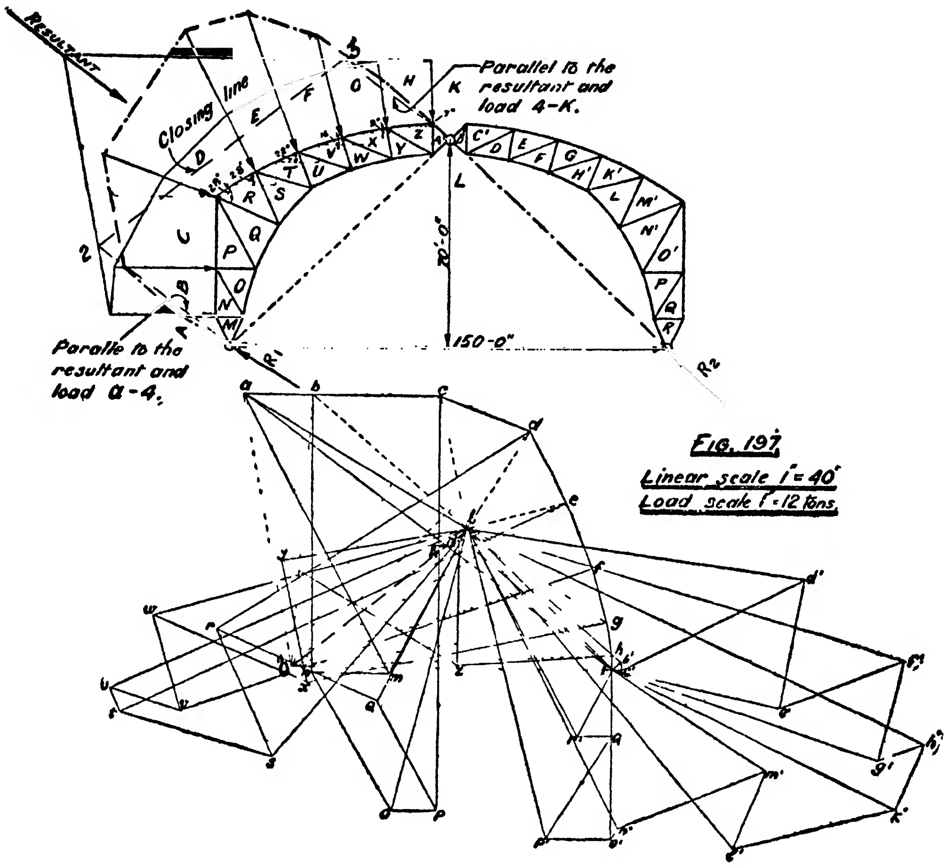


FIG. 197.
Linear scale 1" = 40'
Load scale 1" = 12 tons.

SOLUTION:—The spacing of the arches is 30′. Wind is supposed to act from the left side. The intensity of horizontal wind pressure per square foot of vertical surface is taken as 50 lbs Now commencing from the left hinge the horizontal wind pressure is calculated as follows:—The length of first panel is 18′ and half of this is to be taken for calculating the wind pressure, then the force AB = 9 × 30 × 50 lbs. = 13,500 lbs or 6·02 tons. The second panel length is 21 feet and force BC = (9 + 10·5) × 30 × 50 lbs = 29,250 lbs. or 13·06 tons. The force CD is the resultant of the loads which act on half the length of the panel in the field C and other half length of the panel in the field D = (10·5 × 30 × 50) + (8 × 30 × 38 lbs) = 15750 + 9120 lbs. = 7.03 tons + 4·07 tons. (The panel in the field D is inclined to an angle of 28° with the horizontal and as per Duchemin's formula $N = H \times \frac{2 \sin \theta}{1 + \sin^2 \theta} = 50 \times$

$\frac{2\times\cdot4694}{1+\cdot4694\times\cdot4694}=50\times\cdot7693=38$ lbs. Panel length = 16 feet; and distance centre to centre is 30′.) The resultant of the forces 7·03 and 4·07 tons is the force CD.

Further each panel length is 16 feet and the angle of inclination of each is different from one another as shown. The magnitudes of the forces DE, EF, FG, GH and HK as per Duchemins formula are 8·14, 7·07, 5·57, 3·21 and 1·28 tons respectively.

These loads are plotted separately in the load line and AK is the resultant of all the loads. The left half of the portion of the arch only is loaded and the total load that is the resultant, is to be divided proportionately on these two hinges *v i z* the left and crown hinges. The line of action of the resultant is obtained by means of polar diagram and funicular polygon as shown. The direction of the loads on these two hinges must be parallel to the resultant and the magnitudes of them are $a-4$ and $4-k$. These are obtained as follows.—From pole O″ a line O″-4 is drawn parallel to the closing line $2-3$, of the funicular polygon and thus dividing the resultant $a\ k$ into two parts $a-4$ and $4-k$ which represent the loads to be applied at the left and crown hinges respectively.

Now the whole arch is subjected to two loads $a-4$ and $4-k$ at the hinges and dotted lines are drawn from the supporting hinges to the crown hinge as shown. As we do in the ordinary cases of roof trusses we can go round the crown hinge and resolve the load $4-k$ parallel to the two dotted lines and get the triangle of forces $4-k-l$. Then $k-l$ is the magnitude and direction of the reaction at the right hinge and $l\ a$ the magnitude and direction of the reaction at the left hinge, and l is the real pole to which all the load points are to be connected by rays. From this real polar diagram the curve of equilibrium is to be drawn as shown. The stress diagram can then be drawn as usual. The stress diagram takes a lot of time and ultimately we may or may not succeed in getting the last line close well. The curve of equilibrium may be effectively be utilized in determining the stress of any member as explained previously, see figure 194 and 196.

Note;—Fig. 197 was drawn to the scales 1″=40′ and 1″=12 tons and ultimately the figure was reduced to suit the size of the page of this book. Thescales for fig. 197 are now 1″=65·22′ and 1″=21·36 tons approximately.

CHAPTER XI.

SUSPENSION BRIDGES.

A suspension bridge consists of two cables with vertical rods which carry the platform that supports the road way, pass over towers built on both the ends of the bridge. See the picture view shown in figure 198. The road way is suspended by series of vertical rods and these rods in turn are attached to the main cables. These vertical rods and cables are subjected to tensile stresses which are the greatest advantages in bridge work and hence the bridge is cheaper than ordinary braced girders whose members are subjected to both the compressive and tensile stresses which require more metal and weight. The main disadvantage in this suspension bridge is that this cannot be subjected to heavy traffic such as Railway etc.

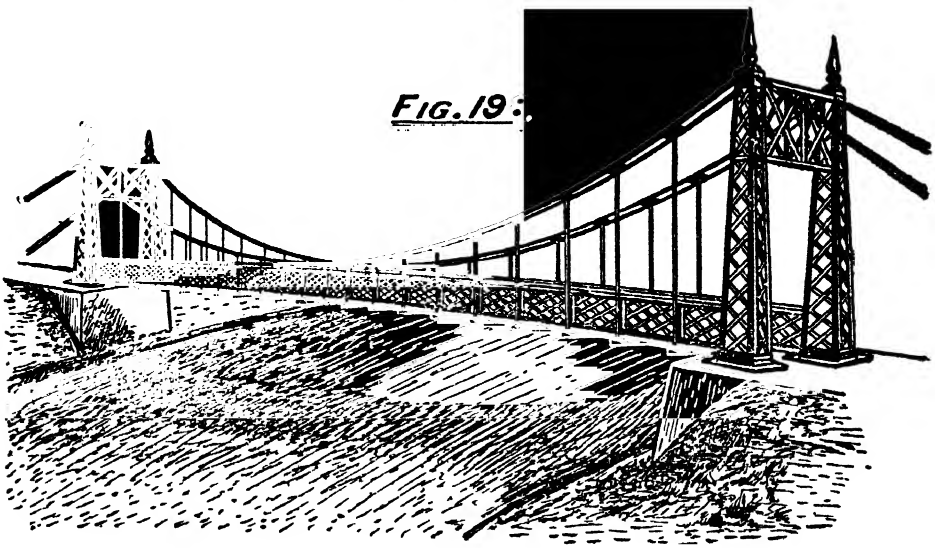
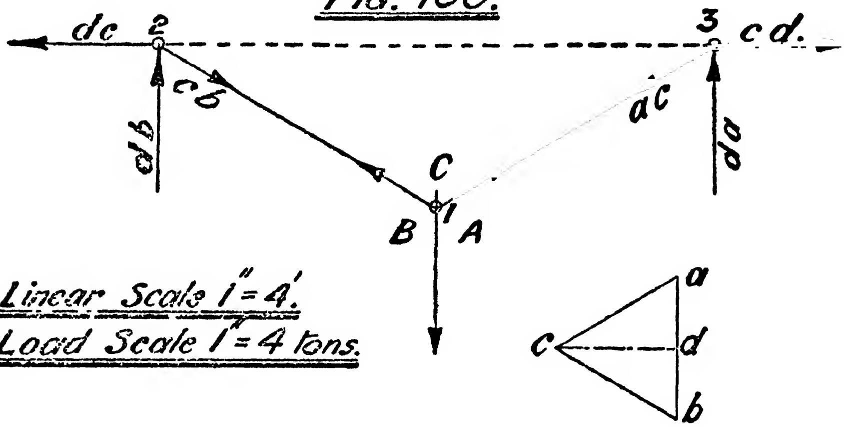


FIG. 19

The theory of suspension is as follows.—To start with take a strong rope or chain and suspend a load AB to it as shown in fig. 199. The point 1 where the load is suspended is kept in equilibrium by three forces namely, the load AB and the tensions in the two portions of the string. Consequently a force triangle *a b c* may be drawn as in fig. 199. The points 2 and 3 are also in equilibrium by a set of three forces at each point and the magnitudes of these forces may be determined by

connecting points 2 and 3 by a straight line usually known as a closing line which is very familiar to the students at this stage. Then from point *c* in the force triangle a line *c d* is drawn parallel to the closing line 2—3 and it is intersected at the point *d* on the load line ***a b***.

FIG. 199.



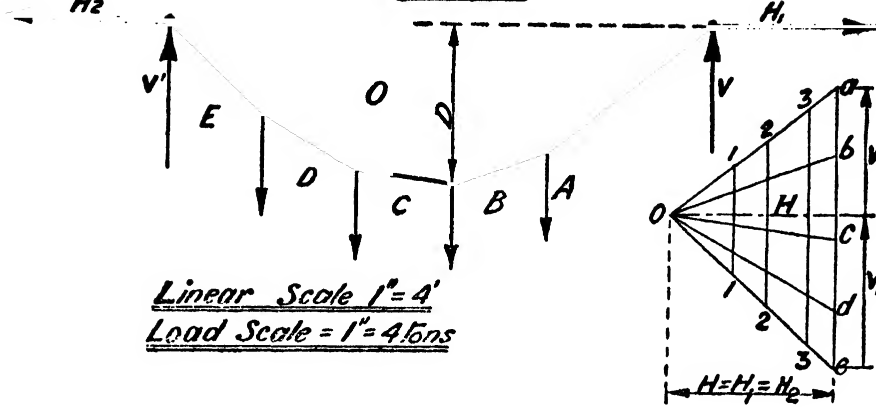
Linear Scale 1" = 4'.
Load Scale 1" = 4 tons.

In this figure three points are in equilibrium and there must be three force triangles, and these are as follows:—The force triangle *a b c* is for the point 1; *b c d* for the point 2 and *a c d* for the point 3. The rays *a c* and *c b* represent the greatest terminal tensions in the chain. The sum of all the horigontal components is equal to zero and sum of all the vertical components is also equal to zero. Hence the system is in equilibrium.

Similarly by hanging series of loads to the chain we have as many force triangles as there are loads on it as shown in figure 200. Since the system is in equilibrium all the sides of the force triangles should converge to the common point O which is called the pole, and the diagram is ealled the polar diagram or the force polygon. The rays *a o* and *e o* will represent the terminal tensions of the chain, and the pole distance H is equal to the horigontal component of the tensions of all the portions of the chain. The chain being flexible and when the loads are suspended, it will take the form of a curve.

By this we find that if any system of loads is known the form in which the chain must hang may be determined or conversely if the form of the chain is known the system of loads may be determined. In any case magnitude of one of the loads must be known.

FIG. 200.



CASE I.

Suppose the form of the chain is known, see figure 200 and the loads are not known; then by selecting any point O, and from it if rays parallel to the segments of the chain are drawn indefinitely we can determine the magnitudes of the system of loading by drawing a vertical line to intersect the rays drawn from pole O. In this way we can draw any number of vertical straight lines intersecting the rays as shown at 1—1, 2—2 and 3—3. All the loads will proportionately be intersected by the rays, and hence the magnitudes of the loads depend on the distance from the pole or the horigonal stress of the terminal tensions of the chain we propose to give. Hence it is necessary to know beforehand the magnitude of one of the loads or the pole distance which is equal to the horigontal stress H of the terminal tensions.

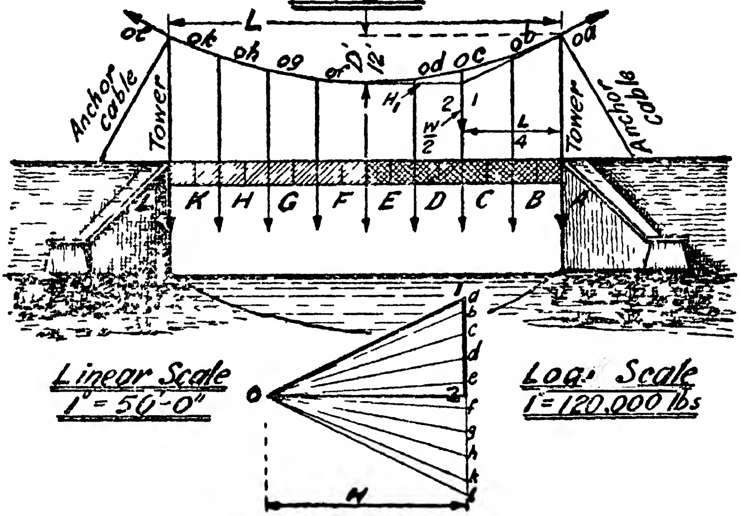
CASE II.

Suppose the system of loads to hang from the chain is known, from this we can have very many forms to the chain by selecting the polar distances at our will. To fix the form of the chain due consideration must be paid to the terminal tensions of the chain or to the dip or sag of the cable or chain. In this way the from may be fixed up. If we like the form of the chain to pass through three given points we can draw very effectively as explained in figures from 17 to 21 of part I.

EXAMPLE 1: —A suspension bridge has a span of 100 feet, width of roadway 12′. The total uniformly distributed load is 200 lbs.

per square foot. Determine the maximum tensions in the cables and draw the parabola of the bridge (See fig. 201.)

FIG. 201.



SOLUTION:—Width of road way 12 feet and span 100 feet. Weight 200 lbs. per square foot. Therefore the total weight on the two cables = $100 \times 12 \times 200$ lbs. = 240,000. On one cable $\frac{240,000}{2}$ = 120,000 lbs. The maximum tension and the horizontal pull can be determined by taking only one half of the cable. The load on one half = 60,000 lbs acting at the centre of gravity of that length as shown. This one half of the cable is kept in equilibrium by three forces *viz.* the horizontal pull H at the greatest dip, the load on one half of the cable 60,000 lbs which is represented by 1—2 acting at $\frac{L}{4}$ and, the terminal tension. Drawing the triangle of forces for these three loads we get the triangle 1—2—0 shown in thick lines.

Here 1—2 represents $\frac{W}{2}$, 0—1 and 0—2 will represent the terminal tension and the horizontal pull of the cable respectively.

The drawing of the parabola of the bridge is as follows:—The platform weight per foot run of the bridge for one cable = 120,000 ÷ 100 = 1200 lbs. Now dividing the span into any number of equal parts say into 8 as shown here, and the panel load will be equal to 12·5 × 1200 = 15,000 lbs. The first and the last load are each equal to 15000 ÷ 2 = 7500 lbs. Filling up those panel loads on the load line as shown

the equilibrium polygon may be drawn commencing from the tower and this will exactly give us the required depth 12′ in the centre.

Note:—The load on the suspension bridge is usually a uniformly distributed one, and with the help of bending moment diagram we can easily determine the horizontal pull of the cable which is always equal to the polar distance H. In the above example dip of the cable is given and is equal to 12′. Assuming the loads are acting on the beam of span L we have for a distributed load the maximum bending moment$=\frac{WL}{8}=$Ordinate$\times$Pole distance H; where W=total load= 120,000 lbs; L=span in feet=100′; and ordinate=12′ as given above. Substituting these values in the equation we have $\frac{120,000\times 100}{8}=12\times H$.

$\therefore H=\frac{120,000\times 100}{8\times 12}=125,000$ lbs. In the graphic solution we got the same result by taking only half the total load on one cable and this, also may be verified algebraically as follows. (See fig. 201.) Half the total load is $\frac{W}{2}$ and the stress at the lowest point of the cable is H. Taking the moment centre at the right support of the cable we have $\frac{W}{2}\times\frac{L}{4}=$ $H\times D$. $\therefore H=\frac{W}{2}\times\frac{L}{4}\times\frac{1}{D}=\frac{WL}{8D}$ as before, and gives you the same result as 125,000 lbs. Ans.

The value of the maximum tension O 1 or O l can be found from fig. 202 as follows:—

$$O\,1^2=(1-2)^2+(O\,2)^2 \text{ where } 1-2=\frac{W}{2} \text{ and } O\,2=H.$$

$$O\,1^2=\left(\frac{W}{2}\right)^2+H^2 \text{ but } H=\frac{WL}{8D}.$$

Substituting the value of H in the above equation we have—

$$O\,1^2=\left(\frac{W}{2}\right)^2+\left(\frac{WL}{8D}\right)^2=\frac{W^2}{4}+\frac{W^2L^2}{64D^2}$$

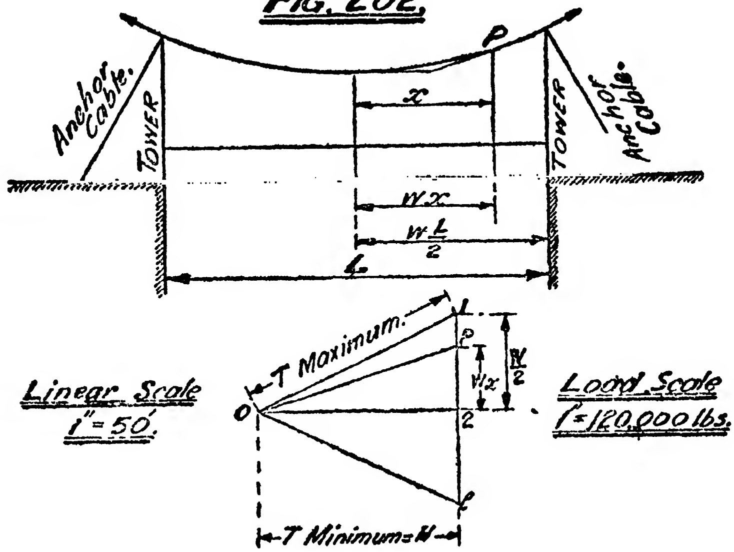
$$O\,1^2=\frac{W^2}{4}\left(1+\frac{L^2}{16D^2}\right)$$

$$\therefore O\,1=\frac{W}{2}\sqrt{\left(1+\frac{L^2}{16D^2}\right)}$$

$$T\,Max: \text{ or } O\,1=\frac{W}{2}\sqrt{\left(1+\frac{L^2}{16D^2}\right)}$$

Again to determine the tension at any point in the chain say at P a tangent line is to be drawn at P, and from pole O a line O P is drawn parallel to the tangent line drawn at P. Now $O\,P^2$ in the right angled triangle $= P\,2^2 + O\,2^2$.

FIG. 202.



Again $1-2 = W \times \frac{L}{2}$ and $P2 = w \times x$, where w = load per foot run; L = span length and x = the distance from the point P to the centre of the bridge.

$$\text{Now } \frac{P\,2}{1-2} = \frac{w\,x}{w\,\frac{L}{2}} = \frac{x}{\frac{L}{2}}$$

$$\therefore\ P\,2 = \frac{x}{\frac{L}{2}} \times 1-2 = \frac{(\,2 \times 1-2\,) \times x}{L} = \frac{W\,x}{L}$$

But $2\,(\,1-2\,) = W$ = Total load on the bridge.

$$\therefore\ P\,O^2 = \left(\frac{W\,x}{L}\right)^2 + H^2.$$

$$P\,O = \sqrt{H^2 + \left(\frac{W\,x}{L}\right)^2}$$

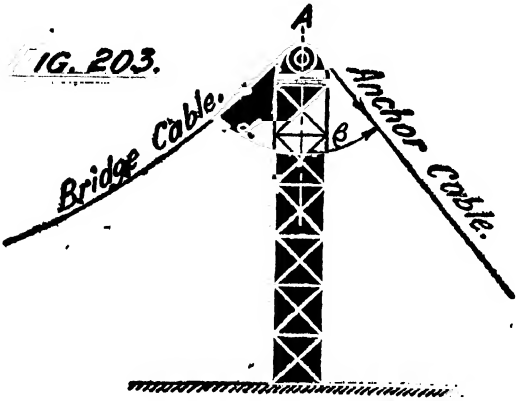
Hence the tension at any point $P = P\,O = \sqrt{H^2 + \left(\frac{W\,x}{L}\right)^2}$.

LENGTH OF CABLE.—The length of the cable may be determined approximately by the following formula—

$S = L + \frac{8}{3} \times \frac{D^2}{L}$, where S = length of the cable; L = Span in feet; D = dip of the cable.

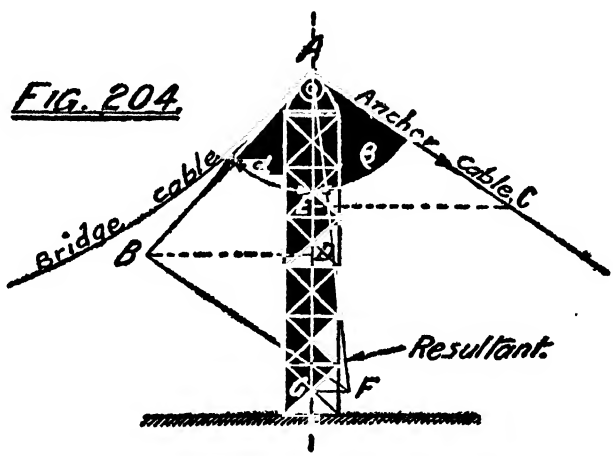
METHOD OF ATTACHING ENDS OF CABLES OR ANCHORAGE.

FIG. 203.



A few important methods are discussed here. First the cable passes over a pulley which is fixed at A on the tower, (see figure 203). In this case the magnitude of the tension in the anchor cable will be always equal to the terminal tension of the bridge cable. If α is equal to β the horizontal components of tensions of both the cables are equal to each other and the line of action of the resultant of these two tensions coincides with the centre line of the tower and hence there will not be any overturning force on the tower.

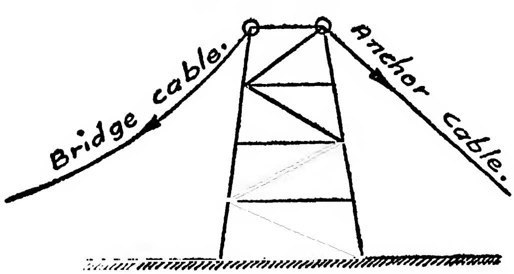
FIG. 204.



If α is not equal to β the resultant of these two pulls will not be vertical and therefore the tower is to resist the horizontal force which is equal to the difference of horizontal components of these two pulls. If the tower were very high the over turning moment will be great (See fig. 204). Here α is not equal to β. The tension AB is equal to AC, but the horizontal component BD is less than the horizontal component of EC; and the difference of these two is equal to GF. The vertical pressure on the tower is equal to AE + AD = AG.

SECOND METHOD.

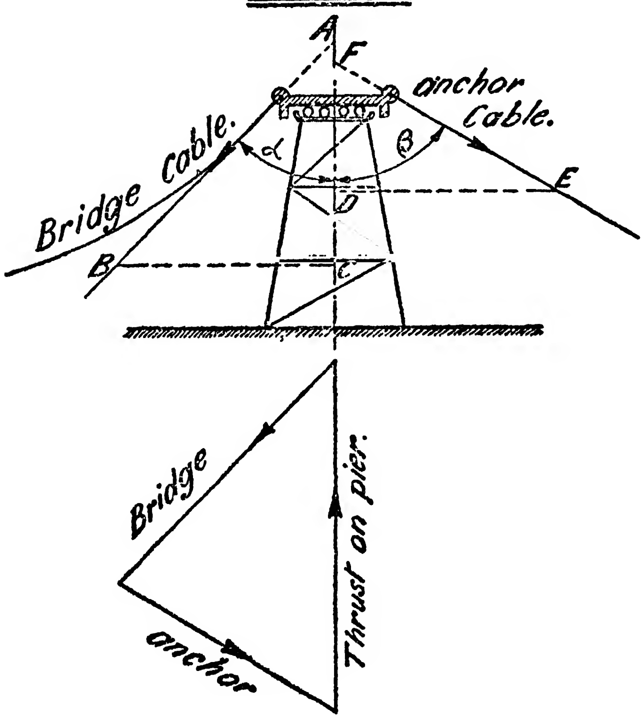
FIG. 205.



In this method the bridge and anchor cables are attached separately to the pulleys which are fixed on the tower as shown in the figure 205. Here the axact calculation is impossible. The tensions in the cables depend on the rigidity of the tower.

THIRD METHOD.

FIG. 206.



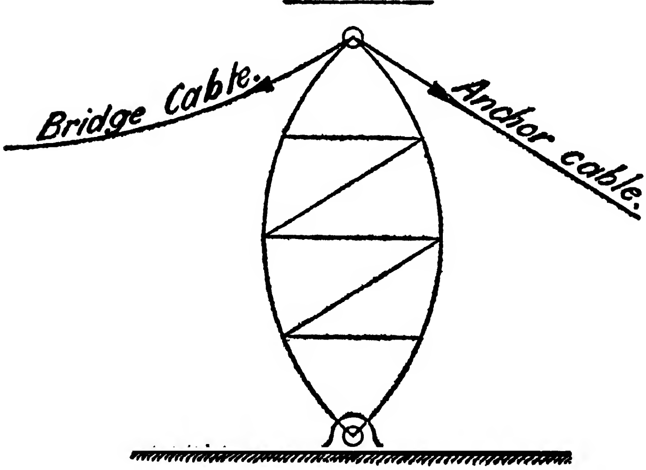
Here the cables are attached to the saddle on roller bearing on top of the tower. There is no friction on top of the towor and the reaction is always vertical. (See fig. 206.) Since the reaction is vertical there is no horizontal force on the tower and this necessitates the horizontal components of the tensions of both the cables to be equal. If α is equal to β the tension in the bridge cable will

always be equal to the tension in the anchor cable, and if the inclinations are not equal, then there will be a difference in the magnitudes of tensions and still the horizonal components of both the tensions must be equal to each other. For example let AB denote the magnitude of the tension in the bridge cable and BC is the horizontal component of it. Then it follows that the horizontal component of the tension of the anchor cable must be equal to BC. Therefore take DE equal to BC and hence EF must represent the magnitude of the tension in the anchor cable.

Note:—The saddle is mounted on roller bearings and the variation of the tensions in the bridge and anchor cable produces the horizontal stress on the tower, and this horizontal stress is to be transmitted through the saddle; but the saddle itself yeilds without transmitting it to the tower and hence the reaction on the tower is always vertical.

FOURTH METHOD.

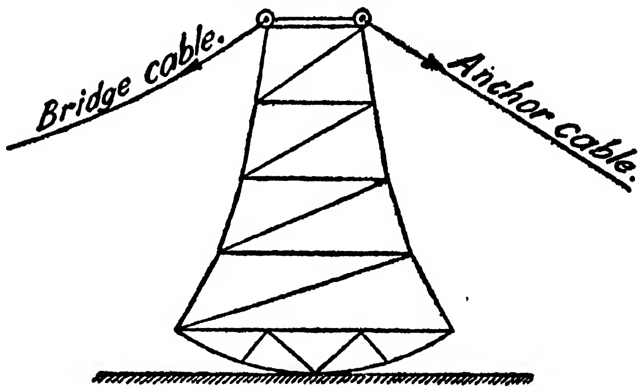
FIG. 207.



In this the tower itself is hinged at the bottom as shown in the figure 207, and the cables are tied to the saddle which is fixed on top of the tower. In this case also the reaction is always vertical, because the tower as a whole yeilds towards the greater pull and thus avoids the horizontal stress caused by the difference of stresses in the cables.

FIFTH METHOD.

FIG. 208.



The bottom of the tower is attached to a rocker bearing as shown in figure 208, and this is more effective than any other methods shown above, but this is usually adopted for very large spans. The weight of the tower itself is enormous and therefore the bottom of the tower is simply resting on the steel bed plate. Any difference in the magnitudes of the tensions in the cables causes the tower to have a gentle motion in the vertical plane. The reaction is always vertical. The tension in the cables in these two latter methods may be determined in the same way as found out in the third method.

SUSPENSION BRIDGE WITH PIN JOINTED STIFFENING GIRDERS.

When the suspension bridge is subjected to moving loads or unsymmetrical dead loads the form taken up by the bridge cables will alter, and when the train with locomotive is allowed to pass over the bridge the alteration of the cables will be more, and oscillations are then set up owing to the impact effect of the engine and train loads. In order to lessen these disturbances some sort of braced girders are generally be used to stiffen the suspension bridges. Owing to these inconveniences, suspension bridges are rarely used for Railways. If the girders are pin jointed at the centre and used for stiffening the suspension bridge, these bridges become actually inverted three hinged arches and the stresses in them may be ascertained in the same way as we did in three hinged arches previously. If the stiffening girders are not pin jointed in the centre, the stresses are more difficult to determine as these become inverted two hinged arches, and consequently

we deal with the stiffening girders that have pin jointed at the centre. We will show now how stiffening girders help the chain to retain its parabolic form under all conditions of loading.

UNIFORMLY DISTRIBUTED LOAD OVER HALF THE SPAN.

EXAMPLE:—Figure 209 shows a Suspension Bridge of 110 feet effective span and a dip of 28′ with hinged stiffening girders. The dead weight of the bridge is assumed to be 1 ton per running foot. This bridge is further loaded with a uniformly distributed load of 2 tons per foot run to cover only one half of the span. This is considered to be the worst position of the loading.

First taking only the dead load of the bridge the polar diagram figure (*a*) is drawn. The determination of the pole O is as per fig. 201 page 152. From this the parabola of the bridge is drawn. The same parabola is taken in fig. (*c*) for the purpose of determining the bending moment in the cable for the distributed load covering half of the span. The determination of pole O′ of fig. (*b*) is as per note of figure 191 page 135. From this polar diagram (*b*), the bending moment diagrams shown shaded are drawn. Here $H_2 = H_3 = H_1 = 27$ tons and tower reaction $R = s\ a' = 41$; tower reaction $R = s\ a' = 41$; tower reaction $R_1 = f'\ s = 14$.

The bending moment at any point P on the cable is equal to the intercept PQ multiplied by pole distance H_1. (See fig. 188.) for proof. These two shaded curves are the bending moment diagrams acting on the girders as well. From the appearance of these two curves we see that the greatest ordinates are at the mid points of these curves. On measuring the mid ordinates TT′ and VV′ to the linear scale we get 7 feet, and pole distance H_1 in load scale is 27 tons : then the maximum bending moment at each point is equal to $27 \times 7 = 189$ tons feet.

From this we understand that the bending moment curves for both the girders are same. The load that covers half the span is 1 ton per foot run, therefore the left girder pulls down on the cable with an intensity of 1 ton per foot run and the right girder is being pulled up by the cable with an intensity of 1 ton per foot run. The reason is, that the ends of these stiffening girders rest on the abutments freely, and are prevented from lifting upwards, but are allowed to move horizontally to a very limited distance, that is, ⅛ inch to every 10 feet of the span length. Therefore the girders distribute the load uniformly

upon the cables and hence these cables retain their original parabolic form under all possible conditions of loading.

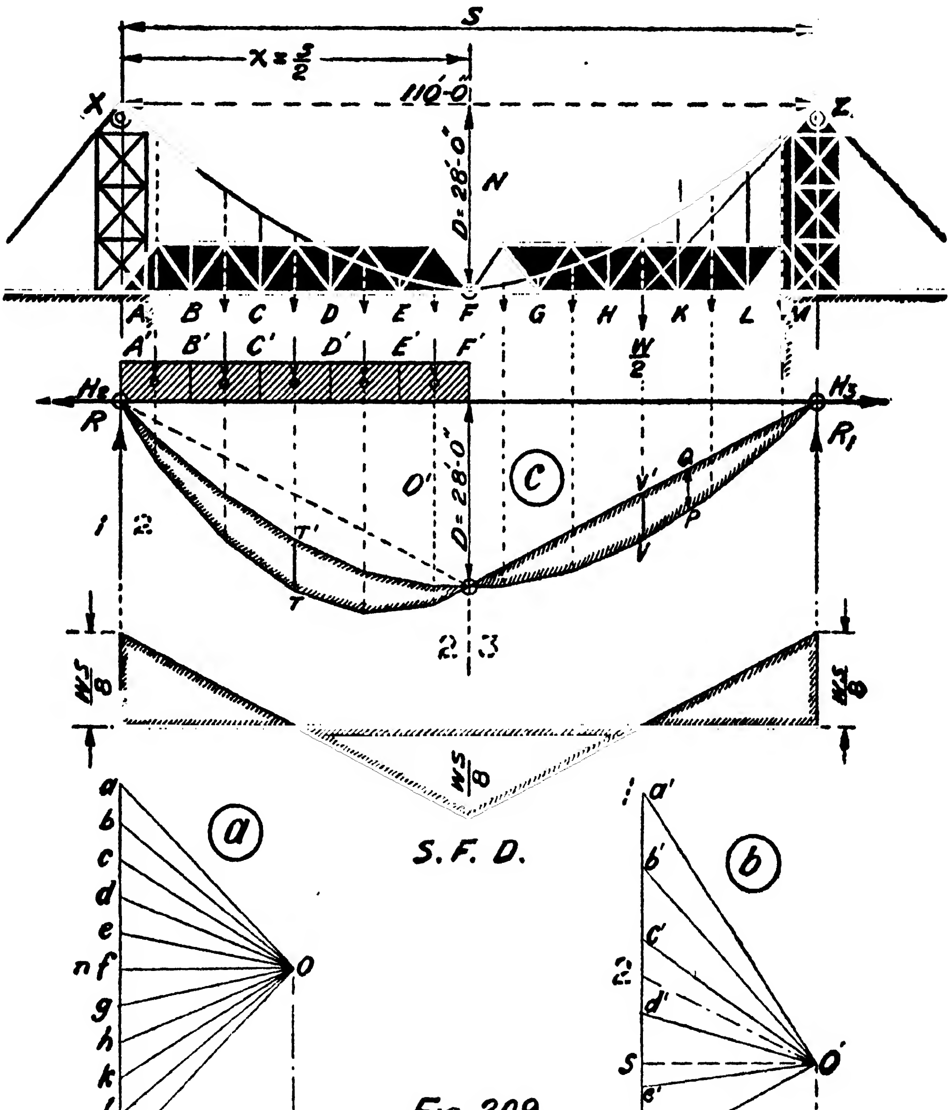


FIG. 209.
Linear scale 1" = 30'
Load scale 1" = 30 tons.

Shearing Force Diagram.—It is possible to draw the shearing force diagram graphically, but it is laborious for the reason that the uniformly distributed load extending over half of the span acts downwards and the upward pull of the suspension rods in the same portion

of the span acts upwards. These two loads acting in the same portion of the bridge act in opposite directions and all these to be plotted in one load line and try the patience of the students.

Therefore the shearing force is to be calculated at the supports and at the central hinge and can be plotted to load scale in a very short time.

Let x = the distance to which the distributed load extends.

w = load per foot run in tons.

s = effective span from centre to centre of bearing in feet.

Take the moment centre at the central hinge, then the resultant of the load w extending over a distance x is $= w\,x$ and its moment about the central hinge $= w\,x\left(\frac{s}{2} - \frac{x}{2}\right) = w\,x\,\frac{s-x}{2}$.

Again the uniform downward and upward pull per foot run due to suspension rods on the left and also on the right half of the bridge $= \frac{w}{2}$ and its resultant for one half of the bridge $= \frac{w}{2} \times \frac{s}{2} = \frac{w\,s}{4}$. Its moment about the central hinge $= \frac{w\,s}{4} \times \frac{s}{4} = \frac{w\,s^2}{16}$.

The equation of the moment on the left is therefore.—

$$R \times \frac{s}{2} + \frac{w\,s^2}{16} - w\,x\left(\frac{s-x}{2}\right) = 0.$$

$$\therefore\ R = \left\{ w\,x\left(\frac{s-x}{2}\right) - \frac{w\,s^2}{16} \right\} \frac{2}{s}$$

$$= w\,x\left(\frac{s-x}{2}\right) \times \frac{2}{s} - \frac{w\,s^2}{16} \times \frac{2}{s}$$

$$R = w\,x\left(\frac{s-x}{s}\right) - \frac{w\,s}{8} \quad \text{..........(1)}$$

On the right.—

$$R_1 \times \frac{s}{2} - \frac{w\,s^2}{16} = 0.$$

$$\therefore\ R_1 = \frac{w\,s^2}{16} \times \frac{2}{s} = \frac{w\,s}{8} \quad \text{..........(2)}$$

Now when $x = \frac{s}{2}$, substituting the value of x in (1) we have

$$R=\frac{w\frac{s}{2}\left(s-\frac{s}{2}\right)}{s}-\frac{w\,s}{8}=\frac{\frac{w\,s^2}{2}-\frac{w\,s^2}{4}}{s}-\frac{w\,s}{8}$$

$$=\frac{w\,s^2}{4\,s}-\frac{w\,s}{8}=\frac{w\,s}{4}-\frac{w\,s}{8}=\frac{2\,w\,s-ws}{8}=\frac{w\,s}{8}.$$

Again let R_c = Reaction at the central hinge. For $x=\frac{s}{2}$. Taking moment centre at the left supporting hinge we have.—

$$R_c\times\frac{s}{2}-\left(\frac{w}{2}\times\frac{s}{2}\right)\frac{s}{4}+\left(w\times\frac{s}{2}\right)\frac{s}{4}=0.$$

$$\therefore\ R_c=\left(\frac{w\,s^2}{16}-\frac{w\,s^2}{8}\right)\frac{2}{s}=\frac{w\,s^2}{16}\times\frac{2}{s}-\frac{w\,s^2}{8}\times\frac{2}{s}.$$

$$R_c=\frac{w\,s}{8}-\frac{w\,s}{4}=-\frac{w\,s}{8}.\quad\therefore\ R_c=-\frac{w\,s}{8}.$$

Note:—These values are plotted in the fig. 209 to the load scale and may be followed very easily. $\frac{w\,s}{8}=\frac{1\times110}{8}=13\ 75$ tons.

To prove that our graphical results are correct let us take the figure 209 (c) separately in figure 210 and calculate analytically as follows. Load covers only half of the span of 1 ton per foot run. Let S = span; D = dip of the cables, R = left reaction, R_1 = right reaction, $H_1=H_2=H_3$ = horizontal components of the pull in the cables. Taking the right hinge as moment centre $R\times S=W\times\frac{S}{2}\times\frac{3}{4}S.$

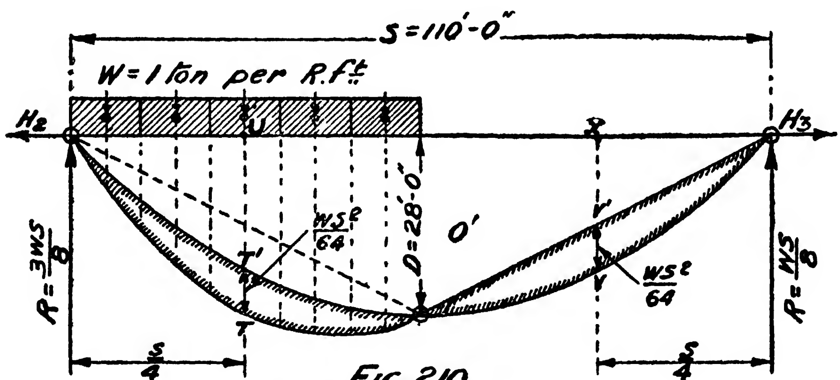


FIG. 210.
Linear scale 1"=30'

$$\therefore\ R=W\times\frac{S}{2}\times\frac{3}{4}S\times\frac{1}{S}=\frac{3\,W\,S}{8}=\frac{3\times1\times110}{8}=41{\cdot}25\text{ tons. Again}$$

taking left hinge as moment centre $R_1 \times S = W \times \frac{S}{2} \times \frac{S}{4}$. $\therefore R_1 = W \times \frac{S}{2} \times \frac{S}{4} \times \frac{1}{S} = \frac{W\,S}{8} = \frac{1 \times 110}{8} = 13{\cdot}75$. The value of H_1 may be determined by taking the centre hinge as moment centre and we know the bending moment at this hinge is zero.

The bending moment at the centre hinge is equal to $\frac{3\,W\,S}{8} \times \frac{S}{2} - W \times \frac{S}{2} \times \frac{S}{4} - H_2 \times D = \frac{W\,S^2}{16} - H_2\,D$ and this must be equal to zero. $\therefore \frac{W\,S^2}{16} = H_2 \times D$. or $H_2 = \frac{W\,S^2}{16\,D} = \frac{1 \times 110 \times 110}{16 \times 28} = 27{\cdot}008$ or say 27 tons as previously got by graphical method.

Bending moment at $T = \left(\frac{3\,W\,S}{8} \times \frac{S}{4}\right) - \left(W \times \frac{S}{4} \times \frac{S}{2}\right) - (H_2 \times UT')$. Here the ordinate UT′ measures to the linear scale 21 feet, which is exactly $\frac{3}{4}$ of D, where D = 28 feet. Substituting the values of H and UT′ we have bending moment at T is equal to $\left(\frac{3\,W\,S}{8} \times \frac{S}{4}\right) - \left(\frac{W\,S}{4} \times \frac{S}{8}\right) - \left(\frac{W\,S^2}{16\,D} \times \frac{3}{4}D\right) = \frac{3\,W\,S^2}{32} - \frac{W\,S^2}{32} - \frac{3\,W\,S^2}{64} = \frac{W\,S^2}{64}$. Substituting the numerical values we get $\frac{W\,S^2}{64} = \frac{1 \times 110 \times 110}{64} = \frac{12100}{64} = 189{\cdot}06$ tons feet and this is exactly the same we got by the graphical method.

Again bending moment at $V = \left(\frac{W\,S}{8} \times \frac{S}{4}\right) - (H_2 \times XV)$. The ordinate XV measures to the linear scale 21′ which is exactly $\frac{3}{4}$ of D. Substituting the values of H_2 and XV we get the bending moment at $V = \left(\frac{W\,S}{8} \times \frac{S}{4}\right) - \left(\frac{W\,S^2}{16\,D} \times \frac{3}{4}D\right) = \frac{W\,S^2}{32} - \frac{3\,W\,S^2}{64} = -\frac{W\,S^2}{64}$, this is the same as before.

UNIFORMLY DISTRIBUTED LOAD OVER THE WHOLE SPAN.

In this case the bending moment curve exactly coincides with the parabola of the cables and hence no bending moment; and the whole behaving as an ordinary suspension bridge shown in figure 201.

SINGLE CONCENTRATED LOAD ROLLING OVER A STIFFENED SUSPENSION BRIDGE.

The same parabola of the bridge of figure 209 is reproduced in figure 211. A′B′ is a single concentrated load rolling over this stiffened bridge AGB. Five points have been taken at regular intervals on the left half of the span. The bending moment diagram for the first position of the load is ACB and the corresponding polar diagram is *a′ b′ c* with the polar distance H_1. (See fig. 190 page. 133.) The magnitude of the bending moment is equal to the intercept *c k* multiplied by the pole distance H_1. (See figure 188 page. 129.) Similarly the bending moment at points 2, 3, 4 and 5 are respectively $DL \times H_2$; $EM \times H_3$; $FN \times H_4$ and zero. When the load is rolling on the left half of the span the values of the bending moment as well as the pole distances varying, and you observe the bending moment diagram on the right half of the span is retaining its parabolic form; but when the load arrives at the 5th point or the centre of the bridge the bending moment in both the halves are of equal value; but the maximum bending moment for this position of the load in these two halves occurs at quarter of the span and the value of this is equal to the intercept PQ multiplied by the pole distance $H_5 = 6{\cdot}9 \times 1 = 6{\cdot}90$ tons feet. Now the polar distance has attained its maximum value.

Again substituting the numerical values for the intercepts and polar distances the following values of the bending moment at 5 points in the left of the bridge are obtained.

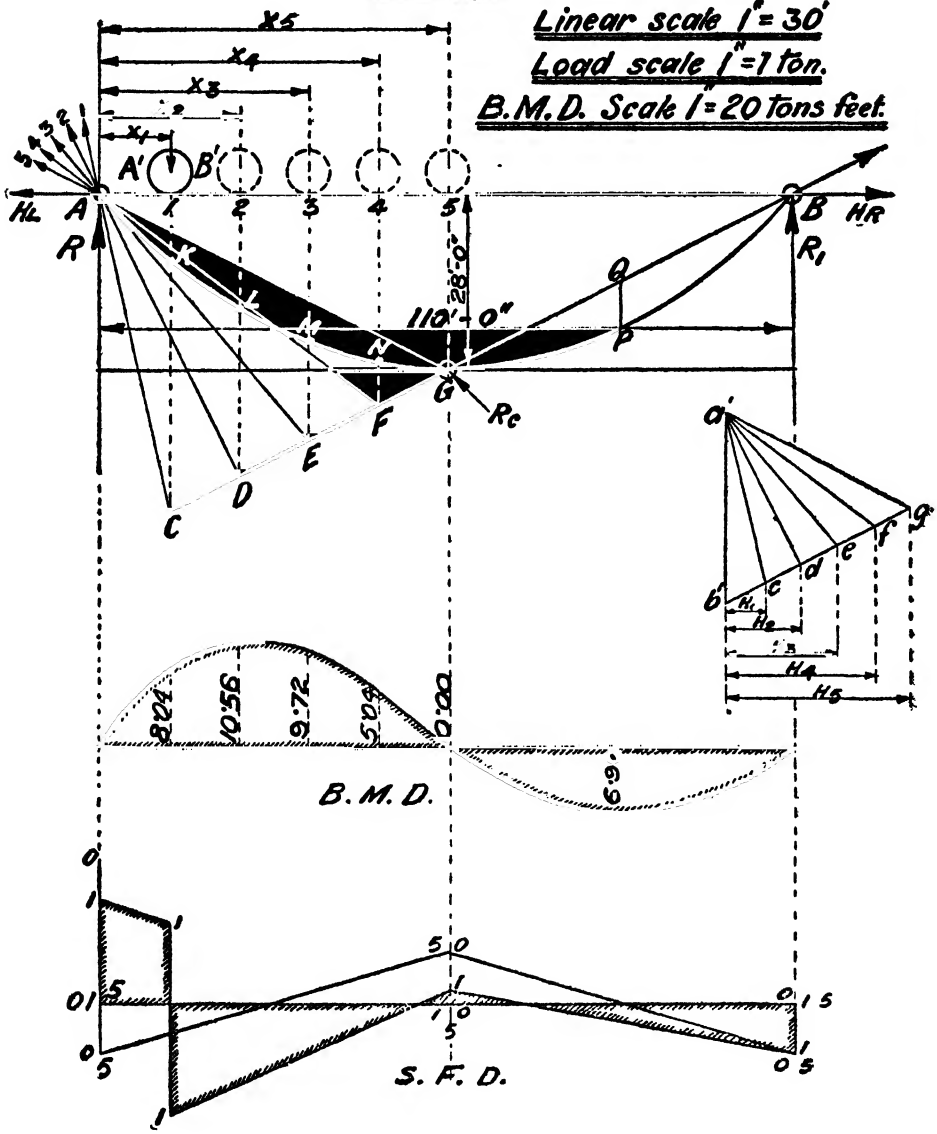
B. M. at $1 = CK \times H_1 = 40{\cdot}2 \times {\cdot}2 = 8{\cdot}04$ tons feet.
,, ,, $2 = DL \times H_2 = 26{\cdot}4 \times {\cdot}4 = 10\ 56$,, ,,
,, ,, $3 = EM \times H_3 = 16{\cdot}2 \times {\cdot}6 = 9{\cdot}72$,, ,,
,, ,, $4 = FN \times H_4 = 6{\cdot}3 \times {\cdot}8 = 5{\cdot}04$,, ,,
,, ,, $5 = 0 \times H_5 = 0 \times 1 = 0{\cdot}00$,, ,,

For this position of the load the B. M. in the right half $= PQ \times H_5 = 6{\cdot}9 \times 1 = 6{\cdot}90$ tons feet.

The above values are plotted separately to a scale of 1″ to 20 tons feet at the bottom of the figure 211. From the above results and as from the bending moment diagram we observe that the maximum bending moment in the left half of the bridge occurs when the load is at 2; and on further observing minutely the maximum bending moment really occurs a little bit towards the right of the point 2; the difference is very little and hence we can safely take the maximum bending moment

as 10·56 or say 10·6 tons feet. From this we can also conclude that the maximum bending moment occurs when the load is ·2 S from A or B; or ·3 S from the centre of the bridge, where S is the span in feet.

FIG. 211.



Shearing force diagram.—Let x_1, x_2, x_3, x_4, and x_5, be the distances in feet from the left abutment to the positions of the moving

load W. S=the span in feet. W=the concentrated rolling load over the span.

The uniform downward and upward pull per foot run due to suspension rods on the left and also on the right half of the bridge $=\frac{W}{S}$ and its resultant for one half of the bridge $=\frac{W}{S}\times\frac{S}{2}=\frac{W}{2}$.

Its moment about the central hinge $=\frac{W}{2}\times\frac{S}{4}=\frac{W\,S}{8}$, (left half).

The moment of the rolling load W about the central hinge $=W\left(\frac{S}{2}-x\right)$

The equation of the moment for the first position of the load on the left is therefore $R\times\frac{S}{2}-W\left(\frac{S}{2}-x_1\right)+\frac{W\,S}{8}=0$.

$$R=\left\{W\left(\frac{S}{2}-x_1\right)-\frac{W\,S}{8}\right\}\frac{2}{S}=W\left(\frac{S}{2}-x_1\right)\frac{2}{S}-\frac{W\,S}{8}\times\frac{2}{S}$$

$$=\left(\frac{W\,S}{2}-W\,x_1\right)\frac{2}{S}-\frac{W}{4}=W-\frac{W\,x_1\,2}{S}-\frac{W}{4}$$

$$\therefore\ R=W\left(1-\frac{2\,x_1}{S}-\frac{1}{4}\right)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

On the right taking the moment centre at the central hinge we have:—

$$R_1\times\frac{S}{2}-\frac{W}{2}\times\frac{S}{4}=0.\quad\therefore\ R_1=\frac{W\,S}{8}\times\frac{2}{S}=\frac{W}{4}\ \ldots\ldots\ldots\ldots(2)$$

On the central hinge Rc, taking moment centre at the left support we have:—

$$Rc\times\frac{S}{2}-\frac{W}{2}\times\frac{S}{4}+W\times x_1=0.\quad\therefore\ Rc=\left(\frac{W\,S}{8}-W\,x_1\right)\frac{2}{S}=\frac{W}{4}-\frac{2\,W\,x_1}{S}.\quad\therefore\ Rc=W\left(\frac{1}{4}-\frac{2\,x_1}{S}\right)\ \ldots\ldots\ldots\ldots(3)$$

Substituting numerical values to above formulæ we get the following magnitudes of the shearing force for different positions of the moving load. Here W=1 ton. When $x=0$ the load will be exactly on the left support.—

$$R=W\left(1-\frac{2\,x}{S}-\frac{1}{4}\right)=1\left(1-\frac{2\times0}{110}-\frac{1}{4}\right)=1-0-\frac{1}{4}=\frac{3}{4}\text{ ton.}$$

$$R_1=\frac{W}{4}=\frac{1}{4}\text{ ton.}\quad Rc=W\left(\frac{1}{4}-\frac{2\,x}{S}\right)=1\left(\frac{1}{4}-\frac{2\times0}{110}\right)=\frac{1}{4}\text{ ton.}$$

Now when W is at a distance of x_1 that is at 11 feet from the left abutment, we get the left reaction.

$$R=W\left(1-\frac{2x_1}{S}-\frac{1}{4}\right)=1\left(1-\frac{2\times 11}{110}-\frac{1}{4}\right)=\frac{11}{20}\text{ tons}=\cdot 55\text{ ton.}$$

$$R_1=\frac{W}{4}=\frac{1}{4}\text{ ton.}$$

$$Rc=W\left(\frac{1}{4}-\frac{2x_1}{S}\right)=1\left(\frac{1}{4}-\frac{2\times 11}{110}\right)=\frac{1}{20}\text{ ton.}$$

Again when the load is at a distance x_5 that is exactly over the central hinge we have:—

$$R=W\left(1-\frac{2x_5}{S}-\frac{1}{4}\right)=1\left(1-\frac{2\times 55}{110}-\frac{1}{4}\right)=-\frac{1}{4}\text{ ton.}$$

$$R_1=\frac{W}{4}=\frac{1}{4}\text{ ton, and } Rc=W\left(\frac{1}{4}-\frac{2x_5}{S}\right)=1\left(\frac{1}{4}-\frac{2\times 55}{110}\right)$$

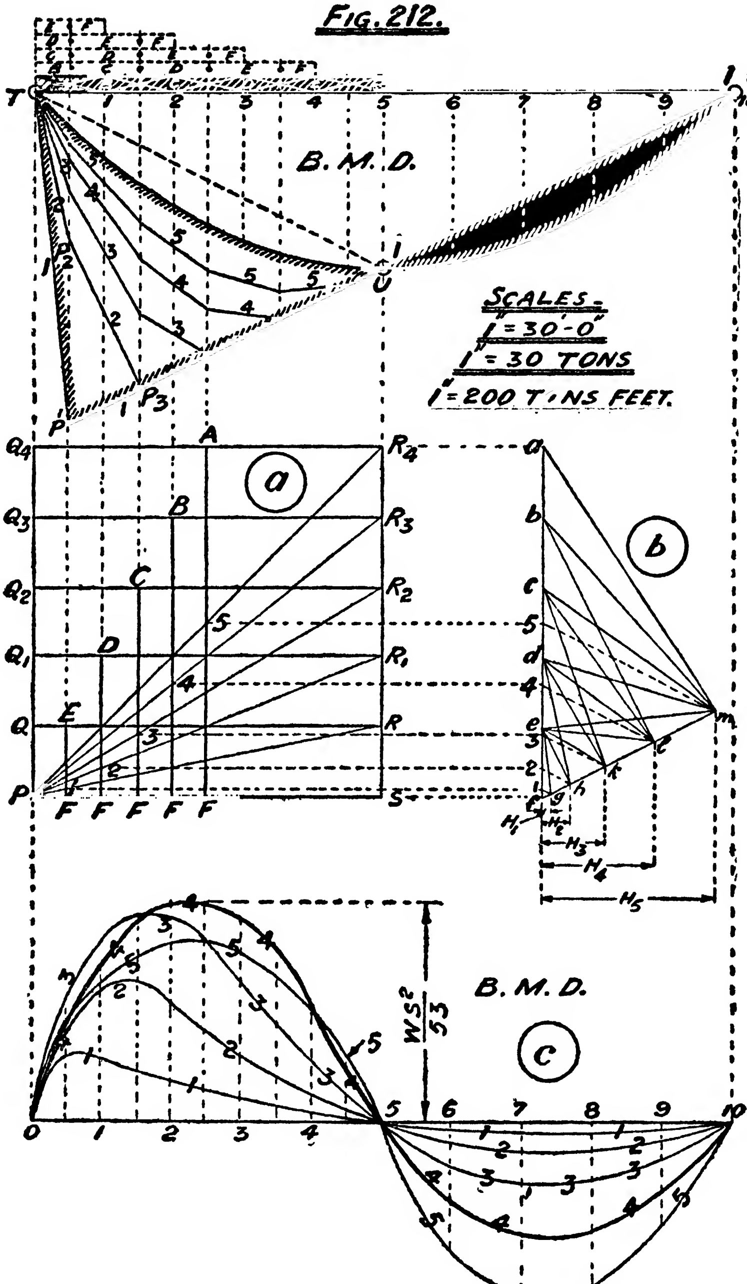
$$Rc=-\frac{3}{4}\text{ ton.}$$

Method of plotting these shearing forces in the diagram.— When the load is at the left support the shear is equal to $\frac{3}{4}$ ton acting upwards and directly over that the moving load W is acting at the instant downwards, therefore net shear is equal to $\frac{3}{4}-1=-\frac{1}{4}$ ton; R_1 and R_c each equal to $\frac{1}{4}$ ton as found above. The shearing force diagram marked 0—0—0—0—0—0 represents the shear when the load is at zero distance. When the load is at x_1 the reaction $R=\frac{11}{20}$ ton, $R_1=\frac{1}{4}$ ton and $R_c=\frac{1}{20}$ ton. At the left support $\frac{11}{20}$ ton is plotted and it is gradually being decreased by an amount of $\frac{W}{S}$ ton per foot run till it reaches the point x_1 and at this point the load W is to be deducted as shown in the diagram and this point is connected to $\frac{1}{20}$ of a ton at the centre hinge and then to a $\frac{1}{4}$ ton ordinate at the right abutment: and this shearing force diagram has been marked as 1—1—1—1—1—1—1 in the figure. Lastly the shearing force diagram for the 5th position that is at x_5, the reaction $R=-\frac{1}{4}$ ton, $RC=-\frac{3}{4}$ ton and $R_1=\frac{1}{4}$ ton. Here at the central hinge the rolling load W is to be deducted from $-\frac{3}{4}+W=\frac{1}{4}$ ton and this has been plotted as shown in the figure and marked 5—5—5—5—5—5. Similarly the shearing force diagrams may be drawn for any position of the moving load over the span.

UNIFORMLY DISTRIBUTED LOAD MOVING OVER A STIFFENED SUSPENSION BRIDGE.

The same bridge represented in fig. 209 is taken for example in figure 212. The uniformly distributed load moving over this span is

FIG. 212.



one ton per foot run. The span is divided into 10 equal parts and each part measures 11 feet since the effective span of the bridge is 110 feet.

The load is assumed to move over the bridge from left to right and occupy successively the first, second, third, fourth and fifth divisions as shown diagrammatically. The dotted rectangles above the load diagram show the positions occupied gradually by the moving load·

Bending moment diagram:—Divide the moving load also into five equal parts and magnitude of each division is 11 tons and assume the front portion EF occupies the first divison, produce the gravity line vertically down and lastly draw the rectangle PQRS as shown in fig. (*a*). Draw the diagonal PR and this diagonal divides the load EF at 1. Now you must remember that F 1 and 1 E are the magnitudes of reactions at central hinge U and left supporting hinge T respectively. (See fig. 53 page 41.) Then join TU and UV and plot the load EF and the reaction point in it as shown in fig. (*b*). The load at U is *f* 1 and draw the force triangle *f 1 g* going round the central hinge; *1 g* is parallel to TU and *f g* is parallel to UV. The point *g* is the pole for the position of the load EF. Then *e f g* is the polar diagram and TP′ UV is the equilibrium polygon or the bending moment diagram for the first position of the load EF. The polar distance is equal to H_1 and measures to the load scale $1\frac{1}{2}$ tons.

Now suppose the moving load approaches the second division then the magnitude of the load is equal to sum of DE and EF, and DF is the resultant. Plot this resultant in fig. (*a*). Draw the rectangle $P\,Q_1\,R_1\,S$ and draw the diagonal PR_1. This will intersect DF at 2 and transfer all these points in fig. (*b*) as shown. As before *f* 2 is the load on the central hinge and *f 2 h* is the force triangle and *h* is the pole for this position of the load. Draw the rays, *d h*, *e h* and *f h* and draw the corresponding funicular polygon $T\,P_2\,P_3$ UV and this represents the bending moment diagram for this position of the load. The polar distance is $H_2=4\frac{1}{2}$ tons; similarly repeat the process for the third, fourth and fifth positions as shown in these diagrams. Now H_3 H_4 and H_5 are the respective polar distances and they measure to the load scale $9\frac{1}{2}$, 18, and 27 tons respectively.

Series of bending moment diagrams are marked 1—1—1, 2—2—2, 3—3—3, 4 4 4 and 5 5 5 for the left half of the bridge but for the right half, the diagram is the same for all the positions of the load as for as the load reaches the central hinge, but the value of the bending moment differs in each case as the distances are varying.

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TABLE SHOWING THE VALUES OF THE BENDING MOMENT FOR THE MOVING LOAD FOR LEFT HALF OF THE BRIDGE.

LOAD.	Polar. Distace.	Magnitudes of the Bending Moment for different positions of the moving load in tons feet.									
		*½	1	1½	2	2½	3	3½	4	4½	5
EF.	1·5 tons	46 × 1·5 69·00	40 × 1·5 60	33 × 1·5 49·5	26 × 1·5 39·0	21 × 1·5 31·5	15 × 1·5 22·50	10 × 1 5 15·00	6 × 1·5 9·00	3 × 1·5 4·50	000 000
DE, EF.	4·5 ”	21 × 4·5 94·5	28 × 4·5 126·0	33 × 4·5 148·5	26 × 4·5 117·0		15 × 4·5 67·50		6 × 4·5 27·0		”
CD, DE, EF.	9·5 ”	12 × 9·5 114·0	17 × 9·5 161·5	23 × 9·5 218·5	21 × 9·5 199·5	20·5 × 9·5 194·75	15 × 9·5 142·5		6·5 × 9·5 61·75		”
BC, CD, DE, EF.	18 ”	5 × 18 90	8·2 × 18 147·6	11·5×18 207·0	12·5×18 225·0	12·70 × 18 228·6	11·5 × 18 207·0	10 × 18 180·0	6·25 × 18 112·50	3 × 18 51·00	”
AB, BC, CD, DE and EF.	27 ”	4 × 27 108·0	5 × 27 135·0		6 75×27 182·25	7 × 27 189	6·5 × 27 175·5		4 × 27 108·0	2·5 × 27 67·5	”

* These numericals refer to the divisions marked along the length of the span of the bridge.

TABLE SHOWING THE VALUES OF BENDING MOMENT FOR THE MOVING LOAD FOR RIGHT HALF OF THE BRIDGE.

Load.	Polar. Distance.	Magnitudes of the Bending Moment for different positions of the moving load in tons feet.					
		*6	7	7½	8	9	10
EF	1·5 tons	5 × 1 5 7·5	6·75 × 1·5 10·125	7 × 1·5 10·5	6·5 × 1·5 9·75	4 × 1·5 6·00	000
DE, EF	4·5 „	5 × 4·5 22·50	6·75 × 4·5 30·37	7 × 4·5 31·50	6·5 × 4·5 29 25	4 × 4·5 18·00	„
CD, DE, EF	9·5 „	5 × 9·5 47·50	6·75 × 9·5 64·12	7 × 9·5 66 50	6·5 × 9·5 61·75	4 × 9·5 38·00	„
BC, CD, DE, EF	18 „	5 × 18 90·0	6·75 × 18 121·50	7 × 18 126·0	6·5 × 18 117	4 × 18 72	„
AB, BC, CD, DE and EF	27 „	5 × 27 135·0	6·75 × 27 182·25	7 × 27 189	6·5 × 27 175·50	4 × 27 108·0	„

* These numericals refer to the divisions marked along the length of the span of the bridge.

The above values of the bending moment have been plotted in fig. (c) and you will observe that the maximum bending moment occurs when the load approaches the fourth division of the bridge, that is at ·4 of the span and its value = 228·6 tons feet nearly. By calculation we get $\frac{WS^2}{53}$ nearly; where W = moving load per foot run in tons; S = span in feet. Substituting the numerical values we have $\frac{1 \times 110^2}{53} = \frac{12100}{53} =$ 228·3 tons feet, but practically no difference. For theoritical calculations refer pages 367 and 368, "Andrews Design of Structures."

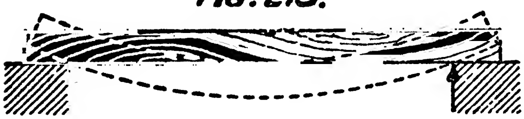
Shearing force diagram will be exactly same as calculated for figure 209.

CHAPTER XII.

FIXED BEAMS.

In chapter IV we dealt with simple supported and inclined beams with bending moment and shearing force diagrams in detail but in this chapter an attempt has been made to show as far as possible clearly, the method of drawing bending moment and shearing force diagrams graphically for fixed beams.

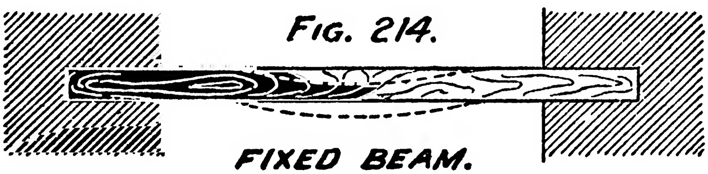
FIG. 213.



SUPPORTED BEAM.

Figure 213 is a supported beam, that is to say it simply rests on the supporting walls and when it is loaded it freely bends as shown in dotted, but there is nothing to prevent its bending action at the supports; whereas the same beam if fixed to the wall properly as shown in figure 214 and then loaded, it bends a little in the central portion as shown is dotted curved line, keeping the left and right fixed portions perfectly horizontal. This horizontality of the fixed portion of the beam is due to the suffiecient weight of the masonry above the fixed end or if there is no convenience to provide the calculated weight above this end a pair of holding down bolts are to be provided at this portion and to be taken down the masonry vertically to a suitable depth. Then only, a perfect fixation of the beam is guaranteed. This method of fixing, sets up the negative bending moment at these points and the area of the negative bending moment diagram must be equal to the bending moment diagram of the freely supported beam. This theorem helps us to draw the bending moment diagram for fixed beams graphically.

FIG. 214.

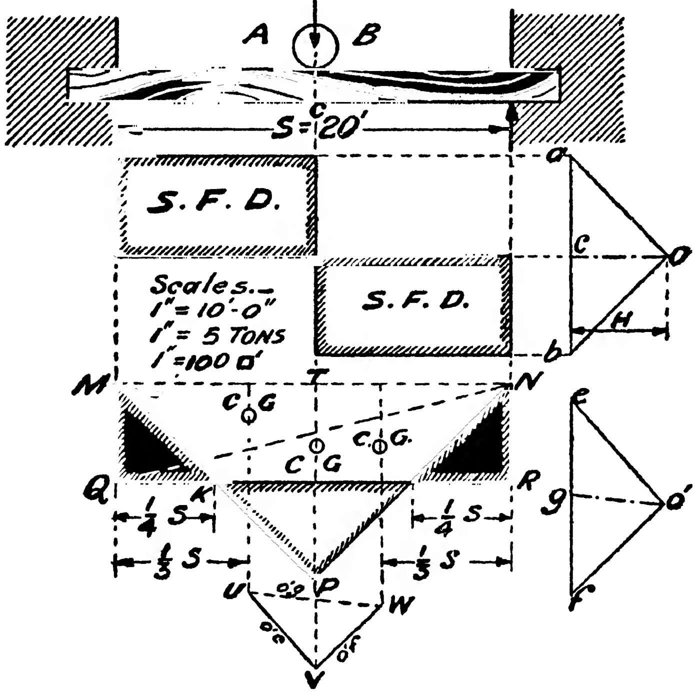


FIXED BEAM.

EXAMPLE 1.—A beam 20′ long fixed at both ends and carries a concentrated load of 5 tons at the centre. Draw bending moment and shearing force diagrams graphically.

SOLUTION.—Draw the load $a\,b$ and you know the magnitude of reaction on each support is equal to $\frac{W}{2} = b\,c$ and $c\,a$ and at right angles to the point c select any pole o and complete the polar diagram. Magnitudes of reactions are known and draw the shearing force diagram as shown, similar to the shearing force diagram of the simple supported beam. Again draw the equilibrium polygon MPN assuming that the beam is a simple supported one. The maximum bending moment is equal to the intercept TP in linear scale multiplied by pole distance H in load scale $= 10' \times 2{\cdot}5$ tons $= 25$ tons feet. Here TP $= 10'$ and H $= \frac{1}{2}$ inch $= 2{\cdot}5$ tons to the load scale. You can verify this by formula as well.—The formula for a supported beam with a concentrated load at centre the maximum bending moment $= \frac{W\,S}{4}$ where W $=$ load and S $=$ span in feet. Substituting the numerical value you have $\frac{5 \times 20}{4} = 25$ tons feet as above.

FIG. 215



The negative bending moment set up by the fixation should be exactly equal in area of the free bending moment diagram MPN. You

know the area of the trangle MPN is equal to $\frac{MN}{2} \times TP$ and the area of the negative bending moment which should be drawn on the same base MN must be equal to $MN \times h$, where h = the height of the rectangle. Then $\frac{MN}{2} \times TP = MN \times h$.

$$\therefore\ h = \frac{MN}{2} \times \frac{TP}{MN} = \frac{TP}{2}.$$

Now take $\frac{TP}{2}$ as the height of the negative bending moment and draw the rectangle MQRN. Eliminating the common portion of the two figures the resulting bending moment diagram is as shewn shaded in the diagram. The bending moment at the fixed end is equal to NR or MQ multiplied by pole distance H = 5′ × 2 5 tons = 12·5 tons feet and this value is exactly one half of the value of free bending moment.

Determination of the height of the rectangle MQRN or the maximum ordinate of the negative bending moment by graphical method is as follows.—Take MQ and NR vertically of equal lengths on the reaction lines and join QN. Now the rectangle has been divided into two triangles MQN and NRQ and the centres of gravity of these two triangles will lie on two vertical lines drawn at $\frac{1}{3}$ the span of the beam as shown.

Note:—The triangles may be small or big such as the triangles shown in fig. 216 the centres of gravity of these four triangles lie on two vertical lines drawn at $\frac{1}{3}$ the span. Here 2 N, 1 Q, 4 R and 3 U are medians of the triangles MQN, NRQ, QUR and RVU respectively. The area of the negative bending moment diagram must be equal to the area of the free bending moment diagram. Remem-

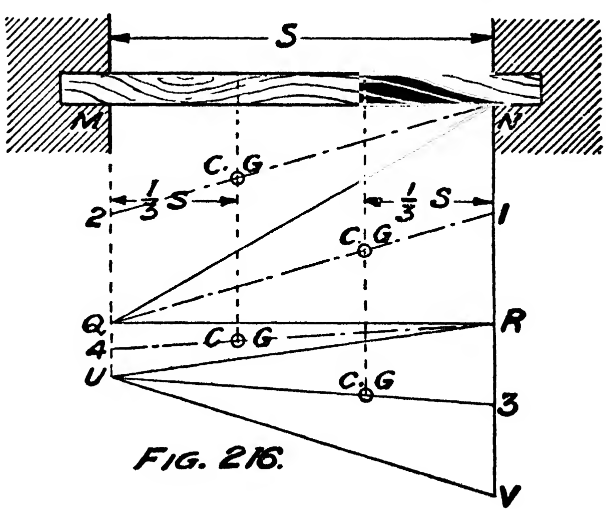


FIG. 216.

er that the centre of gravity line of the area of the negative bending moment diagram must lie on the C. G. line of the free bending moment diagram. Now take $e\,f$ equal to the area of the free bending moment diagram and slect any pole O′. Draw the equilibrium polygon UVW, the first and the last ray of the polar diagram must meet at V on the gravity line of the free bending moment diagram as stated above. Then UW is the closing line of the funicular polygon and from pole O′ the line O′ g is drawn parallel to UW. Now $e\,g$ aud $g\,f$ represent the areas of the negative bending moment to act down on the vertical lines drawn at $\frac{1}{3}$ span.

Now $MQ \times \frac{S}{2}$ = area of the triangle $MQN = e\,g$. $\therefore MQ = \frac{e\,g \times 2}{S}$

Substituting the numerical values in the above equation we have —

$MQ = e\,g \times \frac{2}{S} = \frac{50 \times 2}{20} = \frac{100}{20} = 5$ feet. The area of the free bending moment diagram $= \frac{20}{2} \times 10 = 100$ □′ $= e\,f$ and $e\,g$ is half of $e\,f = 50$□′.

This enables us to draw the negative bending moment diagram as shown shaded. This graphical method is to be thoroughly understood by the students if they desire to follow the further problems on fixed beams, as the same method will be adopted hereafter.

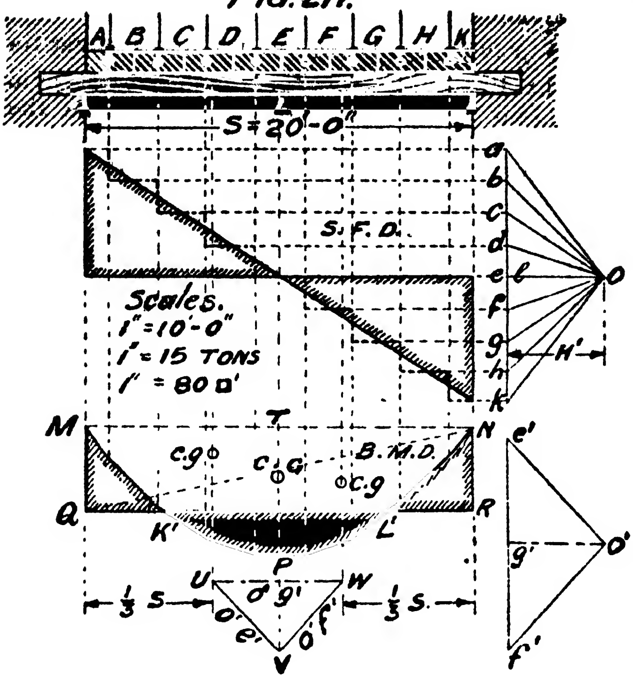
EXAMPLE 2.—A fixed beam of span 20 feet carry a uniform load of one ton per foot run. Draw the bending moment and shearing force diagrams.

SOLUTION:—See figure 217. Divide the distributed load into any number of parts as shown, and draw the bending moment and shearing force diagrams assuming this to be of simple supported beam. Now the area of the negative lending moment diagram must be equal to the area of the free bending moment diagram and the area of the parabola is equal to the base MN multiplied by $\frac{2}{3}$ TP. Then $MN \times \frac{2}{3}\,TP = MN \times h$ where h is the height of the rectangle MQRN. $\therefore h = \frac{MN \times \frac{2}{3}\,TP}{MN} = \frac{2}{3}\,TP$. Take MQ or $NR = \frac{2}{3}$ TP and draw the rectangle MQRN, the resulting figure is the bending moment diagram shown shaded as usual.

Now the maximum bending moment is at the supports and is equal to the intercept MQ or NR multiplied by pole distance H′.

Substituting the numerical values you have $4{\cdot}44 \times 7{\cdot}5$ tons $= 33{\cdot}33$ tons feet. The formula for the bending moment at the centre of the

FIG. 217.



span for simple supported beam with a uniformly distributed load is $\frac{WS^2}{8}$ where W= load per foot run and S=span in feet. Substituting the values you have $\frac{1 \times 20 \times 20}{8}$ $= \frac{400}{8} = 50$ tons feet.

Graphically intercept measures to the linear scale 6·66 feet pole distance in load scale 7·5 tons, then bending moment $= 6·66 \times 7·5 = 49·95$ or say 50 tons feet as above.

The maximum ordinate of the negative bending moment is equal to $\frac{2}{3}$ of the ordinate at the centre of the parabola $= \frac{2}{3} \times 6·66 =$ 4·44 feet, and maximum bending moment $= 4·44 \times 7·5 = 33·33$ tons feet. Again $\frac{2}{3}$ of $\frac{WS^2}{8} = \frac{WS^2}{12} = \frac{20 \times 20}{12} = 33·33$ tons feet as before. At the centre, the ordinate $= \frac{1}{3}$ of $6·66 = 2·22$ and the bending moment at the centre $= 2·22 \times 7·5 = 16·65$ tons feet. You can verify this by the formula $\frac{1}{3} \times \frac{WS^2}{8} = \frac{WS^2}{24} = \frac{400}{24} = 16·66$ tons feet as before. The points of contraflexure are K′ and L′ and at these points the bending moments are zero. The length between K′L′ can easily be found from the following relation $\frac{W(K'L')^2}{8} = \frac{WS^2}{24}$.

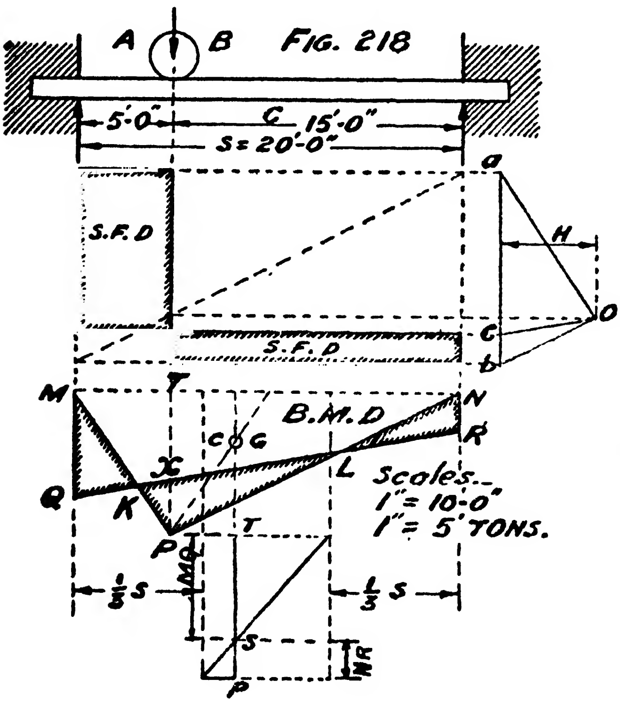
$$\therefore K'L' = \sqrt{\frac{WS^2}{24} \times \frac{8}{W}} = \sqrt{\frac{S^2}{3}} = \frac{S}{\sqrt{3}} = \frac{S}{1·7321} = ·577 S.$$

Then Q $k_1 + L_1$ R $= 1·000 - ·577$ S $= ·423$ S : ∴ Q k_1 or L_1 R $= \frac{·423 S}{2} = ·211$ S. On measuring to the scale you find it measures exactly ·211 S.

Graphical way of getting the negative bending moment.— Take any rectangle MQRN and divide this into two triangles MQN and NRQ. The centres of gravity of these two triangles must lie on two vertical straight lines drawn at $\frac{1}{3}$ span. The sum of the areas of these two triangles should be equal to the area of the parabola MPN. The centre of gravity of the negative bending moment diagram must also lie on the centre of gravity line of the free bending moment diagram.

Now the area of the parabola MPN = MN × $\frac{2}{3}$ TP = 20 × $\frac{2}{3}$ × 6·66 = 88·8 □′. Then the area of the negative bending moment diagram must also be equal to 88·8 □′. Take $e' f'$ equal to the area 88·8 □′ and take any pole O′. Then draw the equilibrium polygon UVW and UW is the closing line, then draw from pole o′ a line $o' g'$ parallel to UW as usual.

MQ × $\frac{S}{2}$ = the area of the triangle MQN = $e' g'$. ∴ MQ = $\frac{e' g' \times 2}{S}$. Substituting the values you have MQ=$\frac{44·4 \times 2}{20}$=4·44, and this is exactly equal to $\frac{2}{3}$ of TP = $\frac{2}{3}$ × 6·66=4 44. The resulting bending moment diargram is as shown in the figure.



EXAMPLE 3 :— Figure 218 represents a fixed beam of 20 feet span with a concentrated load of 5 tons anywhere say at 15 feet from one of the fixed ends. Draw bending moment and shearing force diagrams graphically.

SOLUTION :— Draw the reaction influence line as shown in dotted lines to determine the magnitudes of reactions assuming the beam to be freely supported one. Select the pole O, perpen-

dicular to the reaction. By this you get the closing line M N of the free bending moment diagram exactly parallel to the beam.

Draw the free bending moment diagram M P N as if the beam is freely supported one. Let M Q and N R be the ordinates representing the negative bending moments at the left and right fixed ends respectively. The area of the free bending moment diagram MPN must be equal to the area of the negative bending moment diagram M N R Q. Now the area of the free bending moment diagram $= \frac{MN}{2} \times TP$. Area of the negative bending moment diagram $= \frac{MQ+NR}{2} \times MN$. Therefore $\frac{MN}{2} \times TP = \frac{MQ+NR}{2} \times MN$. Eliminating the common terms we have $TP = MQ + NR$.

The negative bending moment diagram should be divided into two triangles and in this case these two triangles are not equal to each other as the load is not in the centre of the beam, but still the centres of gravity of these two triangles must lie on two vertical lines drawn at $\frac{1}{3}$ the span lengths shown dotted. The gravity line of the free bending moment diagram also is shown.

Now adjust TP in the centre of gravity line of the free bending moment diagram and draw the reaction influence line as shown and the ordinates MQ and NR are directly obtained. For reaction influence line see fig. 236 page 201.

Next draw MQ and NR at the left and right fixed ends of the beam and obtain the resulting bending moment diagram shown shaded. Bending moments at the fixed ends and under the load may be calculated as follows.—B. M at the left fixed end $= MQ \times H = 5{\cdot}4 \times 2{\cdot}5$ tons $= 13{\cdot}5$ tons feet. B. M under the load $= x P \times H = 3 \times 2{\cdot}5$ tons $= 7{\cdot}5$ tons feet and B. M. at the right fixed end $= N R \times H = 2{\cdot}1 \times 2{\cdot}5$ tons $= 5{\cdot}25$ tons feet.

The shearing force is to be drawn as follows —From the pole O draw a line OC parallel to QR and *bc* represents the shear at the right fixed end and *c a* at the left fixed end and the resulting shearing force diagram is as shown shaded.

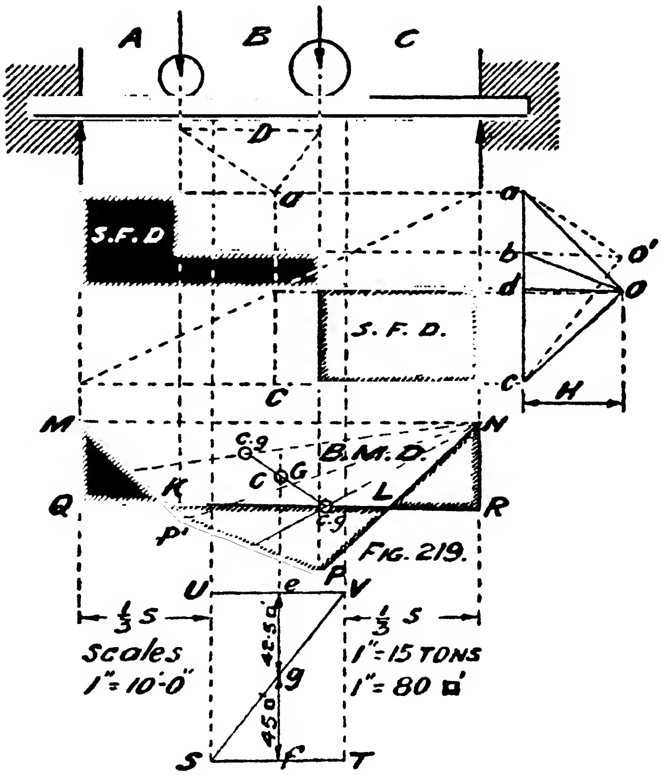
DIAGRAMS FOR UNSYMMETRICAL LOADINGS.

EXAMPLE 4 :—A fixed beam of 20 feet span with concentrated loads 5 and 10 tons placed at 5' and 12' from one of the fixed ends is given in fig. 219. Draw bending moment and shearing force diagrams graphically.

SOLUTION :—Draw the action line of the resultant for the given two loads A B, B C and for this resultant *a c* draw the

reaction influence line as shown in dotted. Select pole O perpendicular to the reaction point to get M N parallel to the beam as usual.

Now draw the free bending moment diagram assuming the beam as freely supported Calculate the area of this bending moment diagram and determine the centre of gravity of the same. As usual draw two vertical lines at $\frac{1}{3}$ the span lengths and the centres of gravity of the two triangular figures of the negative bending moment diagrams must lie on these two lines. Divide the free bending moment diagram into two triangles MP′N and P′PN. Determine the centres of gravity of these two triangles and calculate the areas as well. The centre of gravity C. G. of the free bending moment diagram will be at the intersection of the centre



line of the span with the line joining the centres of gravity of the triangles MP′N and P′PN; sum of the areas of these two triangles is equal to (50 + 37·5) = 87·5□′. Take a line e f equal to this area and adjust this on the C. G. line of the free B. M. D. and draw the reaction influence line STUV as shown. Now MQ × $\frac{S}{2}$ = area of the triangle MQN = e g =

42·5□′. ∴ $MQ = \frac{42·5 \times 2}{20} = 4·25$ feet. $NR \times \frac{S}{2}$ = area of the triangle NRQ = $f\ g = 45$. ∴ $NR = \frac{45 \times 2}{20} = 4·5$ feet. Plot MQ = 4·25′ and NR = 4·5′ and join QR, the resulting bending moment diagram is as shown shaded. $MQ \times H = 4·25 \times 7·5$ tons = 31·875 tons feet is the bending moment at the left fixed end; $NR \times H = 4·5 \times 7·5$ tons = 33·75 tons feet is the bending moment at the right fixed end. At $P = 3·5 \times 7·5$ tons = 26·25 tons feet. Shearing force diagram :—Draw from pole *o* a line *o d* parallel to QR, then *c d* and *d a* are the reactions at the right and left supports respectively. As usual the shearing force diagram is drawn.

EXAMPLE 5 :—A beam of 20′ span is loaded with a uniformly increasing distributed load starting from one of the fixed ends. Total load is equal to 25 tons. Draw bending moment and shearing force diagrams graphically. See fig. 220.

SOLUTION .—The action line of the resultant of these loads should act at the centre of gravity of the loaded diagram and for this resultant load draw the reaction influence line shown dotted. Select pole O perpendicular to the reaction point as usual and draw the free bending moment diagram MPN as if the beam is freely supported. Determine the centre of gravity of the free bending moment diagram as follows. Produce the first and the last ray of the equilibrium polygon and let them meet at X and join X to the middle point c′ of the base MN. This line intersects the parabola at P. Divide the line c′P into five equal parts and the centre of gravity of the parabola will be at

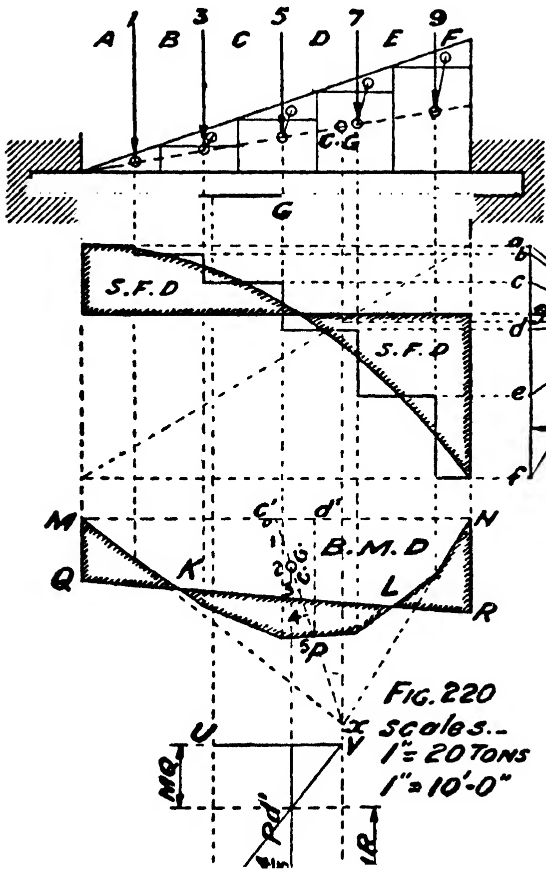


FIG. 220
Scales.–
1″ = 20 TONS
1″ = 10′-0″

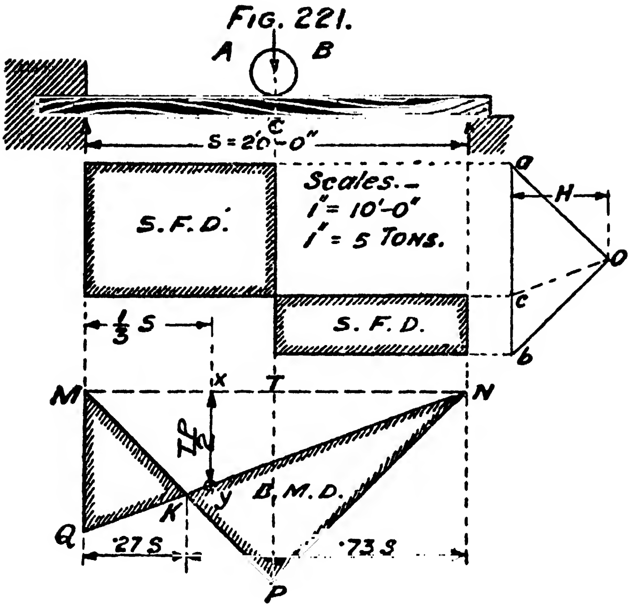
From P draw a perpendicular line $p\,d'$ to meet the base line MN at d'. The area of the parabola MPN is equal to the base MN multiplied by $\frac{2}{3}$ the height of $p\,d'=20\times\frac{2}{3}\times 6{\cdot}37=85\,\square'$ nearly. Then the area of the negative bending moment diagram should be equal to $85\,\square'$. Let MQ and NR be the ordinates representing the negative moments at the left and right fixed ends of this beam. The area of the parabola MPN$=$MN$\times\frac{2}{3}$PD. The area of the figure MQRN$=\frac{MQ+NR}{2}\times MN$; $\therefore MN\times\frac{2}{3}\ PD=MN\times\frac{MQ+NR}{2}$, Leaving out the common terms we have $\frac{2}{3}\,p\,d=\frac{MQ+NR}{2}$, hence $MQ+NR=\frac{4}{3}\,p\,d$. Adjust this $\frac{4}{3}\,p\,d$ in the gravity line of the free bending moment diagram and draw the reaction influence line STUV between the middle third lines. Then the ordinates MQ and NR are determined without any further trouble. Plot these two ordinates MQ&NR at the left and right fixed ends and join QR the resulting bending moment diagram for fixed beam for a uniformly increasing distributed load you get as shown shaded in the figure. Values of bending moment may be calculated as usual by taking the ordinate in linear scale and multiplying the same by pole distance in load scale. Bending moment at the left fixed end$=3{\cdot}3\times10=33$ tons feet, bending moment at the right fixed end $5{\cdot}2\times10=52$ tons feet. K and L are the points of contraflexure as before.

Shearing force diagram.—Draw from pole o a line o g parallel to the closing line QR of the bending moment diagram and the resulting shearing force diagram is shaded.

FIXED AND SUPPORTED BEAMS.

EXAMPLE 6 :—A beam of 20 feet span fixed at one end and supported at the other end. It is to bear a concentrated load of 5 tons in the centre. Draw bending moment and shear force diagrams graphically. See figure 221.

SOLUTION :—Draw the polar diagram $a\ b$ o taking $ab=5$ tons and draw the corresponding equilibrium polygon MPN assuming the beam to be freely supported. The area of the free bending moment diagram MPN is equal to $\frac{MN}{2}\times TP$ and the area of the negative bending momen diagram assuming the beam to be fixed at both the ends is equal to $MN\times h$ where $h=$the height of the rectangle. Therefore $h=\frac{MN}{2}\times\frac{TP}{MN}=\frac{TP}{2}$. When both the ends are fixed the ordinate $\frac{TP}{2}$ is to be taken on the vertical lines drawn at $\frac{1}{3}$ the span length. (See figure 215.) In this



example only the left end is fixed up, so the ordinate $\frac{TP}{2}$ is to be taken on the vertical line drawn at $\frac{1}{3}$ the span near to the fixed end of the beam. Let this ordinate be $x\,y$.

You know the bending moment at the free supported end is zero therefore join the points N & y and continue it to meet the left end reaction line at Q. You observe that the ordinate MQ will be exactly equal to $\frac{3}{4}$ TP. In this case the line NQ is the closing line of the equilibrium polygon or the bending moment diagram. Therefore draw from pole o a line $o\,c$ parallel to this closing line NQ and you get the magnitudes of reactions on the fixed and supported ends of the beam. From this you can draw the shearing force diagram as shown.

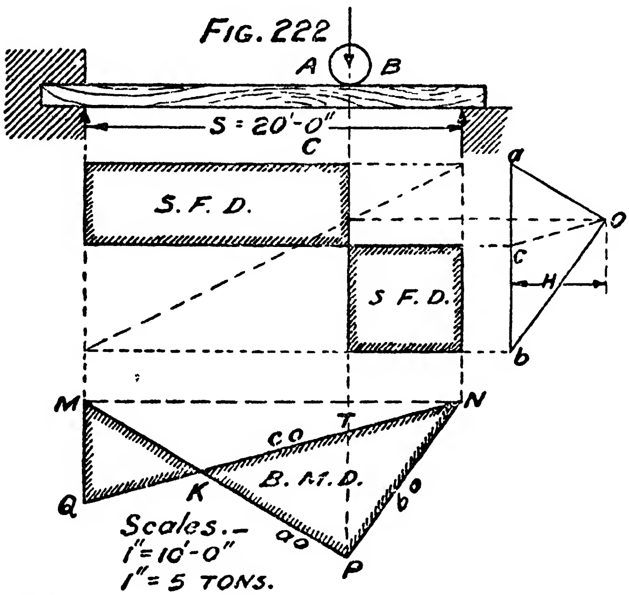
Mathematical reasoning:—Suppose the right end support is removed then this becomes a cantilever with the load at the centre and the deflection for this load on the cantilever is equal to $\frac{5\,WS^3}{48\,EI}$. The free end reaction must be sufficient to resist this deflection to bring the beam horizontal that is, to its original position. The deflection for the cantilever for a load at the free end is equal to $= \frac{PS^3}{3\,EI}$. Now we should find this P which is the same as the reaction at the free end.

$$\frac{PS^3}{3\,EI} = \frac{5\,WS^3}{48\,EI}. \quad \therefore P = \frac{5\,WS^3}{48\,EI} \times \frac{3\,EI}{S^3} = \frac{5}{16}W.$$

Free end reaction is therefore $= \frac{5}{16}$ W and the fixed end reaction is $\left(1 - \frac{5}{16}\right)W = \frac{11}{16}$ W. Substituting the value of W we have $\frac{5}{16}$ W $=$

$\frac{5}{16} \times 5 = \frac{25}{16} = 1\cdot5625$ tons; and $\frac{11}{16} \times 5 = \frac{55}{16} = 3\cdot4375$ tons. From this the point c is fixed in the polar diagram and the bending moment diagram may be drawn very quickly.

The only difficulty is to remember these deflection formulas. On measuring to the scale you find the magnitudes of reactions on both the ends will be exactly equal to 1·56 and 3·44 tons respectively. The contraflèxure point will be at one point K distant ·27 S from the fixed end of the beam.



EXAMPLE 7:— A beam of 20 long fixed at one end and supported at the other end with a concentrated load any where on the beam, say at 6 feet from the freely supported end. Draw bending moment and shearing force diagrams. See figure 222.

SOLUTION:— Draw the polar diagram *a b o* and the bending moment diagram MPN as if the beam is simply a supported one. First determine the magnitudes of reactions for this fixed and suppórted beam by applying the deflection formula as follows.—Suppose the right end support is removed, then this becomes a cantilever with a concentrated load at 6′ from the free end. The deflection $= \frac{W s^2}{2EI}\left(S - \frac{s}{3}\right)$ Where W = load in tons S = span in feet; s = distance from the fixed end to the load and E and I as usual denote modulus of elasticity and moment of inertia respectively.

The reaction at the right support must be sufficient to bring the beam horizontal. Then the deflection for this cantilever for a load at the right free end $= \frac{PS^2}{3\,EI}$. Now we should find this P which is the same as the reaction at the right end.

$$\frac{PS^3}{3\,EI} = \frac{W\,s^2}{2\,EI}\left(S - \frac{s}{3}\right)$$

$$\therefore\ P = \frac{W\,s^2}{2\,EI}\left(S - \frac{s}{3}\right)\frac{3\,EI}{S^3}$$

$$,, = \frac{5 \times 14 \times 14}{2\,EI}\left(20 - \frac{14}{3}\right)\frac{3\,EI}{20^3}$$

$$,, = \frac{980}{2\,EI} \times \frac{46}{3} \times \frac{3\,EI}{8000}$$

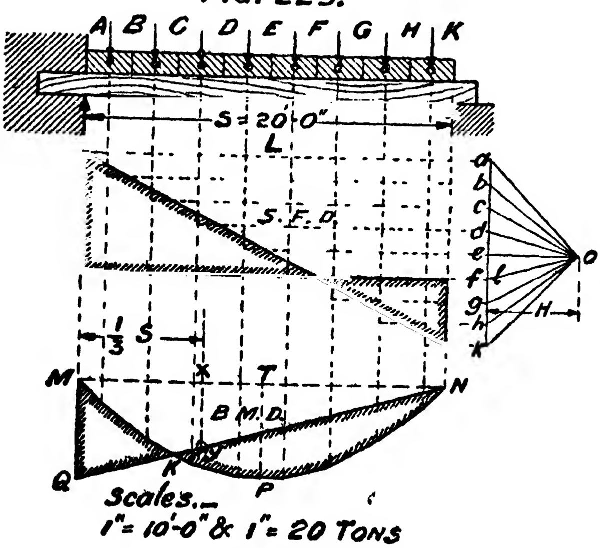
$$,, = \frac{22540}{8000} = 2{\cdot}8175 \text{ tons.}$$

Plotting $b\,c = 2\,8175$ tons in the load line and joining this point to pole O you complete the polar diagram. Select any point P on the action line of the load AB and draw the equilibrium polygon MPNQ which is the same as the bending moment diagram for the fixed and supported beam.

The point of contraflexure is at K. The maximum bending moment is under the load AB and is equal to the intercept TP multiplied by pole distance $H = 6{\cdot}70 \times 2{\cdot}5$ tons $= 16{\cdot}75$ tons feet; at the fixed end $= MQ \times H = 5{\cdot}5 \times 2{\cdot}5$ tons $= 13{\cdot}75$ tons feet. Bending moment at the supported end is zero.

The shearing force diagram is drawn as usual.

FIG. 223.



EXAMPLE 8:— Figure 223 represents a beam fixed at one end and supported at the other end with a uniformly distributed load of one ton per foot run. Draw the bending moment and shearing force diagrams.

SOLUTION:— Draw the free bending moment diagram MPN from a polar diagram $a\,k\,o$ as if the beam is simply supported. Now assume the right

supported end is removed and then this becomes a cantilever with a uniformly distributed load.

The deflection for a cantilever with a uniformly distributed load throughout its length $= \frac{WS^3}{8\,EI}$ where W = total load, S = span in feet E & I are respectively the modulus of elasticity and moment of inertia of the section of the cantilever.

Now the reaction at the right support must be sufficient to deflect the beam upwards to the amount of $\frac{WS^3}{8\,EI}$. The deflection for a cantilever with a concentrated load at the free end $= \frac{PS^3}{3\,EI}$. We will have to determine this P which is the reaction at the supported end. Therefore $\frac{PS^3}{3\,EI} = \frac{WS^3}{8\,EI}$. $\therefore$ $P = \frac{WS^3}{8\,EI} \times \frac{3\,EI}{S^3} = \frac{3}{8}\,W$.

Hence the reaction at the right supported end is $\frac{3}{8}$ W, and plot this $\frac{3}{8}$ W in the load line as *k l*, join *l* to pole O. From N draw a line parallel to *l o* of the polar diagram to meet the reaction line at Q. The shaded diagram represents the bending moment diagram for a fixed and supported beam with a uniformly distributed load throughout its length.

or

Draw a vertical line at $\frac{1}{3}$ the span length from the fixed end and take a line XY equal to $\frac{2}{3}$ the height of the free bending moment diagram MPN. Then join NY and continue this line to meet the action line of the fixed end reaction at Q. Now NQ is the closing line of the bending moment diagram or the funicular polygon, consequently from pole O draw a line *o l* parallel to NQ. Then *k l* and *l a* represent the magnitudes of reactions at the right and left end of the beam.

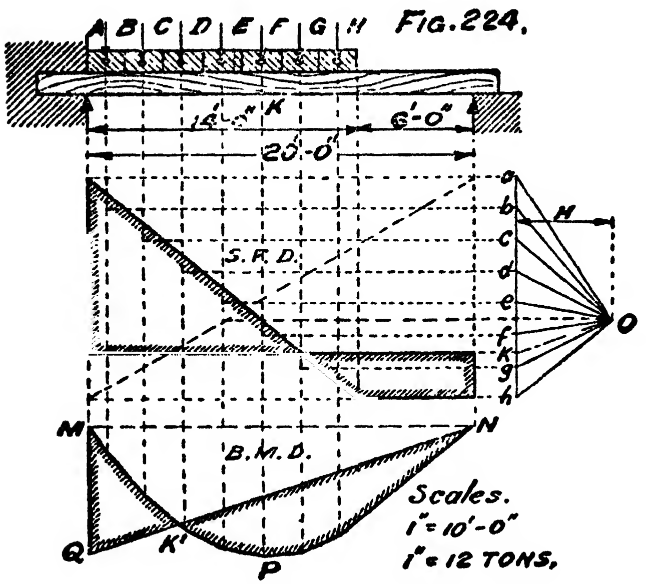
Shearing force diagram may now be drawn as usual as shown.

EXAMPLE 9:—Given a beam of 20 feet span fixed at one end and supported on the other end with a uniformly distributed load of one ton per foot run extending from the fixed end to a point 6 feet from the free end. Draw bending moment and shearing force diagrams. (See fig. 224).

SOLUTION:—Draw the free bending moment diagram MPN from the polar diagram *a h o*. Without determining the magnitudes of reactions at the fixed and free ends it is impossible to draw the negative bending moment diagram for the fixed beam. Therefore by using the deflection formula for the cantilever we can determine the reaction at the supports as before.

Now imagine that the right support is temporarily removed and this becomes a cantilever with a partially distributed load.

The deflection $=\frac{WS^2}{6\ EI}\left(S-\frac{s}{4}\right)$. Where W = total load, s = the distance form the fixed end to the last point of the distributed load and S = span length in feet.



The deflection for a concentrated load at the free end of the cantilever as before $=\frac{P\ S^3}{3\ EI}$. Where P is the concentrated load at the free end.

Then $$\frac{PS^3}{3\ EI}=\frac{Ws^2}{6\ EI}\left(S-\frac{s}{4}\right)$$

$$P=\frac{Ws^2}{6\ EI}\left(S-\frac{s}{4}\right)\frac{3\ EI}{S^3}.$$

$$"=\frac{14\times 14\times 14}{6\ EI}\left(20-\frac{14}{4}\right)\frac{3\ EI}{20^3}.$$

$$"=\frac{2744}{6\ EI}\times\frac{66}{4}\times\frac{3\ EI}{8000}.$$

$$"=\frac{686\times 33}{8000}=2.8287 \text{ tons.}$$

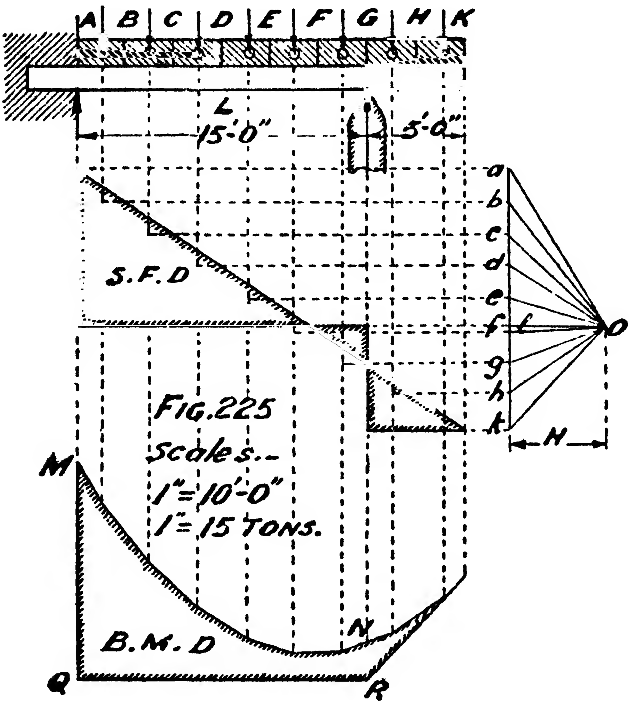
The right end reaction is therefore 2·83 tons and the fixed end reaction = 14 − 2·83 = 11·17 tons nearly.

When the magnitudes of reactions are known you can draw the B. M. D. similar to the B. M. D. for a simple supported beam. The resulting B. M. D. you get without any trouble as shown shaded.

Note :—When you use the deflection formula to determine the magnitudes of reactions, you need not draw the free bending moment diagram at the very start, but wait till you determine the reactions and then you draw the resulting bending moment diagram

as shown shaded. Shearing force diagram is drawn as usual. If you take pole O perpendicular to the point k in load line you get the closing line NQ in the bending moment diagram horizontal.

EXAMPLE 10 :—A beam of 20′ long with a uniform load of 1 ton per foot run is fixed at one end and supported at 5′ from the other end. Draw diagrams of bending moment and shearing force.



SOLUTION :— See figure 225. We can draw the bending moment and shearing force diagrams at once if we know the magnitudes of reactions at the fixed and supported ends similar to the bending moment and shearing force diagrams for ordinary supported beams. Now let us assume the right hand support is removed then this becomes a cantilever with a uniformly distributed load and the diflection is equal to $\frac{WS^3}{12EI}$. at 15′ from the fixed end. The reaction at the right hand support must be sufficient to resist this deflection. The deflection for a concentrated load on the cantilever at 15′ from the fixed end is equal to $\frac{Ps^2}{2\ EI}\left(S-\frac{s}{3}\right)$. Where P=the concentrated load, s = the distance from the fixed end to the concentrated load, and S = span in feet.

These two must be equal if the points at the fixed and supported positions to be of the same level.

$$\text{Therefore } \frac{Ps^2}{2\ EI}\left(S-\frac{s}{3}\right)=\frac{WS^3}{12\ EI}.$$

$$=P\frac{15\times15}{2\ EI}\times15=\frac{20\times20^3}{12\ EI}.$$

$$\therefore P = \frac{20 \times 8000}{12EI} \times \frac{2EI}{225 \times 15}$$

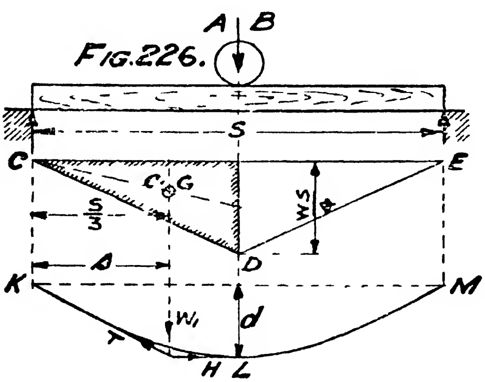
$$= \frac{640}{81} = 7\ 90 \text{ tons}$$

The reaction at the freely supported end is therefore equal to 7.9 tons and at the fixed end 20—7 90=12 10 tons Plotting $k\ l$=7·90 tons in the load line and selecting any pole O at right angles to the point l we get the closing line of the bending moment diagram horizontal. Draw the bending moment & shearing force diagrams as usual similar to the supported beam as shown. The bending moment at the right hand support is equal to NR × H=2 × 7·5=15 tons feet. At the fixed end MQ × H=11 5 × 7 5=85·25tons feet.

DEFLECTION OF BEAMS. (SIMPLE CASES)

In previous examples we simply made use of the deflection formulas without entering into the theoritical investigation how these formulas were arrived at We deal here of the same briefly so that the students remember well when they happen to deal with bending moment and shearing force diagrams for fixed and supported beams, and at times continuous beams -- Take the case of a supported beam of span S with a concentrated load AB at the centre of the beam.

FIG. 226.



The maximum bending moment is equal to $\frac{WS}{4}$ and consider this as the loaded area on the beam. The beam bends and takes up more or less a parabolic form similar to a cable of a suspension bridge, loaded uniformly. See figure 201 page 152. Let KLM be an exaggerated form of the bend of this beam. Let us consider only one half of this cable, and this is kept in equilibrium by three loads similar to the suspension bridge. The load W_1 is equal to the area of half the bending moment diagram; horizontal pull H is equal to the flexural rigidity=(E × I). T= the terminal tension, and d = dip of the cable or the same as the deflection of the

beam. Taking moments about the point K we have $H \times d = W_1 \times s$.

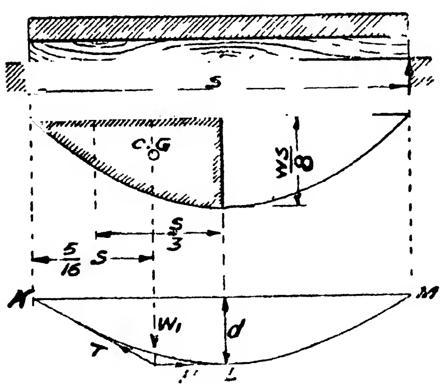
$\therefore d = \frac{W_1 \times s}{H}$ where W_1= the area of the bending moment diagram for the left half of the beam $= \frac{WS}{4} \times \frac{1}{2} \times \frac{S}{2} = \frac{WS^2}{16}$, H=modulous of elasticity multiplied by moment of inertia= $E \times I$, s = the distance from the left support to the centre of area of half the bending moment diagram $= \frac{S}{3}$. Substituting these values in the above we have,

$$d = \frac{W_1 \times s}{H} = \frac{WS^2}{16} \times \frac{S}{3} \times \frac{1}{EI} = \frac{WS^3}{48EI}$$

Students should remember this rule--- The deflection for a beam with concentrated load at the centre is equal to the product of half the area of the bending moment diagram and the distance $\frac{S}{3}$ divided by the flexural rigidity = EI; or in symbols

$$d = \frac{WS^2}{16} \times \frac{S}{3} \times \frac{1}{EI} = \frac{WS^3}{48EI} \quad \text{.............................} (1)$$

FIG. 227.



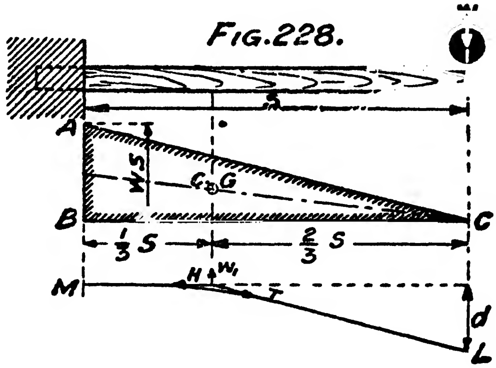
Similarly for a beam with a uniformly distributed load we have (See figure 227) $H \times d = W_1 s$ $\therefore d = \frac{W_1 s}{H}$ where W_1 = area of half the bending moment diagram $= \frac{WS}{8} \times \frac{S}{3} = \frac{WS^2}{24}$, s = distance from the centre of area to the moment centre $= \frac{5}{16}S$; and H as before EI. Substituting these values in the above formula we have $d = \frac{W_1 s}{H} = \frac{WS^2}{24} \times \frac{5}{16}S \times \frac{1}{EI} = \frac{5WS^3}{384EI}$

Therefore the deflection for a beam with a uniformly distributed load is equal to the product of the area of half the bending moment diagram and the distance of its centre of area to the moment centre divided by the flexural rigidity of the beam or in symbols,

$$d = \frac{5WS^3}{384EI} \quad \text{...} (2)$$

DEFLECTION FOR A CANTILEVER WITH A CONCENTRATED LOAD AT THE FREE END.

FIG. 228.

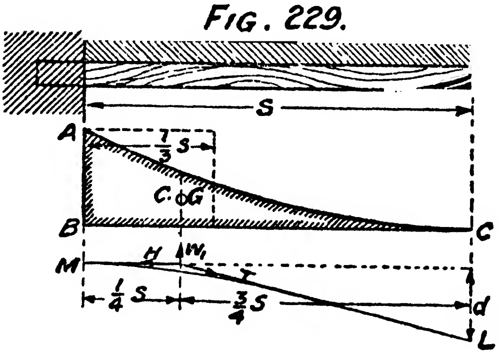


The maximum bending moment is at the fixed end which is equal to WS; and ABC is the bending moment diagram. The centre of gravity of this bending moment diagram is at $\frac{1}{3}$ S. Let ML be the exaggerated form of the bend of the beam. Taking moments about L we have.

$W_1 \times \frac{2}{3}$ S $= H \times d$ Here W_1 = the area of the bending moment diagram. $= \frac{WS}{2} \times S$. This to be assumed to act upwards. $\therefore\ d = W_1 \times \frac{2}{3} S \times \frac{1}{H} = \frac{WS}{2} \times \frac{S}{1} \times \frac{2}{3} S \times \frac{1}{EI} = \frac{WS^3}{3EI}$. $d = \frac{WS^3}{3EI}$ ....(3)

DEFLECTION FOR A CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD THROUGHOUT ITS LENGTH.

FIG. 229.



The bending moment diagram is a parabola and the maximum bending moment $= \frac{WS}{2}$ at the fixed end. As before taking moments about L we have. $H \times d = W_1 \times \frac{3}{4}$ S. Here W_1 = area of the bending moment diagram $= \frac{WS}{2} \times \frac{1}{3} S = \frac{WS^2}{6}$.

$$\therefore\ d = W_1 \times \frac{3}{4} S \times \frac{1}{H} = \frac{WS^2}{6} \times \frac{3\,S}{4} \times \frac{1}{EI} = \frac{WS^3}{8\,EI}.$$

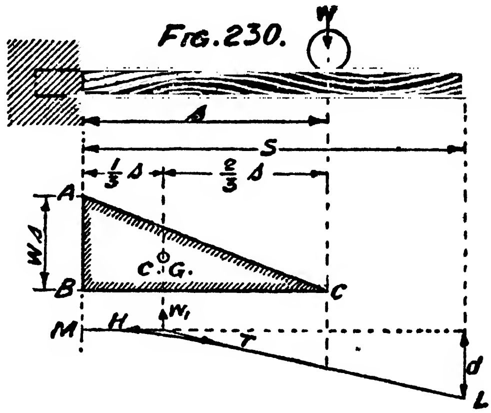
$d = \frac{WS^3}{8\,EI}$ ....(4)

DEFLECTION FOR A CANTILEVER WITH A CONCENTRATED LOAD AT A DISTANCE s FROM THE FIXED END.

Bending moment diagram for this position of the load is ABC and the maximum bending moment is at the fixed end $= W\,s$. As before taking moment about L we have $H \times d = W_1 \times S - \frac{1}{3}s$.

$\therefore\ d = W_1\left(S - \frac{1}{3}s\right)\frac{1}{H}$. where $W_1 =$ area of the bending moment diagram $=$

FIG. 230.

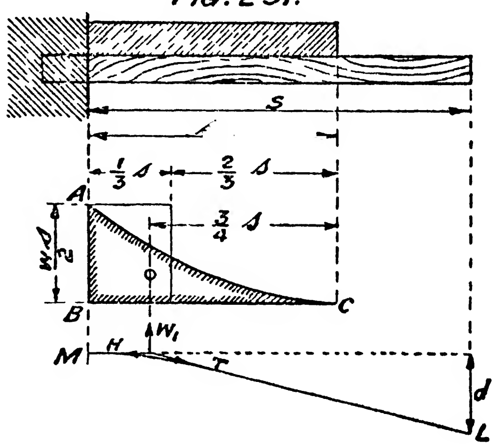


$\frac{Ws}{2} \times s = \frac{Ws^2}{2}$. Hence $d = \frac{Ws^2}{2} \times \left(S - \frac{s}{3}\right) \times \frac{1}{EI} = \frac{Ws^2}{2\,EI}\left(S - \frac{s}{3}\right)$.

$$\therefore\ d = \frac{Ws^2}{2\,EI}\left(S - \frac{s}{3}\right) \qquad \text{.................................} (5)$$

DEFLECTION FOR A CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD EXTENDING TO A DISTANCE s FROM THE FIXED END.

FIG. 231.



The bending moment diagram for the load is the figure ABC and the area of this is equal to $\frac{W\,s}{2} \times \frac{s}{3} = \frac{W\,s^2}{6}$. Now taking moments about the point L we have $H \times d = W_1\left(S - \frac{1}{4}s\right)$ .Where $W_1 =$ area of the bending moment diagram $= \frac{W\,s^2}{6}$.

$$\therefore\ d = W_1\left(S - \frac{1}{4}s\right)\frac{1}{H} = \frac{W\,s^2}{6}\left(S - \frac{1}{4}s\right)\frac{1}{EI}.$$

$$d = \frac{W\,s^2}{6\,EI}\left(S - \frac{s}{4}\right) \qquad \text{..} (6)$$

Note:—Students should remember the above six formulas well as these are freely used to determine the reactions for fixed and supported beams.

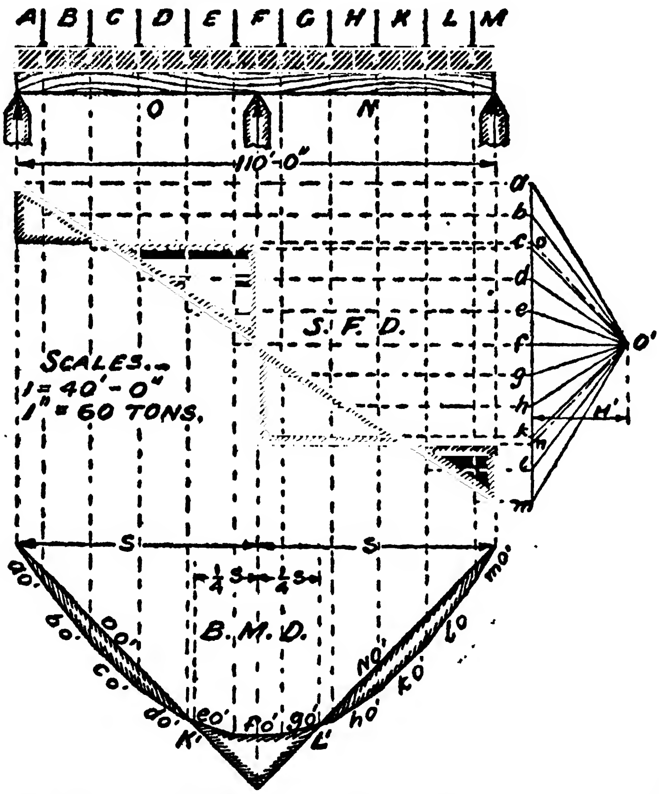
CHAPTER XIII.

CONTINUOUS BEAMS.

A beam is said to be continuous when it is resting on more than two supports. Only the simple cases will be dealt with here, that is to say that all the supports to be of the same level.

EXAMPLE 1:—A beam 100 feet long with a uniformly distributed load of one ton per foot run rests on three supports, one at each extreme end and one at the centre of this beam. Draw bending moment and shearing force diagrams.

FIG. 232.



SOLUTION:—See figure 232. The first main object in these cases is to determine the magnitudes of reactions on the supports, and if you succeed, the bending moment and shearing force diagrams can be easily drawn similar to the ordinary supported beams. Retaining the extreme two supports assume that the support at the centre is removed temporarily, then the deflection for a supported beam with a uniformly distributed load is equal to $\frac{5\ WS^3}{384\ EI}$. (See fig. 227 page 190.) The upward reaction at the centre of the beem must be suffecient to resist this from the centre support. Now you should assume that the uniform distributed load

25

is removed and a concentrated load is placed exactly at the centre, and calculate what would be the deflection, and the deflection for this case$=\frac{PS^3}{48\ EI}$. (See fig. 226 page 189.) These two must be equal to bring the beam at the supports to be on the same level.

$$\therefore \frac{PS^3}{48\ EI}=\frac{5\ WS^3}{384\ EI}.$$

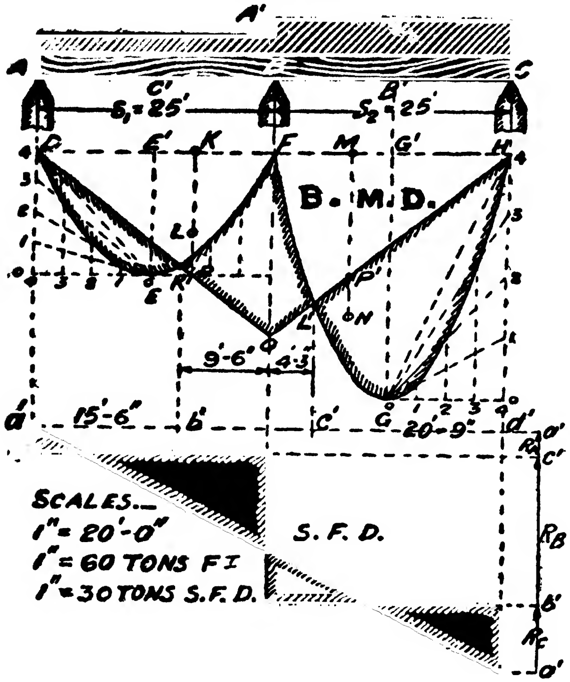
$$P=\frac{5\ WS^3}{384\ EI}\times\frac{84\ EI}{S^3}=\frac{5}{8}\ W.$$

Hence the reaction at the centre support is equal to $\frac{5}{8}$ W, and at the end supports $W-\frac{5}{8}W=\frac{3}{8}W$; then on each end support $\frac{3}{8}\times\frac{1}{2}=\frac{3}{16}$ W. Substituting the numerical values for these we have the magnitude of the reaction at the centre support$=\frac{5}{8}\times 100=62{\cdot}5$ tons, and at the end supports $\frac{3}{16}\times 100=18{\cdot}75$ tons. Plot these loads on the load line and draw the bending moment and shearing force diagrams as shown shaded. The points of contraflexure are at $\frac{1}{4}$ S from the mid support. The bending moment at any point may be calculated as usual by taking the ordinate of the bending moment diagram at that point in linear scale and multiplying it by the pole distance in load scale.

EXAMPLE 2:—A beam 50′ long is supported at the ends and at the centre. It carries a uniform load of half a ton per foot run from the left support to the middle support and 1 ton per foot run from the middle support to the right end support. Draw the bending moment and shearing force diagrams.

SOLUTION:—This can be solved very quickly by the following method. Draw a straight line DFH, parallel to the beam. Draw the bending moment diagram DEF taking the ordinate EE′ equal to $\frac{W_1S_1}{8}$. Where W_1=total load on this span$=25\times\frac{1}{2}=12{\cdot}5$ tons, and $S_1=25$ feet then $EE'=\frac{12{\cdot}5\times 25}{8}=39{\cdot}06$ tons feet. The method of drawing the parabola is shown in dotted lines. Similarly draw the free bending moment diagram FGH for the right span. $\frac{W_2S_2}{8}=\frac{25\times 25}{8}=\frac{625}{8}=$ 78·125 tons feet. Next consider the beam is fixed at the centre support B and simply supported at A & C. Then draw the vertical lines KL and MN at one third the respective spans on left and right of the point F and equal to two thirds the heights of the parabolas DEF and FGH respectively. By trial draw lines HO and DO so that the vertical intercepts PL and P′N must be equal to each other. This trial does

FIG. 235.



not require much of your time, by one or two trials you get the point O without trouble. Because O is the only point which *satisfies the above* condition and no other point. The maximum bending moment is at the centre support and can be measured to the moment scale in which the ordinates EE′ and GG′ are plotted.

The contraflexure points are K′ and L′. If the whole length of the girder is loaded uniformly similar to the example 1, the lines KL and MN would be equal and the lines DO and HO would pass exactly through the points L and N and at a common point O.

Reactions at the supports by this method —The distances from the supports to the contraflexure points are measured to the linear scale and entered in the diagram. Now this becomes an easy matter to calculate the magnitudes of reactions at the supports. The left support A is to bear half the sum of the loads that come on the distance $a'\,b' = \frac{1}{2} \times 15{\cdot}5' \times \frac{1}{2}$ ton $= 3{\cdot}875$ tons, as this portion of the beam is considered to be merely supported.

The centre support B is to bear $\frac{1}{2}$ the load of the distance $a'\,b' =$ $3{\cdot}875$ tons + the load that comes on $b'\,c' = (\,9{\cdot}5 \times \frac{1}{2} + 4{\cdot}25 \times 1\,)$ + half of the load of the distance $c'\,d' = \dfrac{20{\cdot}75 \times 1}{2} = 10{\cdot}375$. Total $3{\cdot}875 + 9 + 10{\cdot}375 = 23{\cdot}25$ tons. This portion of the beam is considered to be the double cantilever.

The third support C is to bear half of the loads that come on the distance $c'\,d' = \dfrac{20{\cdot}75}{2} = 10{\cdot}375$ tons and this portion is considered to be the supported beam.

The values of the bending moment may be verified by Calpeyron's Theorem of Three Moments.

It runs thus.—$M_A S_1 + 2 M_B (S_1 + S_2) + M_C S_2 = 6\left(\frac{A_1 s_1}{S_1} + \frac{A_2 s_2}{S_2}\right)$

Where M_A = Bending moment at the left support.

M_B = „ „ central „

M_C = „ „ right „

S_1 = Length of left span.

S_2 = „ right „

A_1 = Area of the free bending moment diagram for the left span.

A_2 = „ „ „ „ „ „ right „

s_1 = Distance from C. G. of the area of the B. M. D. for the left span.

s_2 = „ „ „ „ „ „ „ right „

Substituting the numerical values for the above we have—$A_1 = 25 \times \frac{2}{3} \times 39{\cdot}06 = 651$; $A_2 = 25 \times \frac{2}{3} \times 78{\cdot}125 = 1302$. $s_1 = s_2 = 12{\cdot}5$. $M_A S_1 + 2 M_B (S_1 + S_2) + M_C S_2 = 6\left(\frac{A_1 s_1}{S_1} + \frac{A_2 s_2}{S_2}\right) = M_A\ 25 + 2 M_B\ (25 + 25) + M_C\ 25 = \left(\frac{651 \times 12{\cdot}5}{25} + \frac{1302 \times 12{\cdot}5}{25}\right)$.

$25 M_A + 100 M_B + 25 M_C = 6 \times 976{\cdot}5 = 5859$. Dividing this by 100 we have $\cdot 25 M_A + M_B + \cdot 25 M_C = 58{\cdot}59$ tons feet. We know the bending moments at A and C are equal to zero, therefore $MB = 58{\cdot}59$ tons feet Ans.

The ordinate FO. below the central support exactly measures 58·60 to the load scale employed in the diagram. This shows the accuracy of the first method. This is the negative bending moment at the central support.

Reactions at the supports.—Take moments about the central support, you have then reaction at $A \times 25 - 25 \times \frac{1}{2} \times 12{\cdot}5 = M_B = -58{\cdot}60$.

$$\therefore R_A = \frac{156{\cdot}25 - 58{\cdot}60}{25} = \frac{97{\cdot}65}{25} = 3{\cdot}906.$$

Again take moments about the central support.—

Reaction at $C \times 25 - 25 \times 1 \times 12{\cdot}5 = MB = -58{\cdot}6$ tons feet.

$$\therefore R_C = \frac{312{\cdot}5 - 58{\cdot}6}{25} = \frac{253{\cdot}9}{25} = 10{\cdot}156 \text{ tons.}$$

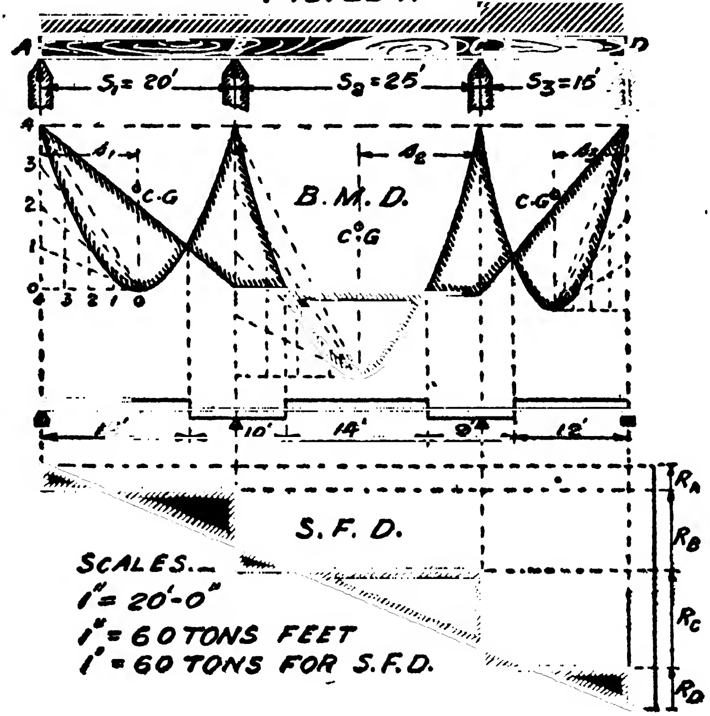
Then $R_B = 37{\cdot}5 - 14{\cdot}062 = 23{\cdot}438$ tons.

The error in the magnitudes of reactions is very small and may be neglected, but if the drawing were to be drawn to a bigger scale and accurate in drawing the profile of the parabola we will not get any difference at all in the values.

The shearing force diagram is drawn as usual and shown shaded in the diagram.

EXAMPLE 3:—A beam is continuous over three spans of 20, 25, and 15 feet and carries a uniformly distributed load of one ton per foot run over the first and second spans and 2 tons per foot run over the third span. Draw the bending moment and shearing force diagrams. Refer fig. 234.

FIG. 234.



SOLUTION.—Draw the free bending moment diagrams for all the three spans and calculate the bending moments at the two intermediate supports by Clapeyron's Theory of Three Moments, as follows.—See figure 234. The supporting points are A, B, C and D, and the free bending moment diagrams are parabolas and the maximum bending moments according to the formula $\frac{WS}{8}$ for each span are 50, 78·125 and 56·25 tons feet respectively. Areas of these three are equal to $20 \times 50 \times \frac{2}{3}$, $25 \times 78{\cdot}125 \times \frac{2}{3}$, $15 \times 56{\cdot}25 \times \frac{2}{3}$; namely 666·66, 1252·08 and 562·5 tons feet² respectively.

Taking the first two spans we have.—

$$M_A S_1 + 2 M_B (S_1 + S_2) + M_C S_2 = 6\left(\frac{A_1 s_1}{S_1} + \frac{A_2 s_2}{S_2}\right)$$

$$M_A\,20+2\,M_B\,(20+25)+M_C\,25=6\left(\frac{666\cdot66\times10}{20}+\frac{1252\cdot08\times12\cdot5}{25}\right)$$

$20\,M_A+90\,M_B+25\,M_C=5756\cdot22$. We know $M_A=O$.

$\therefore\ 90\,M_B+25\,M_C=5756\cdot22$. Dividing this by 25 we get

$3\cdot6\,M_B+M_C=230\cdot25$..(1).

Next taking the second and third spans we have again.—

$M_B\,S_2+2\,M_C\,(S_2+S_3)+M_D\,S_3=6\left(\frac{A_2 s_2}{S_2}+\frac{A_3 s_3}{S_3}\right)=25\ M_B+80$ $M_C+15\ M_D=6\left(\frac{1252\cdot08\times12\cdot5}{25}+\frac{562\cdot5\times7\cdot5}{15}\right)=5413\cdot74$. $25\,M_B+80$ $M_C+15\,M_D=5413\cdot74$. Since $M_D=O$, we have then $25\,M_B+80\,M_C=5413\cdot74$. Dividing this by 25 we get $M_B+3\cdot2\,M_C=216\cdot54$..........(2)

We have obtained row two simultaneous equations (1) and (2) thus

$3\cdot6\,M_B+M_C=230\cdot25$..(1)

$M_B+3\cdot2\,M_C=216\cdot54$ ....(2)

Multiplying (2) by 3·6 and subtracting the same from 1 we get.

$3\cdot6\,M_B+M_C=230\cdot25$ ....(1)

$3\cdot6\,M_B+11\cdot52\,M_C=779\cdot54$ ....(2)

$-10\cdot52\,M_C=-549\,29$

$$\therefore\ M_C=\frac{549\cdot29}{10\cdot52}=52\cdot21 \text{ tons feet,}$$

Substituting this value in (2) we have.—

$M_B+3\cdot2\,M_C=216\cdot54.$

$M_B+3\cdot2\times52\cdot21=216\cdot54.$

$\therefore\ M_B=216\cdot54-167\cdot07=49\cdot47$ tons feet.

Plotting these values in the diagram we get the resulting bending moment diagram shown shaded.

Let R_A, R_B, R_C, and R_D be the reactions at the supports A, B, C, and D, respectively. Taking moments about the support B.—

$R_A\times20-w\times20\times10=-M_B.$

$R_A\ 20-1\times20\times10=-49\cdot47$ tons feet. $\therefore\ R_A=\frac{200-49\cdot47}{20}=\frac{150\cdot57}{20}=7\cdot52$ tons. $R_A=7\,52$ tons.

Taking moments about C we have

$R_A\ (20+25\ )+R_B\times25-(\ w\times45\ )\ \frac{45}{2}=-M_C=-52\cdot21$ tons feet.

$$7\,5\times45+R_B\ 25-\frac{1\times45\times45}{2}=-52\cdot21.$$

$$R_B=\frac{1012\cdot5-337\cdot5-52\cdot21}{25}=\frac{622\cdot79}{25}=24\cdot92 \text{ tons.}$$

Lastly taking moments about the support D we get

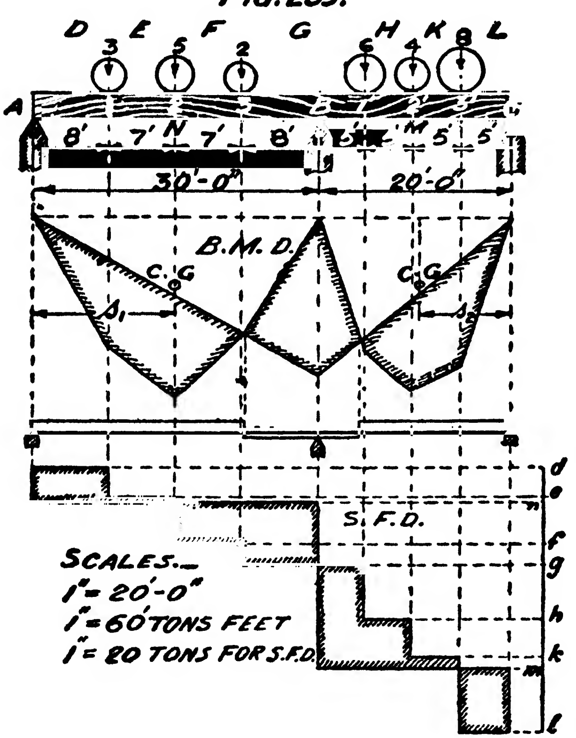
$R_A\ (20+25+15)+R_B\ (25+15)+R_C\ 15-(W_1\times 15\times 7\cdot 5)-(W\times 25\times 27\cdot 5)-(W\times 20\times 50)=M_D$, where W = load per foot run over first two spans = 1 ton per foot run.

W_1 = „ „ „ „ „ the third span = 2 tons „ „ „

M_D = Bending moment at D = O.

$7\cdot 52\times 60+24\cdot 92\times 40+R_C\ 15-(2\times 15\times 7\cdot 5)-(1\times 25\times 27\cdot 5)-(1\times 20+50)=O.$

$$\therefore\ R_C=\frac{-451\cdot 20-996\cdot 8+225+687\cdot 5+1000}{15}=\frac{1912\cdot 5-1448}{15}=\frac{464\cdot 5}{15}.\ R_C=30\cdot 966 \text{ tons.}$$



$R_D=75$ tons $-R_A+R_B+R_C=75-7\cdot 52+24\cdot 92+30\cdot 96=11\cdot 60$ tons. $R_D=11\cdot 60$ tons.

EXAMPLE 4:— A beam continuous over two spans of 30 and 20′ respectively carries concentrated loads as shown in figure 235. Draw bending moment and shearing force diagrams.—

SOLUTION :— Determine the reactions R_A, R_B and R_C as if two beams are independent and merely supported, thus.—$R_A\times 30=3\times 22+5\times 15+2\times 8$. $\therefore\ R_A=\frac{157}{30}=5\cdot 23$ tons. Then $R_B=10-5\cdot 23=4\cdot 77$ tons. Now calculate the free bending moments at different points and plot them as shown. $BM_1=R_A\times 8=5\cdot 23\times 8=41\cdot 84$ tons feet; $BM_2=5\cdot 23\times 15-3\times 7=78\cdot 45-21=57\cdot 45$ tons feet.

$BM_3 = R_B \times 8 = 4{\cdot}77 \times 8 = 38{\cdot}16$ tons feet.

Now coming to the right span we get—

$R_B \times 20 = 8 \times 5 + 4 \times 10 + 6 \times 15. \quad \therefore R_B = \frac{170}{20} = 8{\cdot}5$ tons feet.

$R_C = 18 - 8{\cdot}5 = 9{\cdot}5$ tons feet.

$BM_1' = R_B \times 5 = 8{\cdot}5 \times 5 = 42{\cdot}5$ tons feet,

$BM_2' = 8{\cdot}5 \times 10 - 6 \times 5 = 85 - 30 = 55$ tons feet.

$BM_3' = R_C \times 5 = 9{\cdot}5 \times 5 = 47{\cdot}5$ tons feet.

To any suitable scale plot these values of the free bending moment as shown in the figure. By any convenient method get the centres of gravity of these two bending moment diagrams, and calculate the areas as well.

Then apply Clapeyron's Theorem of Three Moments thus:—

$$M_A\, S_1 + 2\, M_B\, (S_1 + S_2) + M_C\, S_2 = 6\left(\frac{A_1\, s_1}{S_1} + \frac{A_2\, s_2}{S_2}\right)$$

$A_1 = 1006$ tons feet ^{2}, $A_2 = 725$ tons feet^2, $s_1 = 15'$, $s_2 = 9{\cdot}5'$.

$$M_A\, 30 + 2\, M_B\, (30 + 20) + M_C\, 20 = 6\left(\frac{1006 \times 15}{30} + \frac{725 \times 9{\cdot}5}{20}\right).$$

$M_A\, 30 + M_B\, 100 + M_C\, 20 = 5084{\cdot}22$. Dividing the whole expression by 100 we get $\cdot 3\, M_A + M_B + \cdot 2\, M_C = 50{\cdot}84$ tons feet. Since M_A and $M_C = 0$,

$\therefore M_B = 50{\cdot}84$ tons feet Answer.

Plotting this value at the support B we get the resulting bending moment diagram as shown shaded.

Magnitudes of reactions are now to be found out thus.—$R_A \times 30 - 2 \times 8 - 5 \times 15 - 3 \times 22 = -50{\cdot}84$. The bending moment at B is negative.

$$R_A = \frac{16 + 75 + 66 - 50{\cdot}84}{30} = 3{\cdot}54 \text{ tons.}$$

Taking moment centre about B we have.

$R_C \times 20 - 6 \times 5 - 4 \times 10 - 8 \times 15 = M_B = -50{\cdot}84$.

$$R_C = \frac{30 + 40 + 120 - 50{\cdot}84}{20} = 6{\cdot}96 \text{ nearly.}$$

$R_B = 28 - 3{\cdot}54 - 6{\cdot}96 = 17{\cdot}5$ tons.

The only difficulty in this example is to determine the centre of gravity of the free bending moment diagram. The best way to get this point is to cut the templates in tracing paper and suspend the same from two positions alternately and the intersecting point of these two straight lines is the C. G. of the free bending moment diagram.

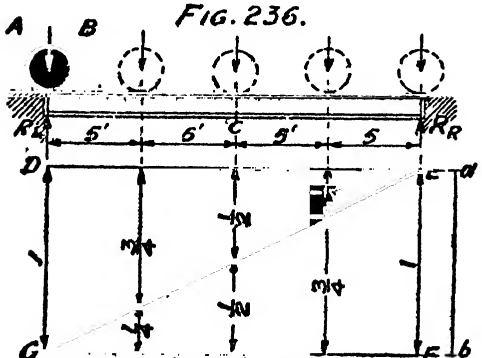
Chapter XIV.

INFLUENCE LINES AND INFLUENCE DIAGRAMS.

Definition:—An influence line is a line which shows the variation of a reaction, bending moment, shear and stress, when a single concentrated load moves over a beam or girder. This influence line is most useful in determining the reactions at the supports and bending moment, shear or stress at a particular point in a beam or girder for a moving load or a system of moving loads.

This can be made clear by the following examples:—Figure 236 shows a beam supported at both ends on a clear span of 20 feet and a single concentrated load of one ton rolls over it. It is required to determine the reactions at the supports for the various positions of this moving load over the beam by an influence line.

FIG. 236.



SCALES.—1"=10'-0", 1"=1 TON.

Here the line *a b* is taken to represent the given load AB to a scale of 1"=1 ton, and a rectangle DEFG is drawn taking DG and EF equal to *a b*. The diagonal GE is drawn. Now consider the load to be on the left support, the reaction at this support is equal to the load itself=1 ton, and the ordinate DG measures 1 ton, the reaction at the right support is zero. When the load moves $\frac{1}{4}$ of the span say 5′ the reaction at the left support$=R_L \times 20 = 1 \times 15$. $\therefore R_L = \frac{15}{20} = \frac{3}{4}$ ton, and $R_R = 1 - \frac{3}{4} = \frac{1}{4}$ ton. You observe these two values $\frac{3}{4}$ and $\frac{1}{4}$ ton directly under the load at $\frac{1}{4}$ of the span. Similarly when the load stops at $\frac{1}{2}$ and $\frac{3}{4}$ of the span the magnitudes of the reactions on both the supports can be determined by the ordinates of the triangles DGE and EFG. Hence for any position of the moving load the magnitude of the reaction at the left support can be measured by the ordinate of the triangle DGE and at the right support from the ordinates of the triangle EFG. The diagonal GE shows the variation of a reaction for this moving load AB, hence this line GE is the reaction influence line for the left

support and the line EG is the reaction influence line for the right support. (See example 30 page 41).

Now suppose that a concentrated load of 10 tons rolls over the same beam instead of one ton. The magnitudes of reactions on both the supports can be obtained by multiplying the load with the ordinates of the unit load thus.—The reaction at the left support when the 10 ton is directly on the left support $= 10 \times 1 = 10$ tons and the reaction at the right support is zero. Again when this load is at $\frac{1}{4}$ of the span the reactions on the left and right supports are $10 \times \frac{3}{4} = 7{\cdot}5$ tons and $10 \times \frac{1}{4} = 2{\cdot}5$ tons respectively. Similarly for every position of the rolling load on the beam the magnitudes of reactions can be easily obtained.

Next, instead of a single concentrated load, let a series of loads such as the locomotive axle loads pass over the bridge as shown in the figures 237 and the magnitudes of reactions on both the supports may be determined. Here D′G′ and E′F′ equal to one ton and E′G′ is the diagonal. The magnitude of reaction on the left support for this position of the locomotive is equal to $(8 \times {\cdot}98) + (8 \times {\cdot}91) + (9 \times {\cdot}84) + (9 \times {\cdot}75) + (9 \times .65) + (8 \times {\cdot}47) + (8 \times {\cdot}38) + (8 \times {\cdot}29) = 44{\cdot}40$ tons, and the reaction at the right abutment $= 67 - 44{\cdot}40 = 22{\cdot}60$ tons.

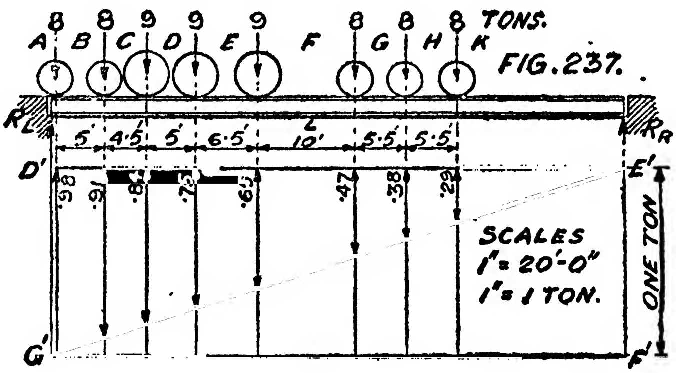


FIG. 237.

Maximum rolling load reaction on the supports will occur when the locomotive axle loads approach towards one end of the girder as shown in figure 237. At times we get the maximum shear when the first load AB is off the girder and the heavier loads approach to one end of the girder. In this position of the axle loads, the reaction may be maximum on the left support, if the same loads were to move towards the right end of the girder, the magnitude of the reaction will not be maximum as the heavier loads are not nearer to this end. (See also plate I fig. 58). By one or two trials we are to determine the maximum shear.

INFLUENCE DIAGRAM AND INFLUENCE LINE FOR BENDING MOMENTS DUE TO A SINGLE ROLLING LOAD OVER A BEAM OR GIRDER.

Figure 238 (*a*) is a supported beam. It is required to find the bending moment at P for every position of a moving load W which is equal to one ton. Let the point P be selected exactly at the centre of the span for the sake of simplicity. Now $R_1' \times S = W \times s_1$. $\therefore R_1 = \frac{W s_1}{S}$. $B.M_P = R_1 \times s = \frac{W s_1}{S} \times s$. Here s and $s_1 = \frac{S}{2}$. $\therefore BM_P = \frac{WS}{S2} \times \frac{S}{2} = \frac{WS}{4}$. Since $W = 1$, $BM_P = \frac{S}{4}$, $\therefore$ the ordinate $c\,d = \frac{S}{4}$. We know the bending moments at the supports are equal to zero. Now the triangle $a\,c\,b$ is the bending moment diagram for a simple supported beam with the load W at P. Next we will prove that for any position of the load W on the beam the bending moment at P will be equal to the ordinate drawn from the load vertically to the triangle $a\,b\,c$ at that position thus.— Suppose the load W moves from P to P′ at a distance $\frac{1}{4}$ S from the right support. Now $R_1 \times S = W \times \frac{1}{4} S$, $\therefore R_1 = W \times \frac{1}{4} S \times \frac{1}{S} = \frac{W}{4}$. $BM_{P_1} = R_1 \times \frac{S}{2} = \frac{W}{4} \times \frac{S}{2} = \frac{S}{8}$. Next we will have to prove that the ordinate $e\,h$ drawn from the load to the triangle $a\,b\,c$ is equal to $\frac{S}{8}$. The two triangles $d\,c\,b$ and $e\,h\,b$ are similar, therefore $d\,c : e\,h :: d\,b : e\,b$. $\therefore c\,d \times e\,b = e\,h \times d\,b$. $\therefore e\,h = \frac{c\,d \times e\,b}{d\,b}$, here $c\,d = \frac{S}{4}$, $e\,b = \frac{S}{4}$ and $d\,b = \frac{S}{2}$. Substituting these values in the above we have

$$e\,h = \frac{\frac{S}{4} \times \frac{S}{4}}{\frac{S}{2}} = \frac{S^2}{16} \times \frac{2}{S} = \frac{S}{8}$$

as before. Hence the ordinate $e\,h$ is equal to the bending moment at P or for the position of W at P_1 the bending moment at P is equal to the ordinate $e\,h$, and for any position of W between A and B the bending moment will be shown by the corresponding ordinate of the triangle $a\,b\,c$.

It is quite clear that the line $a\,c\,b$ is the influence line for the bending moment at P due to the passage of the unit load W across the span and the triangle $a\,b\,c$ is the influence diagram.

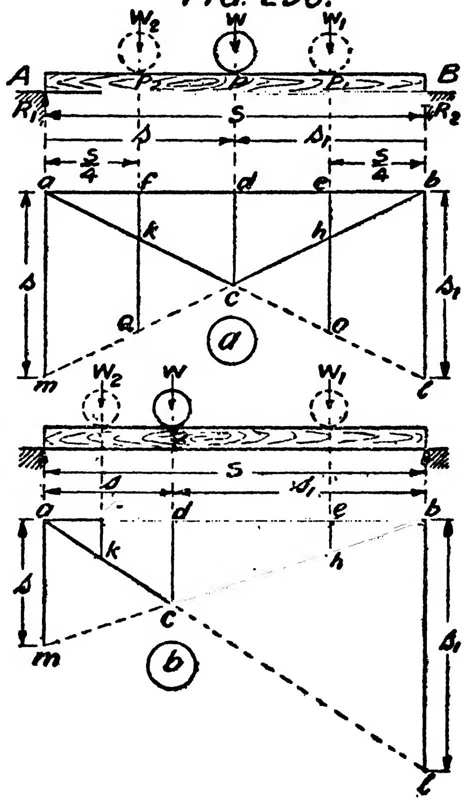
Produce $a\,c$ and $b\,c$ to meet the reaction lines R_2 and R_1 at l and m. Take for instance the straight line $a\,l$, it starts from a, and at a

distance of $\frac{1}{4}$ S measured horizontally it has the vertical ordinate *f k* equal to $\frac{1}{8}$ S, at a distance of $\frac{1}{2}$ S the vertical ordinate is *d c* which is equal to $\frac{2}{8}$ S, at a distance of $\frac{3}{4}$ S the vertical ordinate *e o* should be equal to $\frac{3}{8}$ S and at a distance of S the vertical ordinate is $\frac{4}{8}$ S which is equal to s_1. Similarly the straight line *b c* if produced to meet the reaction line R_1, it will meet the reaction line at *m* and the vertical distance *a m* will be exactly equal to *s*.

From this it is observed that this is the equation of a straight line inclined to the horizontal by an angle whose tangent is $\frac{s}{S}$.

Now we can get the point *c* without calculation by setting down from *b* a depth equal to s_1 and from *a* a depth equal to *s* and joining across, we get the required point *c*.

FIG. 238.



As an illustration see figure 238 (*b*). Now let the point P be selected any where on the beam and we desire to find the bending moment at P for every position of a unit moving load W.

Plot from *b*, a distance *b l* down equal to s_1 and from *a*, a distance *a m* down equal to *s* and join across, the intersecting point is c. Then the triangle *a b c* and the line *a c b* are the influence diagram and the Influence line respectively for the bending moment at P due to the passage of the unit load W across the span.

If instead of one load let there be a group of loads such as exle loads of a locomotive, roll over the span, the total bending moment at P is equal to the summation of all the products of the axle loads and the corresponding ordinates in the influence diagram due to a unit load. For example assume the position of three

axle loads to be at P_2 P P_1 fig. 238 (*a*), and call them W_2 W W_1. Then the total moment at $P = (W_2 \times fk) + (W \times cd) + (W_1 \times eh) = (W_2 \times \frac{1}{8} S) + (W \times \frac{1}{4} S) + (W_1 \times \frac{1}{8} S)$.

When the beam is traversed by a uniformly distributed load extending throughout the span length, the bending moment at P is equal to the area of the influence diagram *a b c* multiplied by the load intensity. The students will get clear conception of all these in the numerical examples which follow shortly.

INFLUENCE DIAGRAM AND INFLUENCE LINE FOR THE SHEARING FORCE DUE TO A SINGLE ROLLING LOAD OVER A BEAM OR GIRDER.

The students are advised to study the shearing force diagram of example 30 page 41, before proceeding to follow the influence line of shear in this page, as the same diagram is again brought in here.

FIG. 239.

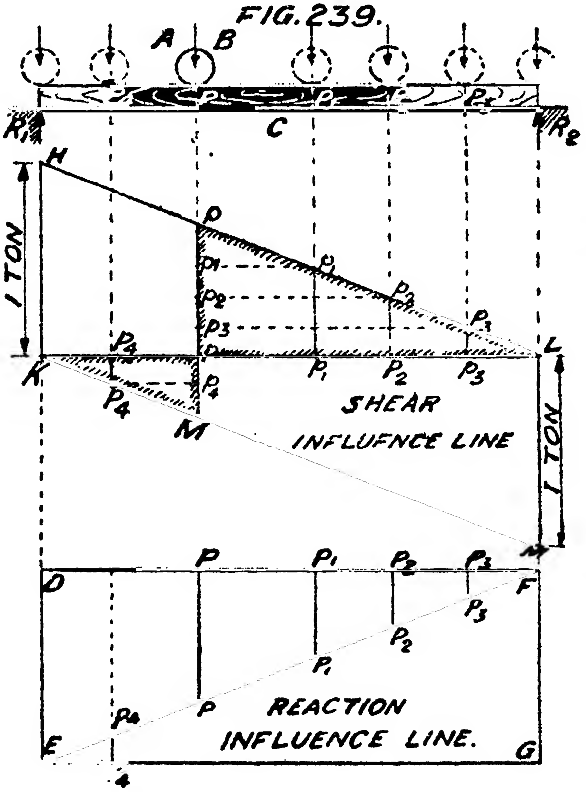


Figure 239 represents a supported beam and a consentrated load AB rolls over it. It is required to find the shearing force at P for every position of moving load AB over it.

Now then, when the moving load is at the left support the shear at P is zero, when it moves to P_4 the shear at P is equal to—P_4 P_4. Similarly when the load moves to P, P_1, P_2 and P_3 the shear at P is equal to the respective ordinates P P, P_1 P_1, P_2 P_2, P_3 P_3; the total shear at P is equal to PP+P_1 P_1+P_2 P_2+P_3 P_3−P_4 P_4 The reaction influence lines also are shown in fig. 239 for the verification of this fact.

The line KMPPL is the shear influence line for the section P in this beam, and the two triangles KPM and PPL together form one influence diagram for the same section.

If instead of a unit load any other such as locomotive axle

loads were to travel, the ordinates got for the unit load are to be multiplied by their respective axle loads.

When a uniformly distributed load travels and covers the whole length of the beam, the shear at the section P is equal to the area of the influence diagram PPL minus the area of the diagram KPM.

EXAMPLE 1 :—A beam of 60' span is traversed by a locomotive whose axle loads are as shown in figure 240. First determine the bending moment and shear at a section P, 25 feet from the left support for the given positions of the loads ; secondly determine the positions of these loads for maximum bending moment and shear at the section P.

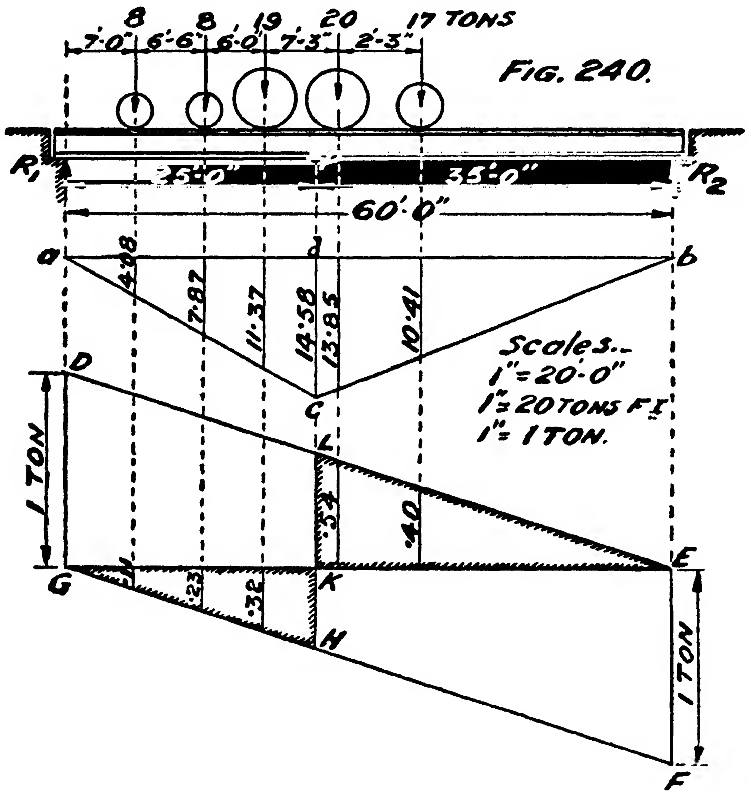


FIG. 240.

SOLUTION :—First draw the influence diagram $a\,b\,c$ placing one ton at P thus—$R_1 \times 60 = 1$ ton $\times 35$, $\therefore R_1 = \frac{35}{60} = \frac{7}{12}$ $R_2 = \frac{5}{12}$.

Then B, $M_P = R_1 \times 25 = \frac{7}{12} \times 25 = 14\ 58$ tons feet. Draw $d\,e = 14{\cdot}58$ tons to the scale. Now the line $a\,c\,b$ is the influence line of moments for the section of the beam at P. The ordinates under the axle loads in the influence diagram give the bending moments in tons feet at the section P due to a unit load at each of these positions. The total bending moment at P for the position of the

axle loads is equal to $(8 \times 4{\cdot}08)+(8 \times 7{\cdot}87)+(19 \times 11{\cdot}37)+(20 \times 13{\cdot}85)+(17 \times 10{\cdot}41)=765{\cdot}6$ tons feet.

The figure DGEF is the shearing force diagram for a unit rolling load, and the influence line for a unit load at P is GHKLE. Total shear at P is equal to $(-8 \times {\cdot}11)-(8 \times {\cdot}23)-(19 \times {\cdot}32)+(20 \times {\cdot}54)+(17 \times {\cdot}4)=-8{\cdot}8+17{\cdot}6=8{\cdot}8$ tons.

POSITION OF THE LOADS FOR MAXIMUM BENDING MOMENT AT P.

To get the position of these loads for the maximum bending moment at P see figure 241. Before locating the position of these loads we will have to know where the line of action of the resultant of this system of loads passes Action line of the resultant must pass between 19 and 20 ton axle loads. We can verify this by drawing polar diagram and its corresponding funicular polygon. Therefore the maximum bending moment at P will occur when 19 or 20 ton load stops at the point P. Now let the axle

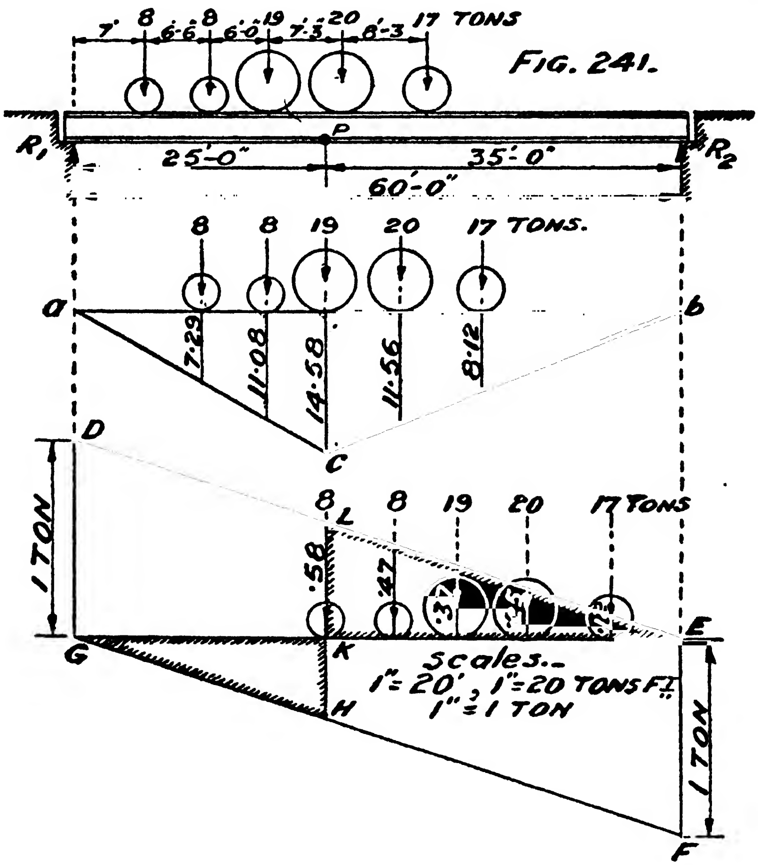


FIG. 241.

loads move to the right so that the 19 ton axle load be exactly at point P, then the ordinates for the unit load under the axle loads 8, 8, 19, 20 and 17, tons are 7 29, 11·08, 14·58, 11·56 and 8·12 tons feet respectively, the total bending moment$=(8 \times 7{\cdot}29)+(8 \times 11{\cdot}08)+(19 \times 14{\cdot}58)+(20 \times 11{\cdot}56)+(17 \times 8{\cdot}12)=793{\cdot}22$ tons feet.

Again bring in the 20 ton axle load under the point P, the ordinates in the influence diagram for the unit load under the axle

loads 8, 8, 19, 20 and 17 tons are 3·06, 6·85, 10·35, 14·58 and 11·14 tons feet respectively and the total bending moment at P=(8×3·06) +(8×6·85)+(19×10·35)+(20×14·58)+(17×11·14)=756·91 tons feet. It is evident now that position of the loads gives us the maximum bending moment, when 19 ton load is at P. Any other position of this system of loads gives us always less.

POSITION OF THE LOADS FOR THE MAXIMUM SHEAR AT P.

Draw the shear influence line for a unit load at P as shown in fig 241. Now take a tracing paper and trace out in a fine pointed pencil the axle loads and vertical ines and place this tracing paper so that the piloting or the front 8 ton wheel be exactly at P. The ordinates for the unit load under these axle loads 8, 8, 19, 20 and 17 tons, are ·58, ·47, ·37, ·25 and ·12 tons respectively and the total shear=(8×·58)+(8×·47)+(19×·37)+(20×·25)+(17×·12)=22·56 tons.

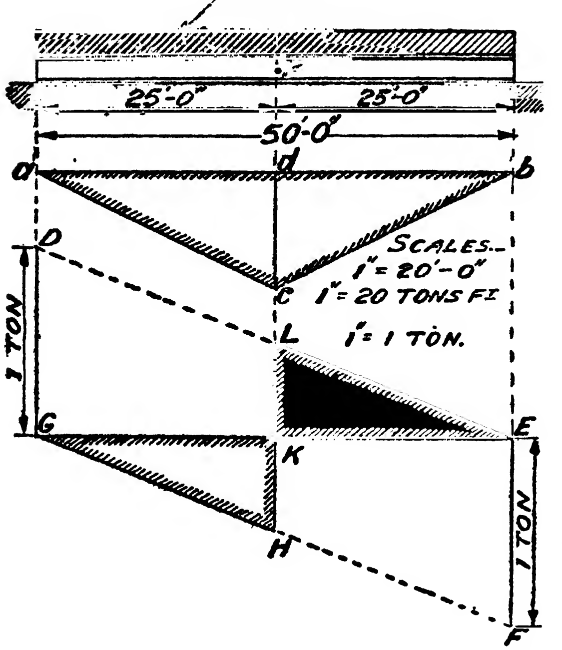
Next bring in the second 8 ton wheel at P, then the front 8 ton wheel will be to the left of the point P and gives you the negative shear and the heavier loads to the right of P give you more positive shear. In this position of the loads, the ordinates in the influence diagram for the unit load under the axle loads 8, 8, 19, 20 and 17 tons are −·3, ·58, ·48, ·36 and ·22 respectively and the total shear at P=(8×−·3)+(8×·58)+(19×·48)+(20×·36)+(17×·22)=24·7−2·4=22·3 tons. Any other position may give you less shear at P and to verify this, bring the 19 ton load at P and the ordinates for the unit load under the axle loads 8, 8, 19, 20 and 19 tons are −·2, −·3, ·59, ·46 and ·34 respectively and the total shear at P=(8×−·2)+(8×−·3)+(19×·59)+(20×·46)+(17×·34)=26·19−4·0=22·19 tons. This is less than the previous one, and the maximum shear at P will occur when the front 8 ton wheel stops at P. The maximum shear at P= 22·56 tons.

EXAMPLE 2 :—A beam of 50 feet span supported at the ends is traversed by a uniformly distributed load of 2 tons per foot run. Determine the bending moment and shear at the centre of the beam when the advancing distributed load covers half and full span. (See fig. 242)

SOLUTION :—The point P is at the centre of the span. Draw the influence diagram *a b c* for a unit load thus. $R_1 \times 50 = W \times 25$. $R_1 = \frac{W\ 25}{50} = \frac{1}{2}$. $BM_P = R_1 \times 25 = \frac{1}{2} \times 25 = 12{\cdot}5$ tons feet. plot $c\ d = 12{\cdot}5$ tons feet $= \frac{WS}{4} = \frac{1 \times 50}{4} = 12{\cdot}5$ tons feet, to a scale of 1″ = 20 tons feet.

Here you observe that every point is loaded and at every point there is an ordinate for that load under the influence diagram for a unit load, and therefore the bending moment for any portion of the distributed load on the beam is represented by that portion of the influence diagram under the distributed load.

FIG. 242.



When the distributed load advances to the centre of the beam, that is 25′ from the left end, the bending moment at P is equal to the area of the influence diagram *a d c* multiplied by the intensity of the load$=\frac{25}{2}\times 12\ 5$ tons feet $\times 2 = 312\ 5$ tons feet.

Again when the advancing distributed load covers the whole span the bending moment at P is equal to the area of the influence diagram *a b c* which is equal to $\frac{50}{2}\times 12{\cdot}5$ tons feet $\times 2 = 625$ tons feet and this should be equal to $\frac{WS}{8} = \frac{50\times 2\times 50}{8} =$ 625 tons feet.

Shearing force:—When the distributed load covers half of the span the shear at P is equal to the area of the shear influence diagram GKH which is drawn for a unit load$=\frac{25}{2}\times\frac{1}{2}\times 2 = 12{\cdot}5$ tons, in this case this is negative shear.

Again when the whole span is covered by the distributed load the shear is equal to the area of the influence diagram LKE—the area of the influence diagram GKH$=$O.

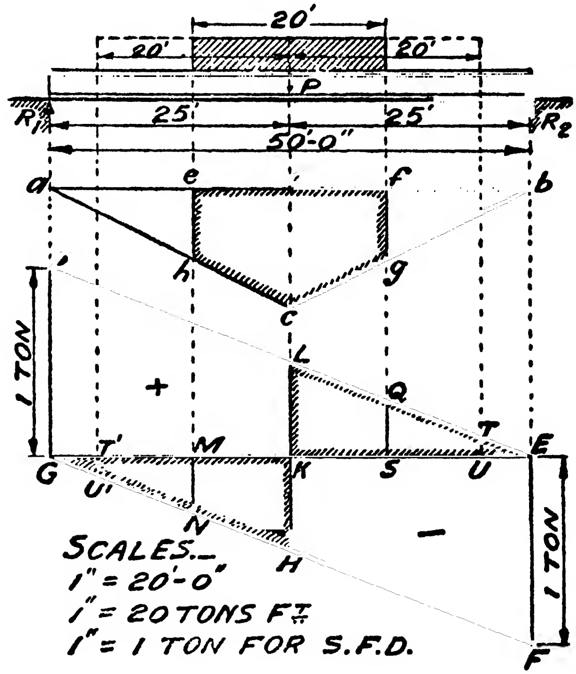
Note:—(1) In computing the areas of the bending moment influence diagram, horizontal distances are to be taken in linear scale and the vertical ordinates in tons feet or tons inches as the case may be.

(2) In shearing force influence diagram the horizontal distances are to be taken in linear scale and verticals in load scale.

EXAMPLE 3:—The same beam of example 2 is traversed by a partially distributed load of 2 tons per foot run extending over a distance of 20 feet. Determine the position of this distributed load which would cause bending moment at P the centre of the beam a maximum. What is the shearing force at P? At what position of the distributed load the shear at P be a maximum?

SOLUTION:—Since the area of the influence diagram under the distributed rolling load represents the bending moment at P, there is no other position for the distributed load but centre of the beam to give the maximum area. Therefore let the centre line of the distributed load coincide with the point P as shown in figure 243. Then the bending moment at P is equal to the area of the figure $c\,f\,g\,c\,h$ multiplied by the intensity of the load. Area of this figure is equal to the area of the rectangle $e\,f\,g\,h$ + the area of the triangle $g\,c\,h = (7\frac{1}{2} \times 20) + (\frac{20}{2} \times 5) = 150 + 50 = 200$. Bending moment at $P = 200 \times 2 = 400$ tons feet.

FIG. 243.



Shearing force at P for this position is equal to the algebraic sum of the areas MNHK and KLQS, which is equal to KLQS − MNHK = O.

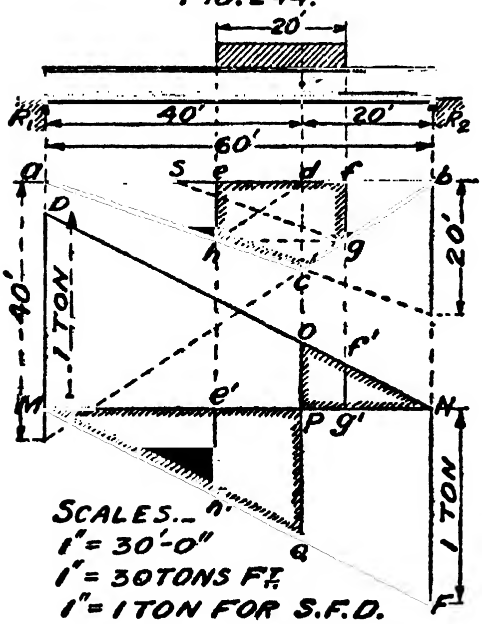
The position of the load for maximum shear at P, is to make the front or the rear end of the distributed load to coincide at P. Assuming the motion of the load is from left to right, the maximum negative shear is represented by the area of the shear influence diagram KHU'T' multiplied by the load intensity $= \frac{\cdot 1 + \cdot 5 \times 20 \times 2}{2} = -12$ tons. We get the maximum positive shear of +12 tons at P when the rear end of the

distributed load coincides with P. In this case the area of the figure LKTU is to be taken.

Note:—In getting the position of the load for maximum bending moment at P when P is not at the centre of the beam, care must be taken that the ordinates projected from the extreme points of the travelling load to the bending moment influence diagram should always be equal viz. *e h* must be equal to *f g* in any case, you will have to shift the load either to right or to left of the point P till this condition is fulfilled. See the next example.

EXAMPLE 4:—Given a supported beam of 60′ span which is traversed by a partial uniformly distributed load of 2 tons per foot run for 20 feet long. Determine the position of this distributed load for a maximum bending moment at P. The position of the point P is 20 feet from the right support. Determine the shearing force for this position.

FIG. 244.



SOLUTION:—Draw the influence diagram *a b c* for a unit load as shown in figure. $R_1 \times 60 = W \times 20$. $R_1 \frac{20}{60} = \frac{1}{3}$. Bending moment at $P = R_1 \times 40 = \frac{1}{3} \times 40 = 13{\cdot}33$ tons feet. Draw $c\,d = 13{\cdot}33$ tons feet. Then the triangle *a b c* is the influence diagram for the unit load W at P.

As suggested in the note of the last example the ordinates *e h* and *f g* projected from the extreme ends of the distributed load must be equal to each other. This truth we learnt from the previous example by locating the point P exactly at the centre of the beam. In this example we will have to calculate by similar triangles the distance *c h* or *c g* as follows.—the two triangles *c h g* and *c a b* are similar, $c\,a : c\,h :: a\,b : h\,g$, $\therefore$ $c\,a \times h\,g = c\,h \times a\,b$. We know $c\,d = 13{\cdot}33$, $a\,d = 40'$ and $c\,a = \sqrt{40^2 + 13{\cdot}33^2} = 42{\cdot}16$ feet. Substituting this value in the equation we have $42{\cdot}16 \times 20 = c\,h \times 60$, $\therefore$ $c\,h = \frac{42{\cdot}16 \times 20}{60} = 14{\cdot}05$ feet. Taking $c\,h = 14{\cdot}05$ feet in the linear scale, we draw the

figure $e\,f\,g\,c\,h$ in the bending moment influence diagram $a\,b\,c$. In this position of the distributed moving load the bending moment at P will be a maximum. By plane geometry we know that if two parallel straight lines are intersected by another two parallel straight lines, this will have parallel segments, and these parallel segments are equal to each other. In this example take $a\,s = 20'$ and $b\,d$ is already equal to $20'$, and from s and d draw straight lines parallel to $a\,c$ and $b\,c$, then $h\,g$ is the common segment and it must be equal to $a\,s$ or $b\,d$. From h and g erect perpendiculars $h\,e$ and $g\,f$, and these must be equal to each other.

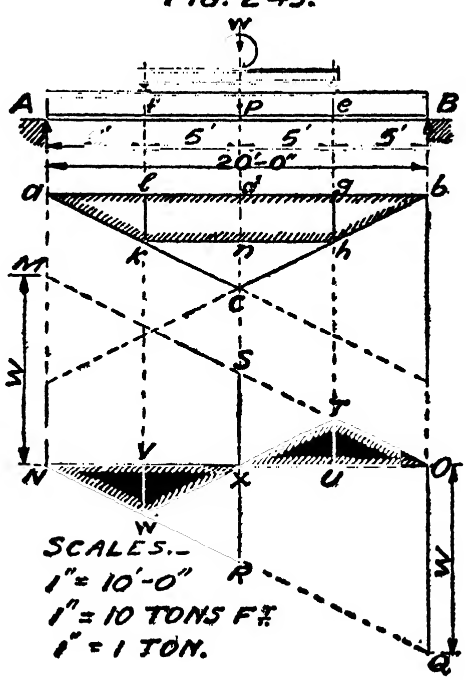
Shearing force:—The shear influence diagram for the point P is as usual the figure MNOPQM. and the value of the shear at P for the position of the distributed load is equal to the area of the figure OP $f'\,g'$ — the area of the figure PQ $h'\,e' = \left(\frac{\cdot 2 + \cdot 36}{2} \times 6{\cdot}5 \times 2\right) - \left(\frac{\cdot 45 + 66}{2} \times 13{\cdot}5 \times 2\right) = 3\,64 - 14{\cdot}98 = -11{\cdot}38$ tons.

EXAMPLE 5:—A beam 20 feet long is supported on both the ends and carries two smaller beams at 10 feet interval in the centre as shown. These two small beams carry a cross beam 10′ long with a concentrated load W = 1 ton placed exactly at its centre as shown in the figure 245.

Draw bending moment and shear influence lines for a point in the centre of the beam and show clearly the shape of the influence diagram.

SOLUTION:—Assume the load W to be exactly at the centre of the bottom main girder, draw the influence line $a\,c\,b$ for this position of the load at P as usual. Now the bending moment at P for any position of the load W between P and B is represented by the ordinate in the triangle $c\,d\,b$ at that position, similarly

FIG. 245.



the bending moment at P for any *position of the load between P and A is represented by the ordinate in the triangle a d c.*

Actual loads on the main girder AB are at *f* and *e* and each is equal to $\frac{W}{2}$, because the load is exactly in the centre of the top beam. Bending moment at P for $\frac{W}{2}$ at *e* is equal to $\frac{W}{2} \times g\ h$, simialrly bending moment at P for $\frac{W}{2}$ at $f = l\ k \times \frac{W}{2}$. Therefore the total bending moment at $P = \frac{W}{2} \times g\ h + \frac{W}{2} \times l\ k = \frac{W}{2}\ (\ g\ h + l\ k\ )$. Substituting the numerical values in these we get the total bending moment at $P = \frac{1}{2}\ (\ 2\frac{1}{2} + 2\frac{1}{2}\ ) = \frac{5}{2} = 2\frac{1}{2}$ tons feet. Plot *d n* equal to $2\frac{1}{2}$ tons feet, then the ordinates *g h*, d *n* and *l k* are equal to one another and a line joining points *k*, *n* and *h* must be a straight line. Hence the bending moment influence line is *a k n h b*. and not *a c b*. In such case as this the bending moment influence line is to be drawn, by first drawing the influence line for a unit load at point P and then projecting vertical lines from *e* and *f* where the loads are actually acting, to intersect the lines *a c*, and *b c* at *k* and *h* respecively and finally points *k* and *h* are to be joined. The line *a k h b* is the influence line for the bending moment at P. The figure shown shaded is the bending moment influence diagram.

Shear influence line:—Draw the shearing force diagram for a travelling unit load, and draw the shear influence line NRSO for a unit load W at P assumed to be on the bottom beam. Now the shear at P for any position of the load W between P and B is represented by the ordinate in the triangle SXO; ordinate in this triangle gives you positive shear. Similarly the shear at P for any position of the load W between P and A is represented by the ordinate in the triangle XRN.

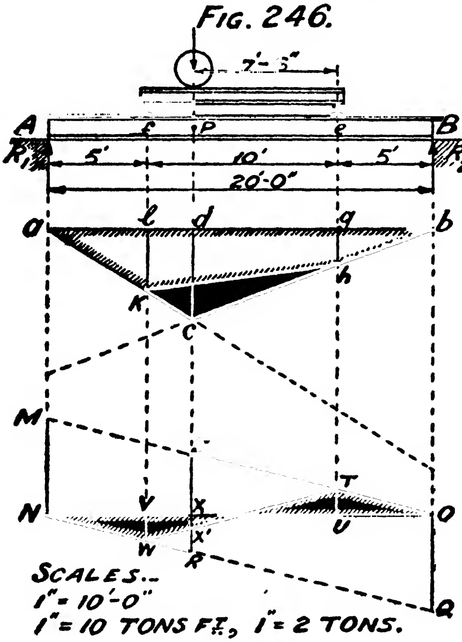
Actually the loads are at *e* and *f* and each is equal to $\frac{W}{2}$ and for these positions of the loads at *e* and *f* the ordinates in the shear influence line are TU and VW respectively, then the total shear at P is equal to $TU - VW = O$. You know the ordinate TU is positive and VW is negative but the ordinate TU is equal to the ordinate VW and hence the total shear at P=O. The positive shear TU gradually reduces itself into nothing at X and from X the negative shear increases gradually from nothing to the ordinate VW at *f*.

The line joining TXW must be a straight line. The shear influence line in this case is NWTO and the line NO is common as usual,

and the shear influence diagram is shown shaded. In such type of example as this, first the shearing force diagram is to be drawn for a travelling unit load, and a shear influence line for the position of the load at P. Laslty from the actual load points such as *e* and *f* vertical lines are to be drawn to intersect the influence line at W and T and these two points are to be joined with a straight line.

Note:—Suppose the load W is not exactly at centre and remains to one side of the beam say nearer to *f*, but still the same constructions and procedure should be followed. For example see figure 246. In this the same method is followed and the shaded figures represent the bending moment and shearing force influence diagrams.

FIG. 246.



EXAMPLE 6:—A Warren girder has 6 panels each 10 feet long and carries a rolling load, greater than the span of 2 tons per foot run.

Find the maximum forces in the members of the second panel from the left end.

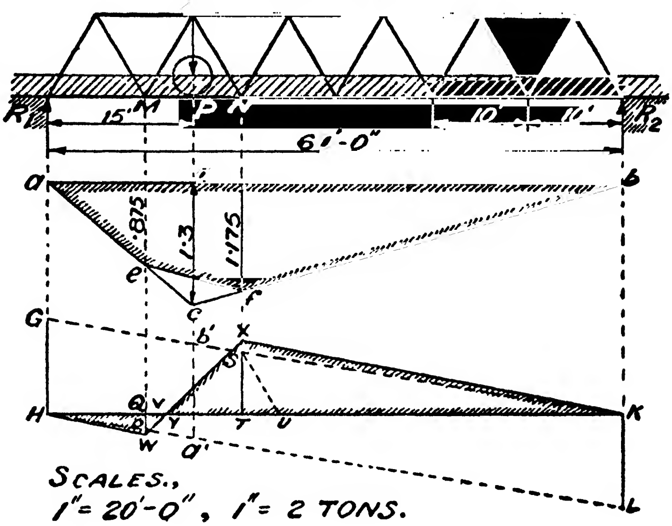
Draw the influence lines for bending moment and shear.

(B. Sc. (Eng.) Part II, 1931.)

SOLUTION;—This is a framed girder and you are required to determine the stresses in the members by means of influence lines, that is from the bending moment and shear force influence lines. See figure 72 page 56. There you observe that the chord stresses are determined from the bending moment diagram and the stresses in the sloping members, from the shearing force diagram. In this diagram of figure 247 you are required to determine the stresses in the members MN, MZ and ZN. Now bring a unit load W at P the centre of the second panel. The bending moment for a unit load at P is to be calculated as follows.— $R_1 \times 60 = W \times 45$ $\therefore$ $R_1 = \frac{45}{60}$ $W = \frac{3}{4}$ W, and $BM_P = R_1 \times 15 = \frac{3}{4}$ $W \times 15 = 11$ $\cdot 25$ tons feet; chord stress is equal to the bending moment divided by the depth of the girder $= 8{\cdot}66'$. Chord stress $= \frac{11{\cdot}25}{8{\cdot}66} = 1{\cdot}3$ tons nearly. Take

$cd = 1{\cdot}3$ tons and draw the influence line $a\ c\ b$, you must remember that the load W is not resting directly on the panel MN, but it rests on the rail bearer or stringer and transmits the pressure on the

FIG. 247.



joints M and N through the floor beams. See figures 245 and 246. Therefore actual loads are at M and N, and consequently draw vertical lines from M and N to intersect the influence line at e and f as shown; the influence line is now, $a\ e\ f\ b$ and the figure $a\ e\ f\ b\ a$ is the influence diagram. Since the distributed load is more than the span the maximum stress in M N is equal to the area of the influence diagram $a\ e\ f\ b\ a$. Area $=\left(\frac{1{\cdot}175+{\cdot}875}{2}\times 10\right)+\left(\frac{40}{2}\times 1{\cdot}175\right)+\frac{10}{2}\times{\cdot}875=38{\cdot}125$ tons. The load intensity is 2 tons per running foot, therefore the stress in M $N=38{\cdot}125\times 2=76{\cdot}25$ tons tension nearly.

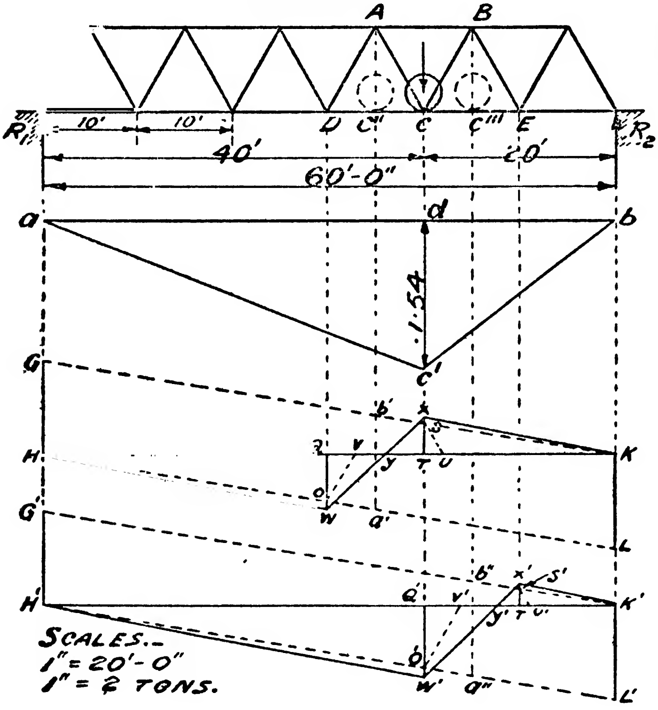
The stresses in ZM and ZN are to be determined from the shear influence line. The shear influence line for the unit load W at P is Ha' b'K shown dotted, assuming this girder to be of solid web.

When the load stops at M the negative shear at P is equal to the ordinate QO and when it moves further on to the point N the positive shear is represented by the ordinate ST, but these two ordinates QO and ST represent the vertical components of the stresses in ZM and ZN respectively. Therefore these two vertical ordinates are to be resolved

parallel to the inclined members ZM and ZN, the lines OY and SU are the resolved parts parallel to the members ZM and ZN respectively. From Q draw QW down equal to OV and draw TX up equal to SU, and join W X, W H and X K. The shear influence line is then HWX K and the figure HWXKH is the shear influence diagram for this second panel.

The maximum stress in ZM or ZN is represented by the area of the influence diagram XVK multiplied by the load intensity and the minimum stresses in these members are represented by the area of the influence diagram HWV multiplied by the load intensity.

FIG. 248.



Maximum stress in ZM or ZN $= \frac{48}{2} \times \cdot 775 \times 2 = 37\cdot 2$ tons. Suppose you are asked to determine the maximum stresses in the members AB BC and CA of the second panel on the top chord by the help of influence lines. Then proceed as follows.—See fig. 248, bring the unit load on the point C and calculate bending moment for this point. $R_2 \times 60 = W \times 40$. $\therefore R_2 = \frac{40\,W}{60} = \frac{2}{3}$ W, $BM_C = R_2 \times 20 = \frac{2}{3}$ $W \times 20 = \frac{40}{3} = 13\cdot 33$ tons feet. Chord stress = bending moment ÷ depth of the girder $= \frac{13\cdot 33}{8\cdot 66} =$ $1\cdot 54$ tons nearly. Plot $c'\,d = 1\cdot 54$ tons, join $a\,c'$ and $b\,c'$, then $a\,c'\,b$ is the influence line for the upper chord member AB.

Maximum compression in AB for the uniformly distributed load of 2 tons per foot run greater than the span is equal to the area of the

influence diagram $a\ c'\ b$ multiplied by the load intensity and is equal to $\frac{60}{2} \times 1\cdot54 = 46\cdot2$ tons. × 2 Tons/Ft

Stresses in AC and BC are to be determined from the shear influence line. The maximum stress in the member BC is to be determined from the shearing force in the second panel of the bottom chord from the right support and similarly the maximum stress in the member AC is to be known from the shearing force in the third panel of the bottom chord. Consequently bring the unit load into the position C″ and draw the influence diagram H a' b' K and draw the ordinates QG and ST from the adjacent joints D and C. Resolve QO and ST parallel to AD and AC and draw QW and TX equal to the resolved parts as usual, then HWXK is the influence line for the members AD, AC. The maximum stress in AC or AD is equal to the area of the influence diagram HWY multiplied by the load intensity $= \frac{36}{2} \times \cdot575 \times 2 = 20\cdot7$ tons.

Again bring the unit load on the centre of the second panel at C‴ and draw the influence line and diagram as shown in the lower figure separately, then the maximum stress in the member BC is equal to the area of the influence diagram H′ W′ Y′ multiplied by the load intensity $= \frac{48}{2} \times \cdot775 \times 2 = 37\cdot2$ tons, the same amount as we got for the members ZM and ZN of figure 247.

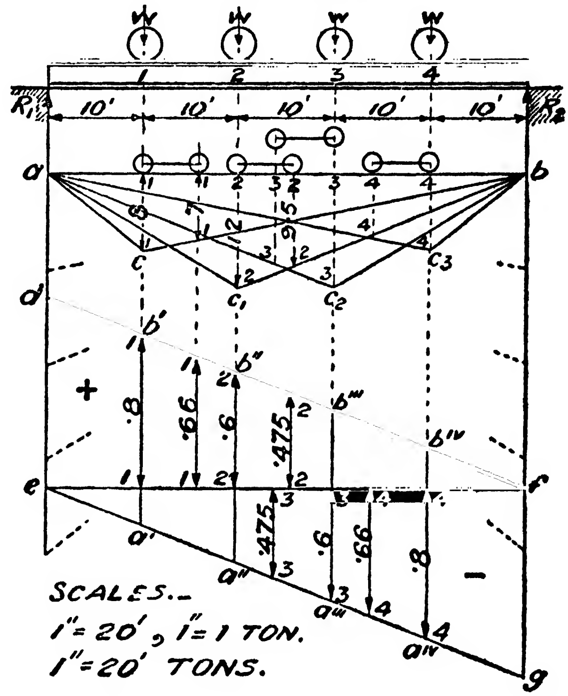
EXAMPLE 7:—One of the girders of a travelling crane of 50 feet span has to carry two rolling loads of 5 tons each spaced 6 feet apart. The wheels rest directly on the plate girder, and there is, therefore, no platform effect. Draw the Shearing Force and Bending Moment influence lines for points 10 feet apart and at these points find the maximum possible Bending Moment and Shearing Force due to the rolling loads.

(B. Sc. Eng. Part I, 1922.)

SOLUTION:—Draw the series of bending moment influence lines for the unit load W for the points 1—2—3—4 marked 10 feet apart as shown in the figure 249. Mark these lines as $a\ c\ b$, $a\ c_1\ b$, $a\ c_2\ b$ and $a\ c_3\ b$. To get the maximum bending moment in the influence line $a\ c\ b$ you will have to bring one of the loabs directly over the point c and the other load is to be brought either to the left or to the right of the point C. By bringing the other wheel to the right you get the maximum ordinates in the influence diagram. The ordinates are 1—1, 1—1 in the influence diagram and in no other position you get the maximum ordinates. Therefore measuring off the ordinates to the scale, you find they measure 8 and 7. The maximum bending moment for the position 1 is equal to

$(8\times 5)+(7\times 5)=75$ tons feet. Similarly the maximum ordinates in the influence diagrams $a\ c_1\ b$, $a\ c_2\ b$ and $a\ c_3\ b$ are $2-2$, $2-2$; $3-3$, $3-3$; and $4-4$, $4-4$ respectively as shown in the figure. Measuring all these ordinates to the scale you get the maximum bending moments at the second, third and fourth divisions $(12\times 5+9{\cdot}5\times 5)$, $(12\times 5+9{\cdot}5\times 5)$ and $(8\times 5+7\times 5)=107{\cdot}5$, $107{\cdot}5$ and 75 tons feet respectively.

FIG. 249.



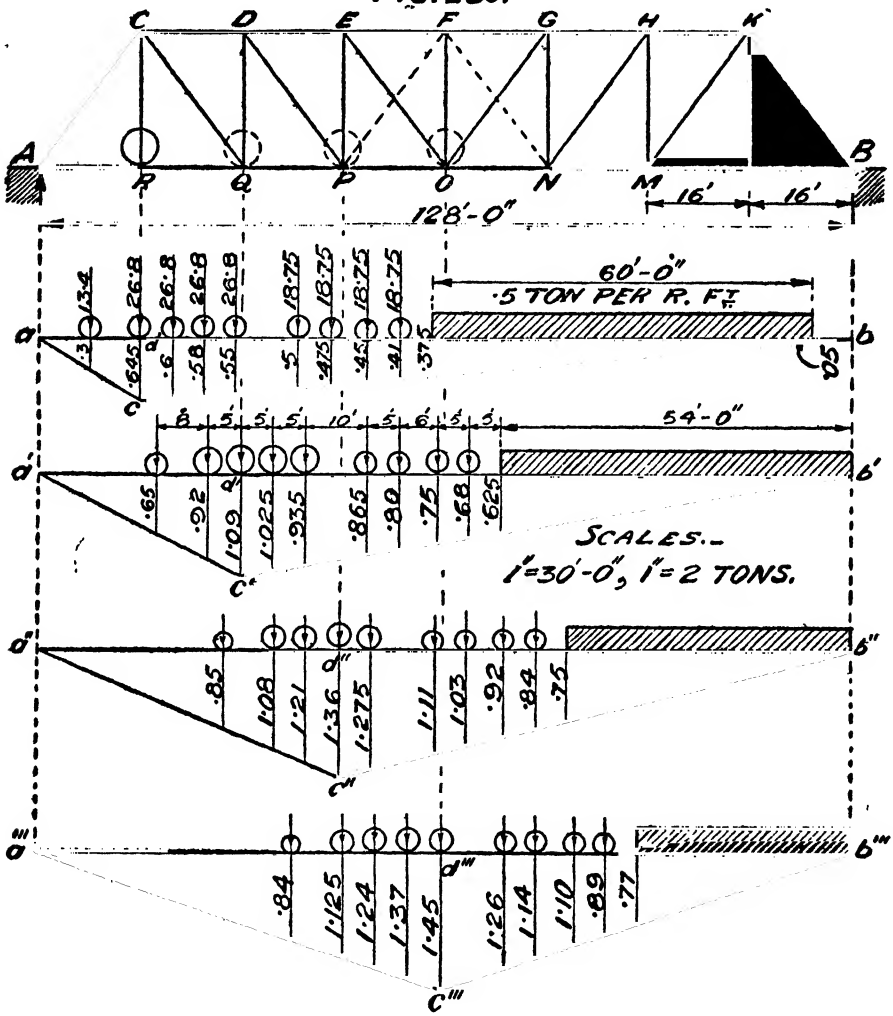
Maximum shearing force.—Shear influence lines for these four points are $e\ a^{I}\ b^{I}\ f$, $e\ a^{II}\ b^{II}\ f$, $e\ a^{III}\ b^{III}\ f$, and $e\ a^{IV}\ b^{IV}\ f$, and the maximum ordinates in these influence diagrams for the first four positions are $(1-1, 1-1)$ $(2-2, 2-2)$ $(3-3, 3-3)$ and $(4-4, 4-4)$. Measuring to the scale you get $(\cdot 8\times 5+\cdot 66\times 5)$, $(\cdot 6\times 5+\cdot 475\times 5)$, $\{(-\cdot 6\times 5)+(-\cdot 475\times 5)\}$, $\{(-\cdot 8\times 5)+(-\cdot 66\times 5)\}=7{\cdot}3$, $5{\cdot}375$, $-5{\cdot}375$ and $-7{\cdot}3$ tons. Minus sign shows negative shear.

EXAMPLE 8:—The live load on a through type Pratt Truss of 128 feet span, depth 22 feet consists of axle loads of a 195·6 ton locomotive followed by a train weighing ·5 ton per lineal foot as shown in the figure 250. Draw the bending moment and shear influence lines and determine the chord and diagonal stresses.

SOLUTION:—First determine the chord stress of the first panel that is, in the member ARQ and the length AQ is considered to be the length of one panel in Pratt Truss. Bring the unit load on the joint R. Then $R_1\times 128=W\times 112$, $\therefore\ R_1=\frac{112}{128}\times W=\frac{56}{63}W$ and $BM_R=\frac{56}{63}\times 16=$ 14·22 tons feet. The chord stress is equal to the bending moment divided

by the depth of the girder $= \frac{14 \cdot 22}{22} = \cdot 645$ tons. Plot $c\,d = \cdot 645$ ton and draw the influence line $a\,c\,b$ and the triangle $a\,c\,b$ is the influence diagram.

FIG. 250.



Next bring in the locomotive axle and train loads over this and see that the first heavy axle load is on the point d and note the values of the ordinates in the influence diagram as shown here. Then calculate the stress in the member ARQ as follows. $(13 \cdot 4 \times \cdot 3) + 26 \cdot 8\,(\cdot 645 + \cdot 6 + \cdot 58 + \cdot 55) + 18 \cdot 75\,(\cdot 5 + \cdot 475 + \cdot 45 + \cdot 41) + \left(\frac{\cdot 375 + \cdot 05}{2} \times 60 \times \cdot 5\right) =$
$4 \cdot 02 + 63 \cdot 65 + 34 \cdot 40 + 6 \cdot 37 = 108 \cdot 44$ tons.

To know exactly which position of the loads gives the maximum stress in the member ARQ bring the second heavy load on the point d and calculate. Then the ordinates under the loads will be, starting from the piloting wheel to the last point of the train are ·125, ·425, ·645, ·6, ·58, ·525, ·5, ·46, ·45, ·41 and ·075, and magnitude of the stress $= (\cdot125 \times 13\cdot4) + 26\cdot8\ (\cdot425 + \cdot645 + \cdot6 + \cdot58) + 18\cdot75\ (\cdot525 + \cdot5 + \cdot46 + \cdot41) + \left(\frac{\cdot41 + \cdot075}{2} \times 60 \times \cdot5\right) = 1\cdot67 + 60\cdot30 + 36\cdot28 + 7\cdot27 = 105\cdot52$ tons.

This position of the loads gives you the stress in the desired member less than the previous one and if you move the loads further on to the left and bring the third heavy load over the point d you get still less that is nearly 100 tons, because the piloting wheel will be out of the span and hence you should decide, that the first position of the loads gives you the maximum stress in the member ARQ. This method of getting the result may appear to the students very tedious but drawing the axle loads on the tracing paper, and then moving the paper on the influence diagram either to the left or right facilitates the method.

Next bring the unit load over the joint Q then determine the stresses in the members QP and CD as follows. $R_1 \times 128 = W \times 96$. $\therefore R_1 = \frac{96}{128} W$ and $BM_Q = \frac{96}{128} W \times 32 = 24$ tons feet. The chord stress $= \frac{24}{22} = 1\cdot09$ tons. Take $c'\ d' = 1\cdot09$, then $a'\ c'\ b'$ is the influence line. Now bring the first heavy load over the point d' and you get the following ordinates in the influence diagram under the loads—·82, 1·090, 1·025, ·975, ·925, ·800, ·750, ·675, ·625, ·550 and O, then multiplying these by the axle and train loads you get 178·74 tons nearly. Next bring in the second heavy axle load over the point d' and note down the following ordinates in the influence diagram under the loads—·65, ·920, 1·090, 1·025, ·975, ·865, ·800, ·750, ·680, ·625 and O. Multiplying these by the corresponding axle loads you get the stress in the member QP or CD = 182·63 tons.

Similarly bring in the third heavy axle load over the point d' and you get the following ordinates ·48, ·750, ·925, 1·090, 1·025, ·900, ·870, ·800, ·725, ·650 and O and the stress in the member QP or CD = 179·37 tons nearly. No trial is needed further, as you observe the stress in the member concerned is decreasing. Hence you are to decide that when the second heavy axle load is over the point d' you get the maximum stress. The maximum stress $= (\cdot65 \times 13\cdot4) + 26\cdot8\ (\cdot920 + 1\cdot090 +$

$1{\cdot}025+{\cdot}975\ )+18{\cdot}75\ \ (\ {\cdot}865+{\cdot}800+{\cdot}750+{\cdot}680\ )+\left(\frac{{\cdot}625}{2}\times 54\times{\cdot}5\right)=8$ ${\cdot}70+107{\cdot}46+58{\cdot}03+8{\cdot}44=182{\cdot}63$ tons. Therefore the tension in the member QP $=182{\cdot}63$ tons and compression in the member CD $=182{\cdot}63$ tons.

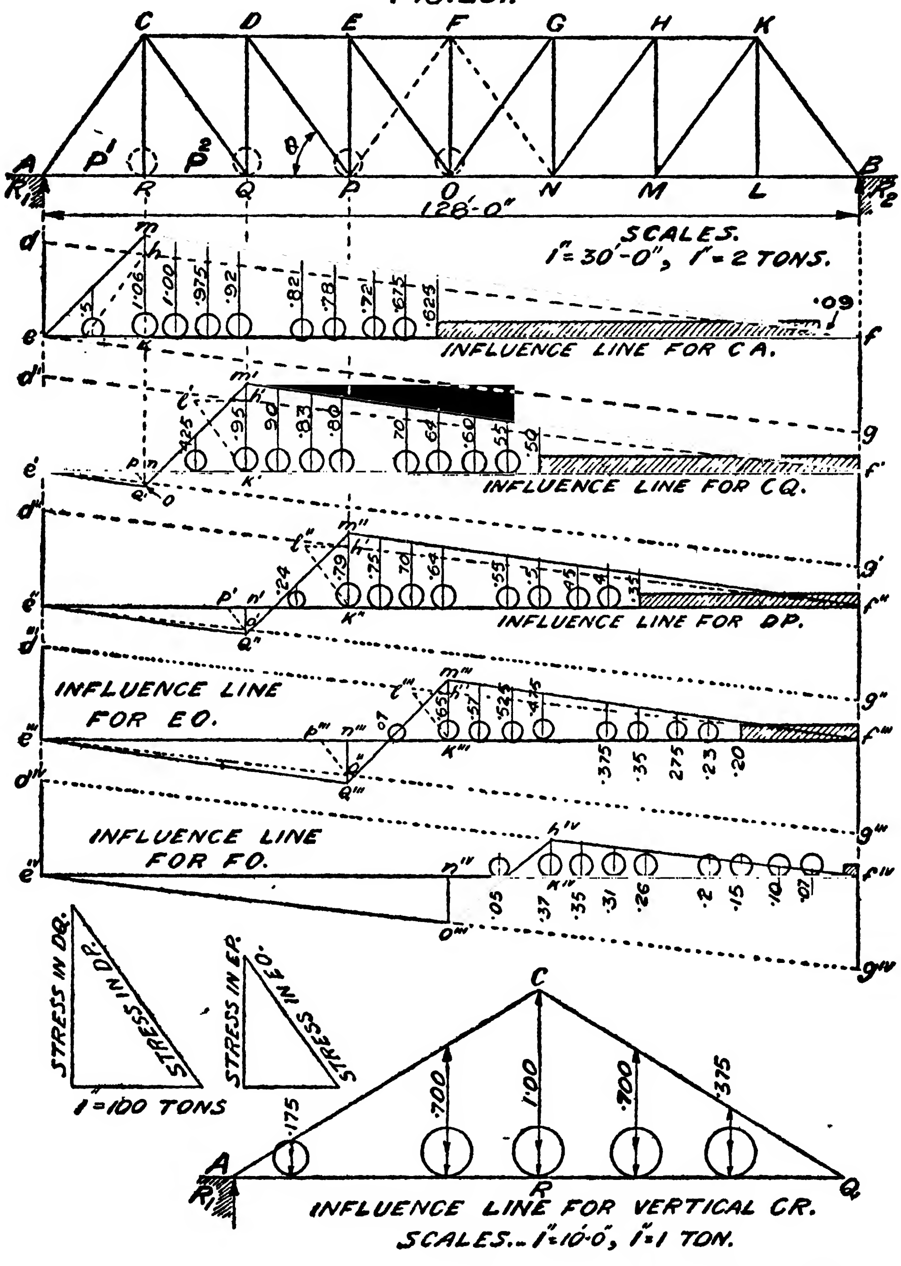
To get the stress in DE or OP bring the unit load over the point P. $R_1\times 128=W\times 80$, $\therefore R_1=\frac{80}{128}W=\frac{5}{8}$. $BM_P=\frac{5}{8}\times 48=30$ tons feet. Stress in DE or OP for a unit load at $P=\frac{30}{22}=1{\cdot}36$ tons. Draw $c^2\ d^2=$ $1{\cdot}36$ tons and $a^2\ c^2\ b^2$ is the influence line. It is clear now that the first and second heavy axle loads cannot give you the maximum stress in the member, so you will have to try the third or the fourth heavy axle load. Therefore bring the third heavy axle load over the point d^2 by means of tracing paper and note down the ordinates and the stress in the member for this position of the loads is therefore equal to $(\ {\cdot}85\times 13{\cdot}4\ )+26{\cdot}8\ \ (\ 1{\cdot}08+1{\cdot}21+1{\cdot}36+1{\cdot}275\ )+18{\cdot}75\ \ (\ 1{\cdot}11+1{\cdot}03+$ ${\cdot}92+{\cdot}84\ )+\left(\frac{{\cdot}75}{2}\times\frac{43}{2}\right)=11{\cdot}39+131{\cdot}99+73{\cdot}12+8{\cdot}06=224{\cdot}56$ tons. By bringing the fourth heavy axle load under the point d^2 you get $({\cdot}8\times 13$ ${\cdot}4\ )+26{\cdot}8\ \ ({\cdot}94+1{\cdot}08+1{\cdot}21+1{\cdot}36)+18{\cdot}75\ \ (1{\cdot}19+1{\cdot}11+1{\cdot}00+{\cdot}92)+$ $\left(\frac{{\cdot}84}{2}\times\frac{48}{2}\right)=10{\cdot}72+123{\cdot}01+79{\cdot}12+10{\cdot}08=222{\cdot}93$ tons. This is less than the previous one and hence the third heavy axle load on the point d^2 will produce the maximum stress in the member DE or OP and is equal to $\pm 224{\cdot}56$ tons.

The stress in EF can be determined by bringing the unit load over the joint O, then $R_1\times 128=W\times 64$. $R_1=\frac{64}{128}=\frac{1}{2}$. $BM_O=\frac{1}{2}\times 64=32$ tons feet and the stress in EF for a unit load at O is equal to $\frac{BM_O}{D}=\frac{32}{22}=1{\cdot}45$ tons. Therefore $c^3\ d^3=1{\cdot}45$ tons and $a^3\ c^3\ b^3$ is the influence line. You get the maximum stress in the member EF by bringing the fourth heavy axle load over the point d^3 and the magnitude of the stress $=({\cdot}84\times$ $13{\cdot}4)+26{\cdot}8\ (1{\cdot}125+1{\cdot}24+1{\cdot}37+1{\cdot}45)+18{\cdot}75\ (1{\cdot}26+1{\cdot}14+1{\cdot}10+{\cdot}89)+$ $\left(\frac{{\cdot}77}{2}\times\frac{33}{2}\right)=11{\cdot}25+138{\cdot}95+82{\cdot}5+6{\cdot}35=239{\cdot}05$ tons compression.

Forces or stresses in the sloping or web members should be determined fromt he shear influence lines. See figure 251. To determine the stress in the member AC, you are to determine the average shear in the panel AR and let P′ be the mid point of AR. Draw the shearing force diagram $d\ e\ f\ g$ for a unit load W passing over the bridge. When the unit load is at A the shear at P′ is zero as the load and reaction neu-

tralize each other and, when the load moves to the joint R the reaction $R_1 = \frac{7}{8}$ W, then the shear at P′ is equal to the ordinate $h\,k$

FIG. 251.



which is equal to the reaction $R_1 = \frac{7}{8}$ W. (See figures 245 and 247)

This ordinate $h\ k$ is the vertical component of the stress in the member AC, consequently resolve this $h\ k$ parallel to the member AC as shown in the diagram, then $h\ l$ is the stress in the member AC. Make $k\ m$ equal to $h\ l$, then $e\ m\ f$ is the influence line for the member AC.

Now bring in the locomotive axle and train loads and locate the first heavy axle load over the point k for the maximum stress and the stress in the member AC is equal to $(\cdot 5 \times 13\cdot 4) + 26\cdot 8\ (1\cdot 060 + 1\cdot 000 + \cdot 975 + \cdot 920) + 18\cdot 75\ (\cdot 820 + \cdot 780 + \cdot 720 + \cdot 675) + \left(\frac{\cdot 625 + \cdot 09}{2} \times 60 \times \frac{1}{2}\right) = 6\cdot 70 + 105\cdot 99 + 56\cdot 15 + 10\cdot 72 = 179\cdot 56$ tons, or say 180 tons compression. If you bring the second heavy load over the point K you get naturally less.

The stress in the diagonal CQ can be determined by bringing the unit load over the joints R and Q. When the load is at R the shear at P^2 is equal to $n\ o$ and when it moves to Q the shear at P^2 is equal to $h'\ k'$. This negative shear $n\ o$ produces compressive stress in the member CQ and the positive shear $h'\ k'$ causes a tensile stress in the same member CQ. These two ordinates $n\ o$ and $h'\ k'$ are the vertical components of the maximum tensile and compressive stresses in the member CQ. Consequently these two ordinates are to be resolved parallel to the member CQ, then $o\ p$ and $k'\ l'$ are the resolved parts. Draw $k'\ m'$ equal to $k'\ l'$ and $n\ q'$ equal to $o\ p$, then $e'\ q'\ m'\ f'$ is the influence line for the member CQ.

Bring in the locomotive axle and train loads within this influence line and let the first heavy axle load be under the point m' to get the maximum tensile stress in the member CQ. Tho maximum tensile stress in the member CQ for the position of the loads is equal to $(\cdot 425 \times 13\cdot 4) + 26\cdot 8\ (\cdot 95 + \cdot 90 + \cdot 83 + \cdot 80) + 18\cdot 75\ (\cdot 70 + \cdot 64 + \cdot 60 + \cdot 55) + \left(\frac{\cdot 50}{2} \times \frac{50}{2}\right) = 5\cdot 69 + 93\cdot 26 + 46\cdot 68 + 6\cdot 25 = 151\cdot 88$ tons or say 152 tons.

Similarly influence lines for DP and EO have been drawn and their stresses are $(\cdot 24 \times 13\cdot 4) + 26\cdot 8\ (\cdot 79 + \cdot 75 + \cdot 70 + \cdot 64) + 18\cdot 75\ (\cdot 55 + \cdot 50 + \cdot 45 + \cdot 40) + \left(\frac{\cdot 35}{2} \times \frac{34}{2}\right) = 3\cdot 21 + 77\cdot 18 + 35\cdot 62 + 3 = 119\cdot 01$ tons and $(\cdot 07 \times 13\cdot 4) + 26\cdot 8\ (\cdot 65 + \cdot 57 + \cdot 525 + \cdot 475) + 18\cdot 75\ (\cdot 375 + \cdot 350 + \cdot 275 + \cdot 230) + \left(\frac{\cdot 20}{2} \times \frac{18}{2}\right) = \cdot 93 + 59\cdot 50 + 23\cdot 16 + \cdot 90 = 84\cdot 49$ or say $84\cdot 5$ tons respectively.

Stresses in vertical members.—The stress in DQ is equal to the

vertical component of the stress in DP. The stress in DP=119·01 tons and the vertical component is equal to 96 tons as per resolution shown in the figure. If you want to determine the same mathematically you can do so. The vertical component is equal to DP × sin θ, but tan $\theta = \frac{DQ}{QP} = \frac{22}{16} = 1{\cdot}375$. On refering to the tables you find $\theta = 53^\circ - 58{\cdot}32'$. Substituting this value in the above you have DP × sin $53^\circ - 58{\cdot}32' = 119{\cdot}01 \times {\cdot}80867 = 96{\cdot}23$ tons, where 119·01 is the stress in the member DP and ·80867 is the value of the sin θ.

Similarly the stress in the vertical EP is equal to the vertical component of the stress in EO. This is equal to 68 tons. The stress in FO is equal to the vertical component of the stress in the diagonal FN. The influence line has been drawn for the member FO and actually this is the influence line for the member FN and the ordinates $h^{IV}\, k^{IV}$ and $n^{IV}\, o^{III}$ are the vertical components of the maximum tensile and compressive stresses of the member FN. Since this diagonal is not meant to take compressive stress, the maximum tensile vertical component is taken to calculate the stress in the member FO. The maximum tensile stress in the vertical FO is equal to $(-{\cdot}05 \times 13{\cdot}4) + 26{\cdot}8\ ({\cdot}375 + {\cdot}350 + {\cdot}31 + {\cdot}26) + 18{\cdot}75\ ({\cdot}20 + {\cdot}15 + {\cdot}10 + {\cdot}07) + \left(\frac{{\cdot}025}{2} \times 2 \times \frac{1}{2}\right) = -{\cdot}67 + 34{\cdot}70 + 9{\cdot}75 + {\cdot}01 = 43{\cdot}79$ or say 44·00 tons.

The stress in the vertical CR is to be calculated as follows.— The tension in this member is equal to that portion of the loads between the supporting point and the second panel joint that is carried by the road way to the floor beam. Therefore these heavy axle loads are to be broughht on this space to cause the stress in this member a maximum. Draw the reaction influence line ACQ for the first two panels and CR equal to 1 ton, then the triangle ACQ is the influence diagram. On a tracing paper draw out the axle loads with their spacings to the same linear scale and slide the same on the influence diagram. You observe at the very first trial that the piloting axle load and front four heavy axle loads with the second heavy axle load over the point R will give you the maximum reaction over the floor beam fixed at the point R and the maximum reaction is equal to $({\cdot}175 \times 13{\cdot}4) + 26{\cdot}8\ (700 + 1{\cdot}000 + {\cdot}700 + {\cdot}365) = 2{\cdot}34 + 74{\cdot}37 = 76{\cdot}71$. tons.

The tension in the member C R=76·71 tons.

EXAMPLE 9:—Draw the bending moment and shear influence lines

for the members of the Bow String Girder, span 70 feet and depth 12 feet, for a unit load passing over the girder.

SOLUTION:—This is the case of a girder of variable depth. (See figure 252). Bending Moment Influence Lines.—To draw the influence line for the member AP proceed as follows.—For the position of the unit load at A the stress in AP is zero and therefore move the load to P, then $R_1 = \frac{60}{70} W = \frac{6}{7} W$. $BM_P = R_1 \times 10 = \frac{6}{7} \times 10 = \frac{60}{7}$ tons feet. The stress in AP is equal to the bending moment divided by the depth of the girder $= \frac{60}{7} \times \frac{1}{6 \cdot 42} = 1 \cdot 335$ tons, plot $c\ d = 1 \cdot 335$ tons and the line $a\ c\ b$ is the influence line for the member AP.

Influence line for the member PO—Place the unit load at O. Then $R_1 = \frac{5}{7}$ W. $BM_O = \frac{5}{7} \times 20 = \frac{100}{7}$. Stress in $OP = \frac{100}{7} \times \frac{1}{10} = \frac{10}{7} = 1 \cdot 428$ tons, where 10′ is the depth of the girder at that point. Plot $c'\ d' = 1 \cdot 428$ tons and and $a\ c'\ b$ is the influence line for the member PO.

Influence line for ON.—Unit load at N, $R_1 = \frac{4}{7}$ W, $BM_N = R_1 \times 30 = \frac{4}{7} \times 30 = \frac{120}{7}$ tons feet and the stress in $ON = \frac{120}{7} \times \frac{1}{12} = \frac{10}{7} = 1 \cdot 428$ tons, where 12′ is the depth of the girder. Plot $c_2\ d_2$ equal to 1·428 tons and $a\ c_2\ b$ is the influence line for the member ON.

The stress in MN for the unit load at M is equal to $R_1 \times 40 \div 12 = \frac{30}{70} \times 40 \times \frac{1}{12} = \frac{10}{7} = 1 \cdot 428$ tons, same as the stress in ON. The influence line for this is $a\ c^3\ b$.

The influence line for the member BC for the unit load at P is the same as above, and the depth of the girder is not 6 25 but 6·00 feet, that is, the perpendicular distance from the line BC to the joint P. Then the stress in BC for a unit load at P is $\frac{60}{7} \times \frac{1}{6} = \frac{10}{7} = 1 \cdot 428$ tons. Plot $c^4\ d^4$ and $a'\ c^4\ b'$ is the influence line for the member BC.

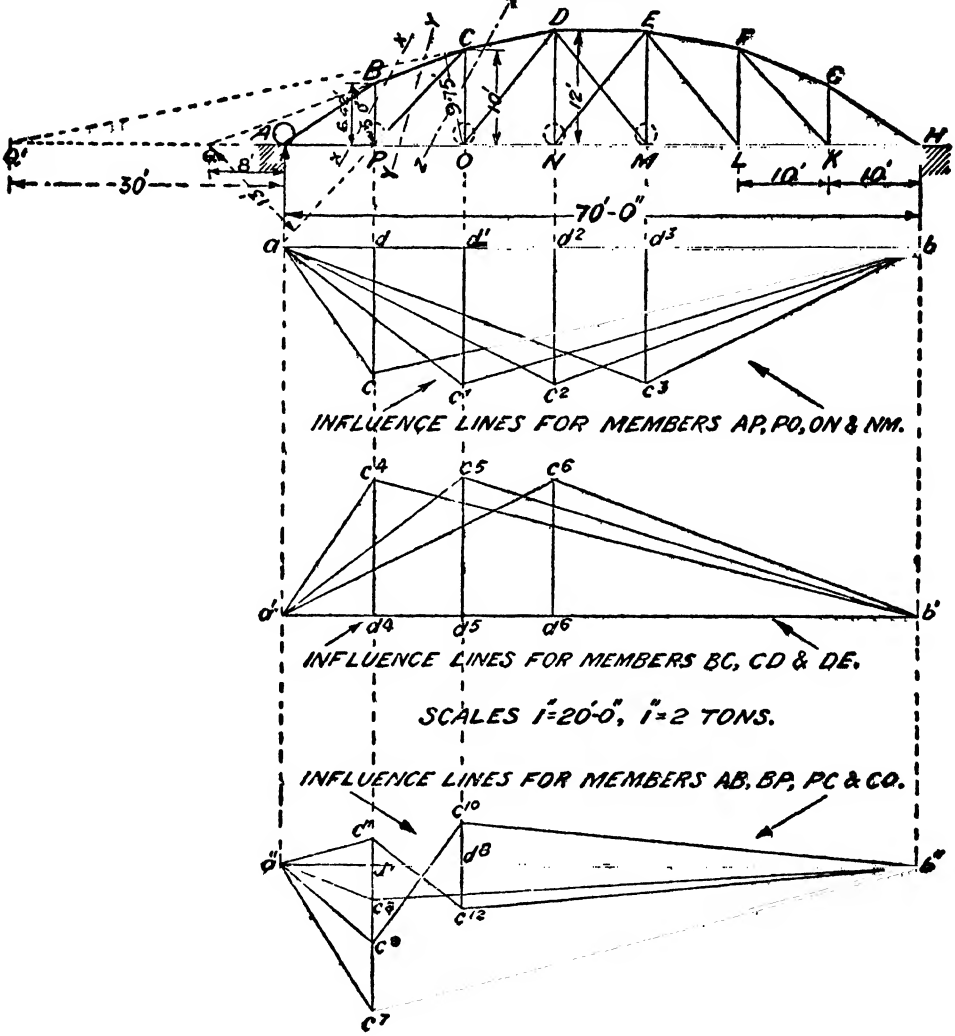
Similarly influence line for CD is to be drawn as follows.—Stress in $CD = \frac{100}{7} \times \frac{1}{9 \cdot 75} = 1 \cdot 465$ tons, plot $d^5\ c^5$ equal to 1·465 tons and $a'\ c^5\ b'$ is the influence line. 9·75 feet is the depth of the girder for the member CD.

The stress in the member DE is the same as the stress in the member ON. Plot $c^6\ d^6 = 1 \cdot 428$ tons and $a'\ c^6\ b'$ is the influence line for the memeber ON.

Influence Lines for Vertical & Diagonal Members.—Influence line for the member AB.—For a unit load at P the stress in AB is to

be calculated by taking P as moment centre. $R_1 \times 70 = W \times 60$ $\therefore R_1 = \frac{60}{70} = \frac{6}{7}$ W. $R_1 \times 10 +$ stress in AB$\times 5\ 5 = 0$. Therefore stress in AB $= -\frac{6}{7} \times 10 \times \frac{1}{5 \cdot 5} = -1 \cdot 55$ tons. Here the minus sign shows that the dire-

FIG. 252.



ction of the force in AB has anticlockwise moment about the moment centre, therefore the arrow head points towards the joint A and hence the stress in the member AB is compressive. Plot $c^7\ d^7$ equal to 1·55 tons and $a''\ c^7\ b''$ is the influence line for the member AB.

For the vertical member BP. Take a section XX cutting the

members AP, BP and BC and produce CB and PA to meet at Q, then AQ measures 8'. Take Q as moment centre and the unit load at A. $R_1 = \frac{70}{70} = 1$. Then $-R_1 \times 8 + W \times 8 =$ stress in BP$\times$18. Therefore stress in BP$= -1 \times 8 + 1 \times 8 \div 18 = 0$. Move the load to the point P, then $-R_1 \times 8 =$ Stress in BP$\times$18, $R_1 = \frac{6}{7}$. Stress in BP$= -\frac{6}{7} \times 8 \div 18 = -\frac{48}{7} \times \frac{1}{18} = -{\cdot}38$ ton. Here minus sign shows that the arrow head is towards the joint and hence the stress in BP is compressive. The influence line for the member BP is $a'' \; c^8 \; b''$.

Influence line for the member PC—Take a section $y\,y$ cutt.ng the members BC, PC and PO, and by producing CB and OA, you get the same point Q for their intersection. Take Q as moment centre and the unit load at P. Then $R_1 = \frac{60}{70}W$. $-R_1 \times 8 + W \times 18 =$ stress in PC$\times$13. Stress in PC$= -\frac{6}{7} \times 8 + (1 \times 18) \div 13 = {\cdot}85$ ton. Plus sign shows that the arrow head is towards the joint P and hence the stress in the member PC is in compression. Now move the load to the joint O, and taking moment about Q, you get—$R_1 \times 8 =$ stress in PC$\times$13. $\therefore$ Stress in PC$= R_1 \times 8 \div 13$. but $R_1 = \frac{5}{7}W$ Then $-\frac{5}{7} \times 8 + 13 = -{\cdot}44$ ton. Minus sign shows that the arrow head is away from the joint and hence the stress is in tension. Plot $c^9 \; d^7$ and $c^{10} \; d^8$ join $c^9 \; c^{10}$ to $a'' \; b''$ and $a'' \; c^9 \; c^{10} \; b''$ is the influence line for PC.

Influence line for the vertical CO—Take a section ZZ cutting the members PO,CO, and CD and produce CD and NA to meet at Q_1. Then taking Q' as moment centre with a unit load at P, you have $-R_1 \times 40 + W \times 50 =$ stress in CO$\times$60. Therefore stress in

$$CO = -\frac{R_1 \times 40 + W \times 50}{60} = -\frac{6}{7} \times 40 + 1 \times 50 \div 60 = \frac{110}{420} = {\cdot}26 \text{ ton.}$$

Plus sign shows that the arrow head is away from the joint c and hence the stress in the member is in tension.

Move the unit load to O, then $-R_1 \times 40 =$ stress in CO$\times$60. Then unit load is in the right hand side ot the section which is supposed to be removed. Stress in CO$= -\frac{5}{7} \times 40 \div 60 \; \frac{-200}{420} = -{\cdot}47$ ton. Here minus sign shows the arrow head is towards the joint C, so the stress in CO is in cempression. Plot these ordinates as shown in the diagram and $a^{11} \; e^{11} \; c^{12} \; b^{11}$ is the influence line for the member CO. Similarly you can proceed on for the rest of the members. If you desire to know the maximum stress in any member for a uniformly distributed or for a locomotive axle loads, you can make use of these influence lines and determine as shown in example 8.

CAPTER XV.

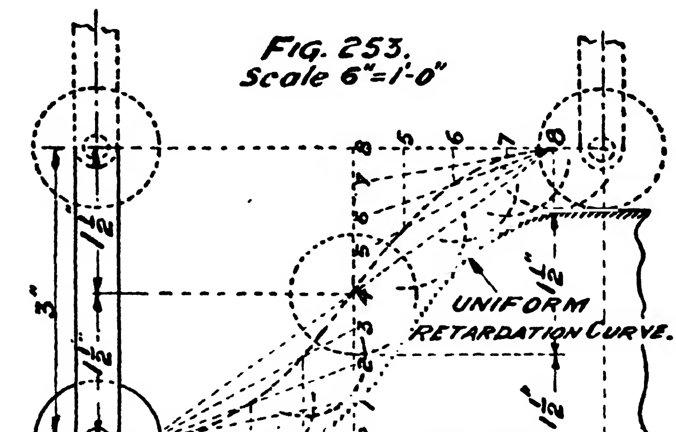
CAMS.

Cams are generally flat metal pieces of different shapes and some are usually fixed to shafts called cam shafts. These give reciprocal or to and fro motions to other pieces of metal called followers usually move in guides vertically and some move about fixed axes. Cam and follower always have line contact. The follower will be in contact with the cam owing to its own gravity weigt and generally it is provided with some sort of spring to enable it to rest always over the edge of the cam. You can observe these cams in weaving, printing and very many other machines Cams may have reciprocating or rotary motion and the follower also have reciprocating or angular motion

There are three different kinds of cams (1) Plane sliding cams, (2) Plane rotating cams and (3) Cylindrical cams. Out of these the sliding and rotating cams will be dealt with in this chapter.

EXAMPLES ON SLIDING CAMS.

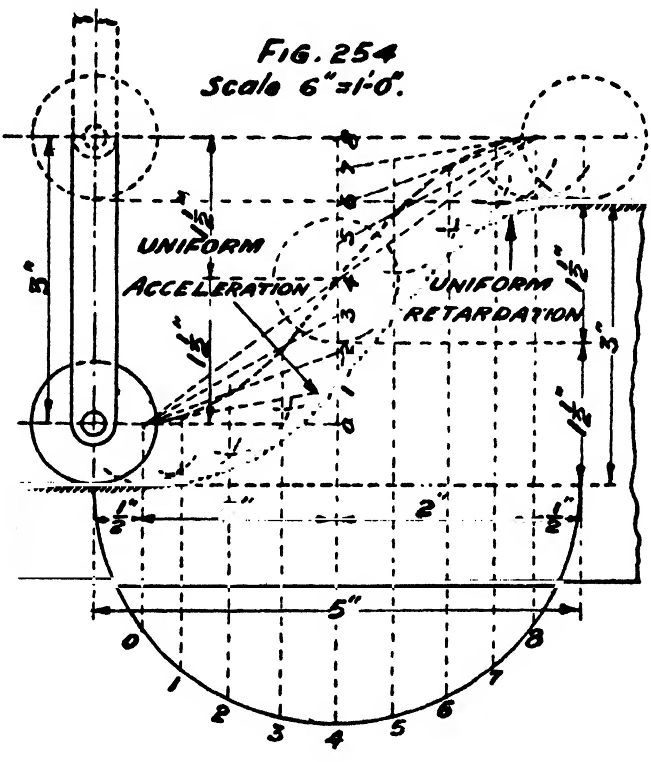
EXAMPLE 1 :—A plane reciprocating cam has uniform motion and a stroke of 5 inches. The follower reciprocates at right angles to the line of stroke of the cam and in the plane of the cam. For the first half inch of the forward stroke the follower is at rest at the bottom of the stroke. For the next 2″ of the cam stroke the follower rises $1\frac{1}{2}''$ with uniform acceleration. For



the next 2″ of the cam stroke the follower rises $1\frac{1}{2}''$ with uniform

retardation, and then remains at rest until the cam has completed its forward stroke. The follower is provided with a roller $1\frac{1}{4}''$ diameter which works on the cam. Draw the outline of the cam. (*LOW*).

SOLUTION :—The cam has uniform motion and a stroke of 5 inches, periods of rest at the beginning and end of stroke. Draw a straight line 5 inches long and divide this into any number of equal parts or say 10 equal parts for convenience, then each division measures $\frac{1}{2}$ inch. For the first half inch of the stroke the follower is at rest and next 2 inches of the stroke the follower rises $1\frac{1}{2}$ inches with uniform acceleration, therefore draw a vertical line 4—4 equal to $1\frac{1}{2}$ inches and divide this into 4 equal parts. Draw the parabola as shown. For the next 2 inches of the stroke, the follower rises $1\frac{1}{2}''$ with uniform retardation, therefore draw 4—8 equal to $1\frac{1}{2}$ inches and divide this into 4 equal parts and draw similarly the parabola as shown. Then leave the last half inch horizontal. The outline of the cam is as shown shaded in the figure.

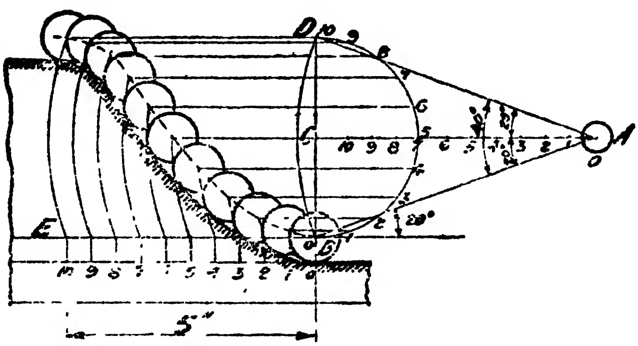


EXAMPLE 2 :- A plane reciprocating cam has simple harmonic motion, and a stroke of 5 inches. The follower is at rest for the first half inch of the stroke, and rises with uniform acceleration $1\frac{1}{2}$ inches for the next 2 inches of the stroke. Again it rises $1\frac{1}{2}$ inches with uniform retardation for the next 2 inches of the cam stroke. The remaining half inch of the cam stroke the follower is at rest. Draw the outline of the cam. Diameter of the roller $1\frac{1}{4}''$.

SOLUTION :—In this example the follower rises with uniform acceleration and retardation as in the last example, but the cam has simple harmonic motion. For 5 inches of the stroke draw a semicircle with a radius of $2\frac{1}{2}''$ and divide that portion of the arc which is intersected by the vertical dotted lines drawn at 4" apart into 8 equal parts as shown in the figure 254. Draw 0—4 and 4—8 each equal to $1\frac{1}{2}$ inches and divide 0—8 into 8 equal parts. Draw two parabolas for the centre line of the follower as shown in the diagram and the shaded portion is the outline of the cam.

EXAMPLE: 3—A straight lever oscillates in the plane of a sliding cam about an axis at one end, through angles of 20° on opposite sides of a line parallel to the line of stroke of the cam. The lever has simple harmonic motion, and one complete oscillation of the lever is performed during two strokes of the cam. The stroke of the cam is 5''. The cam works against a roller 1" in diameter whose axis is at the free end of the lever and 6 inches from the axis about which the lever swings. Assuming that the cam has uniform motion draw its contour.

FIG. 255.
SCALE $\frac{1}{4}$ SIZE.

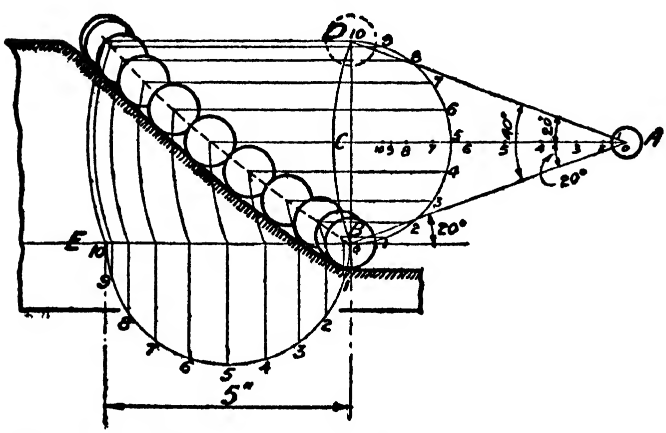


SOLUTION :—
The cam has uniform motion and therefore draw a line 5 inches long and divide that into 10 equal parts. On the zero point draw the roller of one inch diameter. Call the centre of the roller as B and from B draw a line BA six inches long at 20° inclination to the horizontal. From A draw a line AC horizontal and again from A draw another line AD at 20° to AC. Then BD is the complete lift of the lever, and since the lever has simple harmonic motion draw a semicircle with centre C and radius CD or CB and divide this into 10 equal parts as shown. Draw the horizontal lines from these ten points as shown in the diagram, From the point B draw another horizontal line BE to left and project ten points of the bottom parallel line on to it. On AC mark out these ten divisions, then take a length AB through a compass and strike arcs as shown in the figure, without changing the radius AB. The needle point of the campass is to be placed on the divisions in AC and pencil.

point is to be on the divisions in BE. The outline of the cam is as shown shaded.

EXAMPLE 4:—This is the same as example 3 but the cam has simple harmonic motion instead of uniform motion. Draw the contour of the cam. (See fig. 256.)

FIG. 256.
SCALE ¼ SIZE.



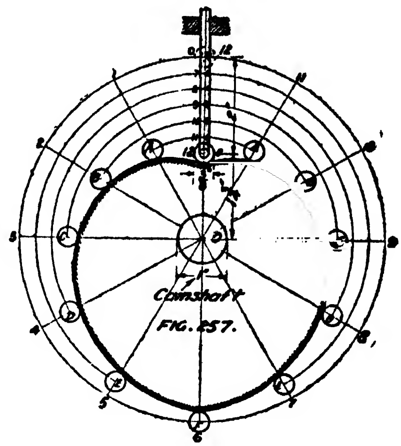
SOLUTION:—In this example the cam also has simple harmonic motion, therefore on the 5″ stroke of the cam describe a semicircle and divide it into ten equal parts as shown. Project all these points on the line BE, and by means of a compass with a radius AB strike arcs adjusting the pencil point exactly on the projected points in the line BE and needle point on the line AC. The outline of the cam is as shown shaded.

EXAPLES ON ROTATING CAMS.

EXAMPLE 1:—Design an edge cam to give a uniform upward motion of 2 inches during one half of its revolution, and a uniform return the next half. Least distance between centre of cam shaft and centre of roller 1¾″; diameter of cam shaft one inch. Diameter of roller 3/8″.

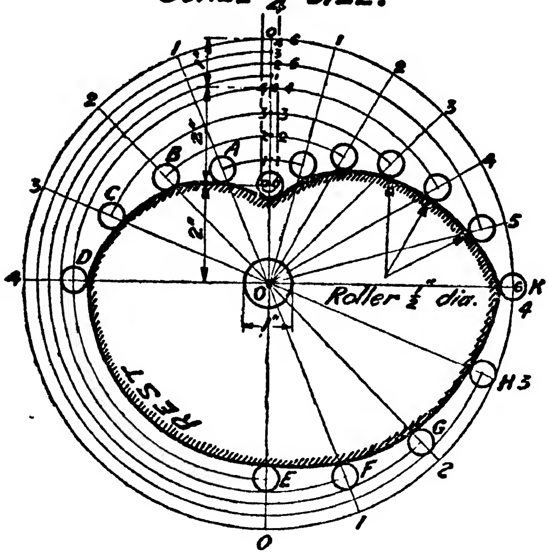
SOLUTION:—Draw vertical and horizontal lines through the centre O of the camshaft. (See fig. 257). Take a distance of 1¾″ to centre of roller and a further distance of 2 inches. Divide the distance of 2 inches into any number of equal divisions, preferably 6 or 8 as the circle also can be divided into 6 or 8 divisions conveniently. From centre O with a radius of 3¾ inches draw a circle. Divide each semicircle into six equal parts as shown. From the centre of camshaft O with radii O—1, O—2, O—3, O—4, O—5 and O—6 strike arcs and get them intersected at

AA, BB, CC, DD, EE and F respectively as shown. Since the diameter of roller is $\frac{3}{8}''$ the edge of the cam is to be $\frac{3}{16}$ inch below these intersected points. A fair curve is to be drawn by hand as shown, and the outline of the cam is shaded.



EXAMPLE 2:—Design a cam to give a uniform upward motion of 2 inches during $\frac{1}{4}$ of a revolution, a period of rest during the next quarter, a further uniform rise of one inch during the third quarter, and then a uniform return through the whole distance of 3 inches during the last quarter revolution. Least distance from centre of shaft to centre of roller, 2 inches. Diameter of camshaft one inch, diameter of roller $\frac{1}{2}$ inch.

FIG. 258.
SCALE $\frac{1}{4}$ SIZE.

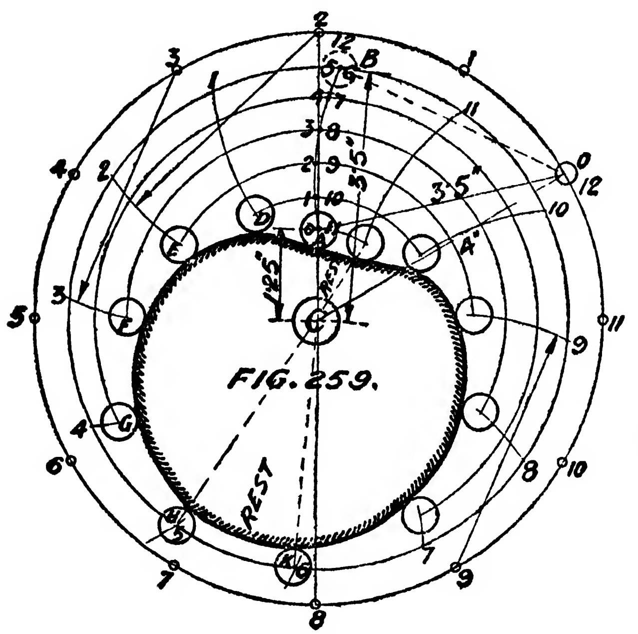


SOLUTION.—From the centre O of the cam shaft (See fig. 258) draw vertical and horizontal lines. Plot 2 inches on the vertical line the distance from centre of camshaft to centre of roller and further distances of 2 inches and one inch. From centre O with a radius of 5 inches describe a circle. Divide the first quarter of the circumference into 4 equal divisions. Divide the distance of inches which is the upward motion during the first quarter revolution into four equal divisions. Now strike arcs from centre O with radii O—1, O—2, O—3 and O—4 to intersect at A, B, C and D repectively On joining up these intersection points you obtain the first portion of the required curve of the cam.

The next quarter revolution is the period of rest and therefore describe an arc with O—4 as radius and get it intersected at E. In the next quarter revolution there is a movement of one inch more. Divide this one inch and also the third quarter of the circle each into 4 equal parts. Swing round the points as before and get them intersected at E, F, G, H and K.

In the last quarter revolution the follower must fall through the full distance of 2 + 1 = 3 inches. Divide this 3 inches into say 6 equal parts and last quarter circle also into 6 equal parts. Swing round the points as before. Join up these intersected points and you get the profile of the cam.

EXAMPLE 3:—O is the axis about which an arm OA swings. OA = 3·5 inches. A is the axis of a roller, ·5 inch in diameter, carried by the arm, and this roller works against a cam which rotates with uniform velocity, and whose axis C is 4 inches from O. The greatest and least distances of A from C are 3·5 and 1·25 inches respectively. Design the cam so that the arm shall have uniform angular velocity when swinging, and periods of rest at each end of the swing corresponding to one-twelfth of a revolution of the cam.



SOLUTION:—From the question you understand that ACO is the triangle and whose sides measure 1·25, 4 and 3·5 inches respectively. Therefore construct the triangle ACO as shown in the figure 259 and the points A, C, and O represent in turn the centre of the roller, the cam centre and the centre of the axis of the arm OA respectively. From centre C and CO as radius discribe a circle and divide the circumference into 12 equal parts. Then the greatest distance from the cam centre C to the centre of the roller is 3·5 inches,

and therefore from points C and O strike arcs CB and OB taking the radius of each equal to 3 5 inches and let them intersect at B.

With centre C and CB as radius describe another circle and divide this circumference also into 12 equal divisions. The follower is to have a period of rest at each end of the swing corresponding to one-twelfth of a revolution of the cam, consequently the arc AB of the arm OA is to be divided into 5 equal divisions and by this you can give a period of rest equal to one-twelfth revolution of the cam.

Now imagine the cam is fixed up and the arm OA with its roller is rotated about the centre of the cam shaft so that it may occupy the various positions shown, that is, the axis O of the arm OA is to move on the outer circle of 4″ radius and the axis of the roller is to move on the inner circle of 3·5 inches radius. For instance to get the points D, E, F, G and H strike arcs from the centre C with radii C—1, C—2, C—3, C—4 and C—5 and strike another set of arcs with a radius OA commencing from the divisions on the outer and inner circumferences bearing the same numbers such as 1—1, 2—2, 3—3, 4—4 and 5—5, to intersect the five arcs in D, E, F, G and H as shown in the figure.

The distance from H to K represents the period of rest and then you will have to repeat similar constuction for another 5 points with a further period of rest at the last. The outline of the cam is shown shaded.

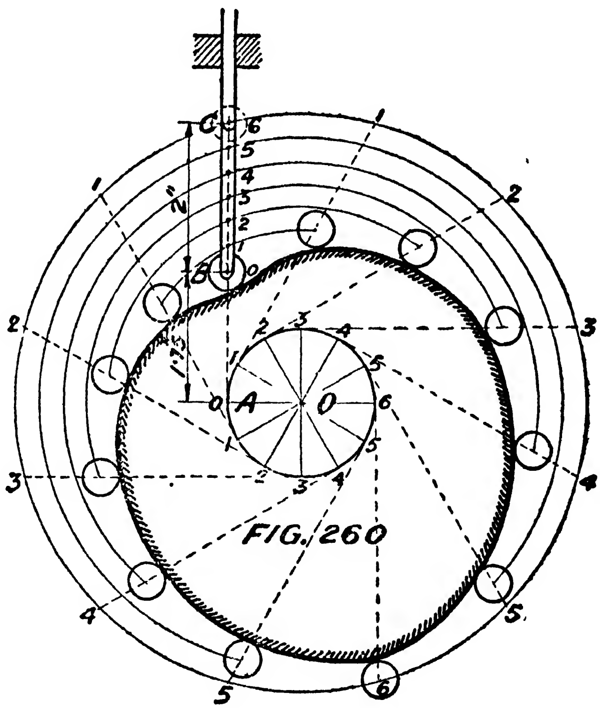


FIG. 260

EXAMPLE 4:— Design a cam to give a uniform upward motion of 2 inches during one-half of its revolution and a uniform return the next half. The vertical path of the follower is not in a line with the centre of

camshaft, but one inch away from it. The distance between camshaft and centre of roller is $1\frac{3}{4}$ inches, and the diameter of roller is $\frac{1}{2}''$.

SOLUTION:—(See fig. 260) Since the path of the follower is one inch to left of cam shaft, describe a circle from centre O with a radius of one inch. Let the path of the roller touch this circle tangentially at the zero point at A. Divide the circumference of this circle into 12 equal parts commencing from the point A. From centre O with a radius OC describe a circle and divide the circumference of this circle also into 12 equal divisions commencing from C. Then connect the points of divisions of the circumference of the small circle with the corresponding divisions of the circumference of the outer circle as shown. You observe that these straight lines are connected tangentially because the path of the follwer touches this circle tangentially if produced. Suppose the cam is rotated either clockwise or anticlockwise, all these 11 straight lines coincide with the straight line AC in turn.

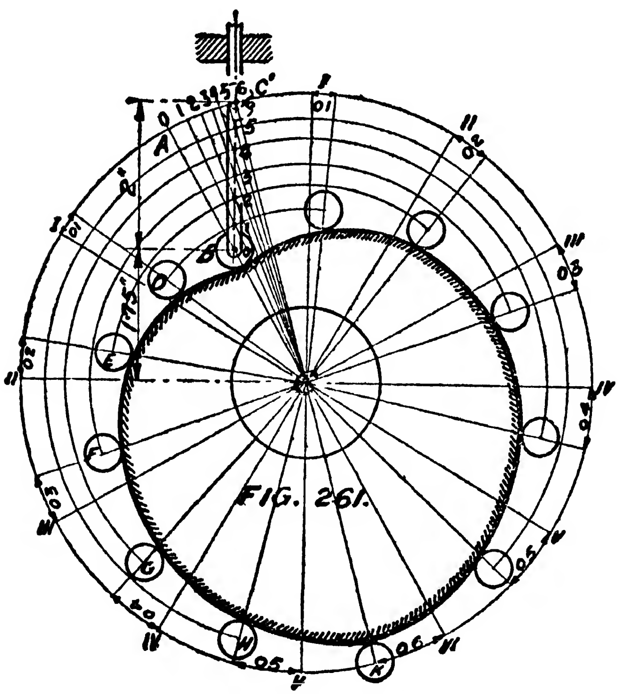


FIG. 261.

Divide the distance of 2 inches which is the upward motion of the follower into 6 equal divisions and swing round from the centre of cam shaft these six points of divisions along the line BC to meet the corresponding tangential lines numbered in the same order. After drawing the rollers at those intersected points you can draw the outline of the cam as shown in the figure.

ALTERNATIVE METHOD.

See fig. 261. The alternative method is as follows.—Divide BC′ into 6 equal parts, and with centre C and radius C 6 or CC′ discribe a circle. Connect the divided points 1, 2, 3, 4, 5 and 6 with centre C and

produce C O, C 1, C 2, C 3, C 4, and C 5, to meet the circumference of the circle at O, 1, 2, 3, 4 and 5. Then divide the circle into 12 equal divisions by straight lines from the point A in the circumference. At the point I take a distance O 1 and join that point to the centre of the circle and swing round C 1 and get it intersected at D. Again at II, take a distance O 2 and swing round C 2 to intersect at E. Similarly at III, IV, V and VI take distances O 3, O 4, O 5 and O 6 and swing round C 3, C 4, C 5 and C 6 and get them intersected at F, G, H and K respectively and repeat the same for the other half of the revolution of the cam, as shown in the diagram.

EXAMPLE 5:—Draw the profile of the cam to do the following work.—It has to lift a bar vertically with uniform velocity, the length of the travel of the bar being 6″; it then has to allow the bar to descend again with uniform velocity, but at one half the speed of the ascent. The two movements occupy one revolution of the uniformly rotating cam. The diameter of the roller working on the cam is $\frac{1}{2}$ inch, and the least thickness of metal round the cam centre must be 2 inches. The line of stroke of the moving bar passes through the cam centre. (B. E.).

SOLUTION:—Let t be the time of one revolution, $t\,a$ = time of ascent, $t\,d$ = time of descent, then $t\,a + t\,d = t$................(1). Again time of descent equals twice the time of ascent, therefore $t\,d = 2\ t\,a$, because the speed of descent is one half of the speed of ascent.

Substituting this value in (1) we have $t\,a + 2\,t\,a = t$, $\therefore t = 3\ t\,a$. Hence $t\,a = \frac{t}{3}$ and $t\,d = 2\ t\,a = 2 \times \frac{t}{3} = \frac{2}{3}\,t$.

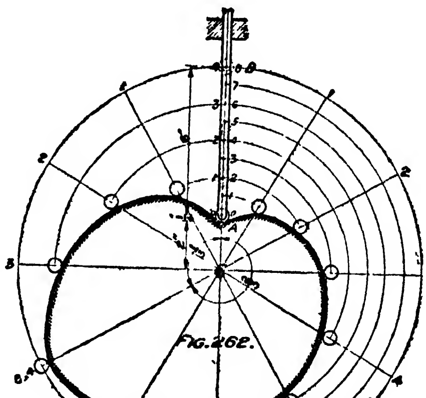


Fig. 262.

Now take 2″ from the cam centre to the bottom of the roller and take 6″ from the centre of roller upwards representing the travel of the bar. Describe a circle from centre C with a radius CB and divide this circle into three equal parts. Arrange for the first one third of the revolution of the cam to give the vertical ascent of 6 inches and for the remaining two thirds of the revolution of the cam, the vertical descent of the same amount. Divide $\frac{1}{3}$ of the circle into 4 equa

parts and the vertical lift also into 4 equal parts and obtain the points as shown. Divide the remaining $\frac{2}{3}$ of the circle into 8 equal parts and the vertical lift of 6 inches also into eight equal parts, and obtain the points as shown in the figure.

EXAMPLE 6:—Set out the form of a plane cam, rotating with uniform velocity to give a bar reciprocating motion of the following character. During each stroke the bar is to have simple harmonic motion. The out-stroke is to be performed while the cam makes one half of a revolution, and the in-stroke while the cam makes one third of a revolution. There are to be equal periods of rest at each end of the stroke. Stroke of bar 3 inches. Line of stroke, $\frac{1}{2}$ inch to one side of axis of cam. Diameter of roller which works on cam, 1 inch. Minimum distance between axis of cam and axis of roller, 2 inches. If the cam makes 30 revolutions per minute, what is the maximum speed of the bar, in feet per minute, (a) during the out stroke, (b) during the in stroke?

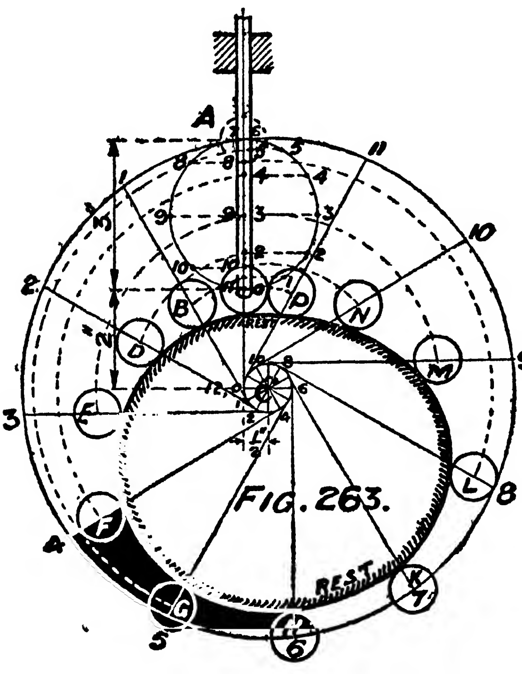


FIG. 263.

SOLUTION:—Distance from camshaft to the line of stroke of the bar is $\frac{1}{2}$ inch. Draw a circle from camshaft centre C with a radius equal to $\frac{1}{2}$ inch and divide this circle into 12 equal parts. The line of stroke of the bar is tangential to this circle and touches tangentially at zero point as shown in figure 263. Take minimum distance of 2″ from camshaft centre to the centre of roller and from this a further distance of 3 inches is to be taken to represent the stroke of the bar.

From camshaft centre C with a radius CA the highest point of the stroke, describe a circle and divide this also into 12 equal parts, as shown the zero point being at A. Join all these 12 points of the outer

circle to the corresponding 12 points of the small circle of ½ inch radius tangentially. The stroke of the bar is 3 inches and the bar is to have a Simple Harmonic Motion, therefore draw a semicircle on the length of 3 inches and divide this into six equal parts, as the follower is to rise 3″ during one half of a revolution of the cam. Strike arcs with the radii C 1, C 2, C 3, C 4, C 5 and C 6 to intersect the tangential lines 1—1, 2—2, 3—3, 4—4, 5—5 and 6—6 at B, D, E, F, G and H respectively as shown. Then a period of rest equal to $\frac{1}{12}$ of a revolution of the cam is to be given, that is from H to K.

In one third of a revolution the follower is to come down 3 inches with S. H. M. Divide the semicircle into 4 equal parts and from the cam centre C with radii C 8, C 9, C 10, and C 11 strike arcs to intersect the tangential lines 8—8, 9—9, 10—10, and 11—11 at L, M, N, and P respectively as shown. Lastly give another period of rest equal to $\frac{1}{12}$ of a revolution of the cam from 11—O. The form of the cam is shown shaded.

Maximum velocity of the bar during the out stroke.—The motion of the bar is simiple harmonic and therefore the maximum velocity of the bar is at the centre. The cam makes 30 revolutions per minute, then its angular velocity (which is generally denoted by the Greek letter ω) is $\omega' = \frac{2\pi N}{60} = \frac{2\pi \times 30}{60} = \pi$, Where N = number of revolutions per minute. Amplitude $a = 1\frac{1}{2}''$ or $\frac{3}{2}''$ and $x = o$. If the motion of the cam be reduced to an equivalent of circular motion, we find in this case $\omega' = \omega$, for the outstroke is completed in half the revolution of the cam, and so it is in the circular motion. Let V be the velocity of the bar per minute and we have $V = \omega\sqrt{a^2 - x^2}$

$V = \omega a$ per sec

,, $= \pi\frac{3}{2}$,, ,,

,, $= \pi \times \frac{3}{2} \times \frac{1}{12} \times 60$ per miuute

,, $= \pi \times 7{\cdot}5$,, ,,

,, $= 3{\cdot}1416 \times 7{\cdot}5 = 23{\cdot}562'$ per minute

Maximum velocity of the bar during the instroke.—

In this case ω' is not equal to ω for the instroke is completed in $\frac{1}{3}$ revolution of the cam, but in circular motion it is completed in $\frac{1}{2}$ revolution.

$\frac{1}{3}$ revolution of the cam is completed in $\frac{60}{30} \times \frac{1}{3} = \frac{2}{3}$ sec. In $\frac{2}{3}$ second there is $\frac{1}{2}$ revolution in circular motion. The nubmer of revolution per minute is equal to 45.

$$\therefore \omega = \frac{3\pi}{2}.$$

$$\text{Now } V = \omega\sqrt{a^2 - x^2} = \frac{3\pi}{2}a = \frac{3\pi}{2} \times \frac{3}{2}.$$

$$= \frac{9\pi}{4} \text{ per sec.}$$

$$= \frac{9\pi}{4} \times \frac{1}{12} \times 60 \text{ per minute.}$$

$$= \frac{9 \times 3{\cdot}1416 \times 60}{48} = 35{\cdot}343' \text{ per minute.}$$

EXAMPLE: 7.—A cam mechanism is shown in figure 264 (a). The cam C rotates uniformly about O, actuates the slider S by means of the bent lever LL. The slider has an intermittent motion as follows: (a) A period of rest while the cam turns through 150°. (b) The upward half of a simple harmonic motion from A to B while the cam turns through the next 120°. (c) The downward half of another simple harmonic motion while the cam turns through 90°. Set out the true shape of the cam profile, working to the given dimensions and not copying the diagram. [B. E.]

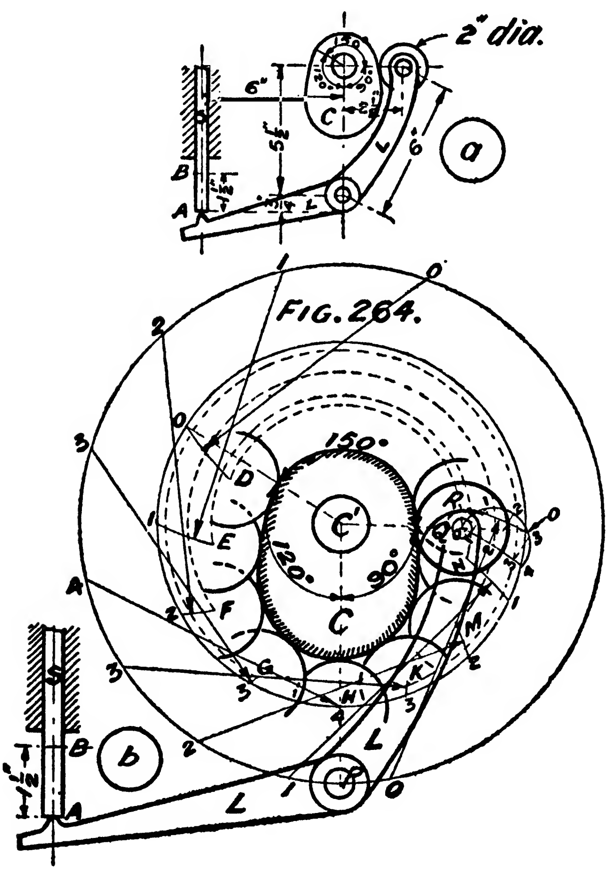


FIG. 264.

SOLUTION.—The slider S rises 1½ inches by the rotation of the cam C, the lower and the upper ends of the lever L also move radially 1½ inches with P as centre.

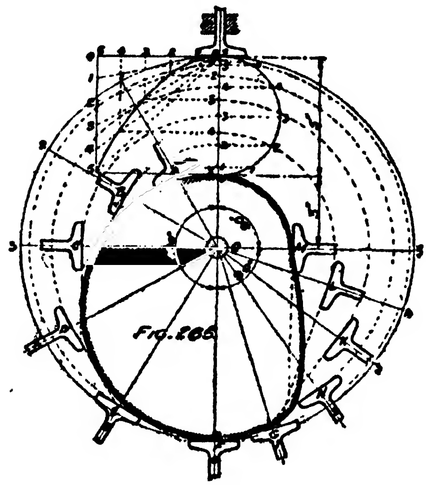
At the roller end of the lever take 1½ inches radially and describe a semicircle and divide that into 4 equal parts as shown. Draw the path of the axis of the lever with C′ as centre and C′ P as radius as shown. With C′ as centre and C′ 4 as radius describe a circle and divide the arc of this circle which subtends an angle of 120° into 4 equal parts as shown. From the centre C′ strike arcs with radii C′ O, C′ 1, C′ 2, C′ 3 and C′ 4 to intersect the arcs discribed from the centres on the path of the axis of the lever with a common radius equal to the length of the lever arm PQ at points D, E, F, G and H respectively as shown in the diagram. Now the slider has moved up 1½ inches, and the slider is to come down 1½″ while the cam turns further through 90°.

Again divide the arc of the circle which subtends 90°, of radius C′ 4 into 4 eqnal parts as shown and repeat the same construction used above for 120° arc and get the arcs intersected at K, M, N and R as shown in the diagram. The shaded figure shows the profile of the cam. Figure 246 (a)is the given question and figure (b) is the answer.

EXAMPLE 8—A vertical bar with a flat horizontal foot is driven upwards with simple harmonic motion and lowered with uniform acceleration, by a cam mounted on a horizontal shaft, and having uniform angular velocity. The upstroke of the bar is performed while the cam turns through an angle of 180°, and the downstroke while the cam turns through an angle of 90°. The bar is at rest at the bottom of its stroke while the cam turns through an angle of 90°. In its lowest position, the sole of the foot of the bar is 3 inches above the axis of the cam and the stroke of the bar is 5 inches. Draw the outline of the cam.

SOLUTION:—The least distance from the cam centre to the sole of the foot of the bar is 3 inches and the storke of the bar is 5 inches. From cam centre take 3″ and 5″ respectively to represent the least distance and the stroke of the bar. The bar is driven upwards with simple harmonic motion, and therefore draw a semicircle on the 5 inch length and divide the semicircle into say 6 equal parts. Project these points on the vertical 5 inches as shown. Next from the cam centre take a radius equal to $3+5=8$ inches and describe a circle. Divide half of this circle into 6 equal parts and from cam centre O with radii $O-1$, $O-2$, $O-3$, $O-4$, $O-5$, and $O-6$ strike arcs to intersect the radial lines at A, B, C, D, E, and F respectively as shown in figure 265. At the intersecting points A, B, C, D, E and F draw the flat horizontal

foot of the bar at each successive point and produce the lines as shown. Tangential to these lines draw a fair curve to represent the outline of the cam for turning through an angle of 180°.



For the next movement of 90° the flat bar is to be lowered with uniform acceleration. Therefore take lengths of 5 inches horizontal and vertical to the left of 5 inch stroke of the bar and draw a uniform acceleration curve as shown here. Divide the arc of 90° into five equal parts as the vertical and horizontal lines of uniform curve are divided into 5 equal parts. Then project the points of the uniform acceleration curve to right on to the vertical line above the cam centre as 1, 2, 3, 4 and 5. Strike arcs from cam centre O with radii O – 1, O – 2, O – 3, O – 4 and O – 5 to intersect the radial lines at G, H, K, L and M respectively. In setting out the outline of the cam in this portion, a difficulty arises. The outline of the cam is to touch tangentially all the straight lines drawn from the flat foot of the bar. In this case it is impossible to have tangential connections, and if attempted it is possible only to have tangential connections for two points G and M only, as shown in dotted, but the outline does not touch the tangential lines drawn from points H, K and L and fail to obtain the desired motion.

In such difficulties as these the best way is to draw a mean curve to touch all the lines as shown in the figure. The next 90° turning of the cam the bar is at rest. The final outline of the cam is as shown shaded.

EXAMPLE 9:—A vertical reciprocating piece moves in guides under the action of a cam attached to a shaft which rotates uniformly; the centre of the cam lies in the line of motion of the reciprocating piece. Suppose a friction roller used of diameter equal to $\frac{1}{8}$th stroke and suppose also that the least radius of the cam is $\frac{1}{4}$th stroke. Trace the form of the cam that the piece may slide uniformly and make one complete movement in each revolution. (I. Sc. Eng. Part II, 1923.)

SOLUTION.—Let us take one inch as the stroke of the cam, then the diameter of the roller and the least radius of the cam should be $\frac{1}{8}''$ and $\frac{1}{4}''$ respectively. (See fig. 266).

In one revolution of the cam the vertical piece is to make one complete movement, that is to say that the vertical piece is to be raised one inch from its position and lowered to the same position in one revolution of the cam.

Take one inch vertically and divide the same into 6 divisions to raise the piece, and the same divisions may conveniently be used to lower the piece as shown in the figure. From cam centre C a radius epual to CB describe a circle and divide the same into 12 equal parts. Then take through the pencil sweep the lengths C 1, C 2, C 3......C 12 on the vertical line and get them intersected with the radial lines C 1, C 2 C 3 ......C 12 of the major circle These intersected points are the centre points of the roller, therefore after drawing the rollers at these points draw a tangential line connecting all these small circles and you get the form of the cam as shown in the figure.

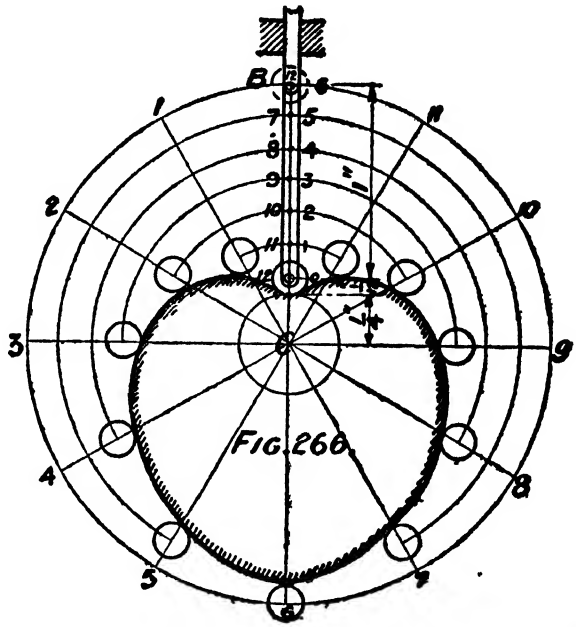


FIG. 266

EXAMPLE 10:—A vertical spindle is supplied with a plane horizontal face at its lower end. The face is actuated by a cam keyed to a shaft, the axis of which is in the line of stroke. Design the cam to raise and lower the plate under the following conditions:—

The valve to be raised through the first half of its stroke with uniform acceleration and through the second half with uniform retardation· Simple harmonic motion is given on the return stroke. The least radius of the cam is 2″, and the travel of the spindle is 1¼″. The spindle is raised in 2/5, lowered in ½, and remains at rest during the remainder of a complete revolution of the cam shaft.

(1. Sc. Eng. Part II, 1925.)

SOLUTION :—Select anywhere the cam centre C. (See fig. 267). Take 2 inches the least radius of the cam vertically and another $1\frac{1}{2}$ inches for the travel of the spindle. From cam centre C and radius CL describe a circle and divide the circumference into 10 equal parts. The reason of dividing the circle into 10 equal parts is as follows.—In one revolution of the cam the spindle is to be raised in $\frac{2}{5}$ of a revolution lowered in $\frac{1}{2}$ and the remainder to be in rest. If you reduce these fractions to a common dinominater you have $\frac{4}{10}$, $\frac{5}{10}$ and $\frac{1}{10}$, and sum of these is equal to one revolution, Therefore the circumference is divided into 10 equal parts. Next construct the uniform acceleration and retardation curves for the height of $1\frac{1}{2}$ inches as shown here by dividing the height of $1\frac{1}{2}''$ into 4 equal parts, and the reason is that in $\frac{4}{10}$ of a revolution of the cam the spindle is to be raised $1\frac{1}{2}$ inches. Obtain the points 1, 2, 3 and 4 and with radii C – 1. C – 2, C – 3 and C – 4 strike arcs and intersect the radial lines at A, B, C and D as shown. Then the return stroke to be of S. H. M. Therefore draw a semicircle on $1\frac{1}{2}''$ and divide it into 5 equal parts and obtain points E, F, G, H and K as usual. The balance $\frac{1}{10}$ of the revolution of the cam is a period of rest and it is shown there. The outline of the cam is as shown shaded.

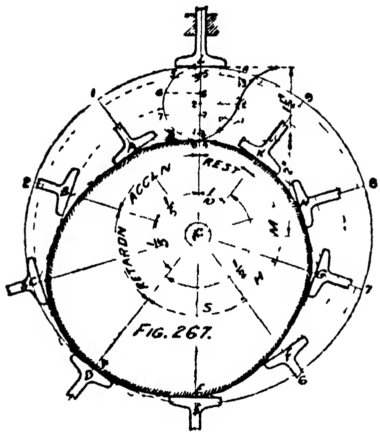


FIG. 267.

EXAMPLE 11 :—Find the profile of a cam to raise a tappet, having a roller 2" diameter, through 2" lift, the motion being S. H. M. Rise takes place in $\frac{1}{4}$th revolution, then rest for $\frac{1}{12}$th revolution, and fall during the next $\frac{1}{3}$rd revolution of the cam.

Diameter of the cam shaft=2", least thickness of metal=1". The line of action of the tappet passes through the cam centre, and the cam rotates with uniform angular velocity. (1, Sc. Eng. part 11, 1926).

SOLUTION :—From cam centre C take 1 inch, 1 inch, 2 inches and I inch, representing the radius of the cam shaft, least thickness of the cam the lift of the tappet and the radius of the roller respectively as shown. Then describe a circle with a radius CA. Divide the top left hand quarter of the circle into three equal parts and on 2 inches vertical line draw the semicircle and divide that one also

into three equal parts. With radii C – 1, C – 2, and C – 3 intersect the radial lines drawn from C the cam shaft centre at B, D & E. Mark out $\frac{1}{12}$th of the circumference of the outer circle from the point E and consider that as a period of rest. Further mark out $\frac{4}{12}$th of the circumference of the outer circle in 4 – 5 – 6 & 7. In this $\frac{1}{3}$rd revolution of the cam the tappet is to fall and therefore draw another semicircle and divide this into 4 equal parts. With radii C – 4, C – 5, C – 6 and C – 7 intersect the radial lines drawn from the cam centre at G, H, K, and L respectively. The profile of the cam is as shown shaded in the figure. In drawing the profile of the cam care must be taken

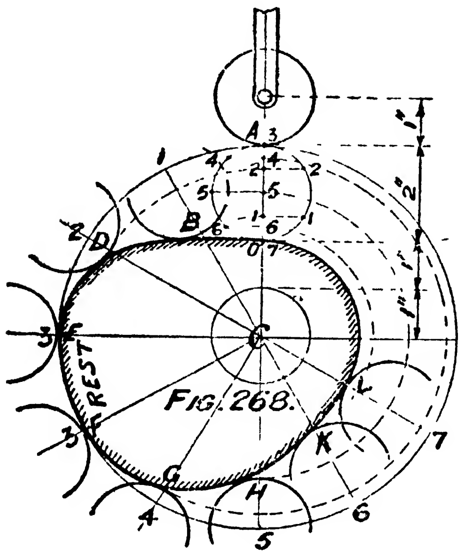


FIG. 268.

that the edge of the cam is to tuch the roller tangentially. For instance the points of intersection at B, D and H are away from the profile of the cam but the edge of the cam touches the rollers tangentially at different points.

EXAMPLE 12 :—The vertical line of stroke of the valve spindle of an oil engine is 2″ from the centre line of the horinzontal cam shaft. The minimum radius of the cam surface is 1½″ and the maximum is 3″, and these parts of the cam surface are concentric with the cam shaft. The working face of the cam which lifts the valve spindle is part of an involute of a circle, 4" diameter; which is concentric with the camshaft. There is a roller on the valve spindle, 2" diameter, which engages with the cam surface. Draw full size the profile of the working face of the cam, and determine (a) the maximum lift of the valve, and (b) the angle through which the cam rotates while lifting the valve. (1936).

SOLUTION :—Select any point C as for the centre of the cam shaft and with this centre draw a circle in dotted with the minimum radius of 1½" and with the same centre draw a portion of the circle with the given maximum radius of 3" as shown. The working face of the cam is a part of an involute of a circle 4" diameter and

therefore with the same centre C describe a third circle in dotted with a radius of 2″ and draw the involute curve starting at any point say from O. This curve should be drawn as far as this intersects the maximum radius of the cam at P. Then O—P is the involute curve and continue this curve in free hand to meet the minimum radius of the cam at the point D. Then the working face is D O P.

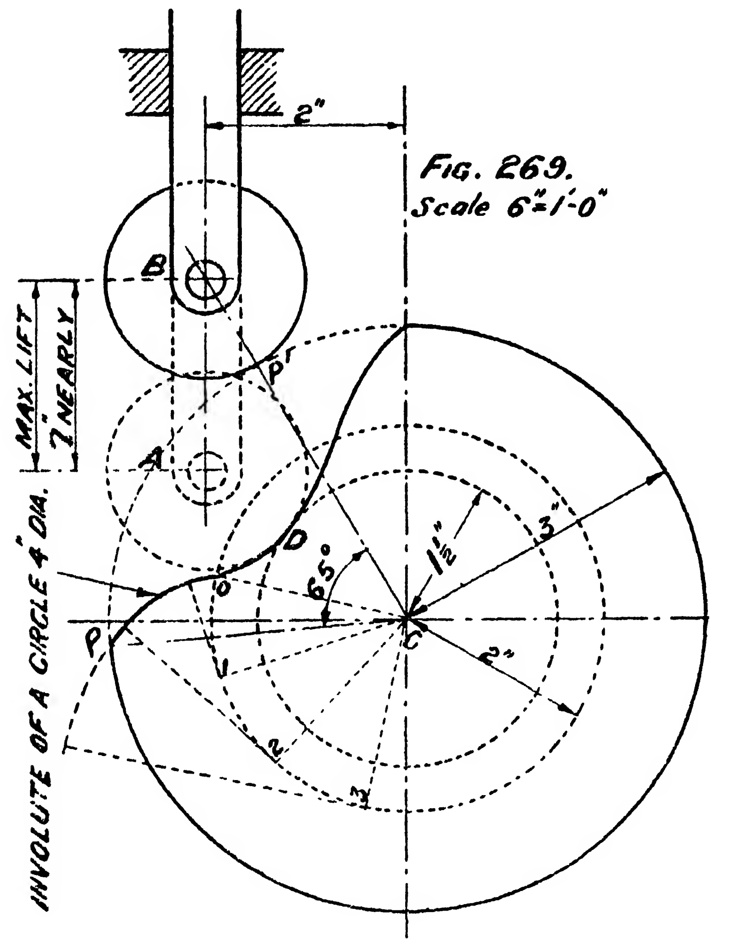


FIG. 269. Scale 6"=1'-0"

Next draw the vertical line of stroke of the valve spindle at 2″ from the cam shaft and attach a roller of 2″ diameter to it as shown. This roller touches tangentially the minimum thickness of the cam at D and the maximum thickness at P′. When the cam is revolved in the clockwise direction the involute face of the cam lifts the roller from the position A to B. When the centre of the roller reaches the point B the point P of the cam touches the roller tangentially at P′.

The maximum lift is 1″ nearly and the cam rotates at $\theta = 65^\circ$.

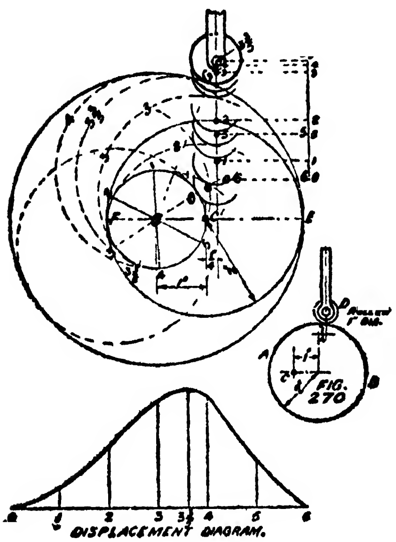
EXAMPLE 13 :—Figure 270 shows a circular cam AB fixed to a shaft whose axis is at C ; it rotates uniformly and gives a vertical motion to the tappet rod and roller D.

Draw on a time base a full size diagram showing the displacement of the centre of the roller D from its lowest position for one

revolution of the cam. Take a base line 6 inches long to represent the time of one revolution.

(I. Sc. Eng. Part II, 1928.)

SOLUTION :—In the enlarged figure C′ is the centre of the cam and C is the centre of the rotation of the cam. CF is the least radius of the cam. With C as centre and CE the greatest radius describe a circle and let the circle touch the roller tangentially at G. The centre of the roller at this position gives you the greatest displacement.



Next with centre C take the least radius CF and describe a circle and let the roller touch this circle tangentially at O and the centre of the roller gives you its lowest position here. Then divide this small circle into six equal parts as shown. Now observe that the centre point C′ of the cam will be moving on the circumference of this small circle when the cam is rotated about C the centre of its rotation. In smaller circle, six points have been marked on the circumference at equal distances apart as stated above. When the center C′ of the cam comes at point 3 you get the lowest position of the roller and the position of the cam is shown in dotted circle for reference. When C′ comes on the point 4 the roller is raised and its centre is marked as 1. Similarly you get the points 2, 3, $3\frac{3}{5}$, 4, 5 and 6 showing the displacement of the centre of the roller for different movements of the cam.

These points are shown on the vertical line to the right of the figure. Lastly on a distance base of 6″ the required displacement diagram has been drawn as shown.

EXAMPLE 14 :—A valve and spindle are actuated by means of a cam, the axis of which is in the line of stroke. The valve is raised through 2 inches in 1/6th of a revolution of the cam, lowered in the following 1/6th and remains at rest during the remainder of the revolution.

Least thickness of metal, $1\frac{1}{2}$ inches. Diameter of roller, 2 inches. Design the cam so that the motion of the valve shall be simple harmonic. (See figure 271). (I. Sc. Eng. Part II, 1929.)

SOLUTION :—Select a point C the centre of the revolution of the cam, take $1\frac{1}{2}$ inches the least radius of the cam and another 2 inches for the lift of the spindle. Then with C as centre and CA = 3·5 inches

as radius describe a circle as shown. In 1/6th of a revolution of the cam the valve is to be raised 2″ and another 1/6th of a revolution the valve is to be lowered. Therefore divide 1/3rd circumference of this circle into 12 equal parts as shown. Since the motion of the valve is simple harmonic, draw a semicircle on 2 inches lift and divide that into 6 equal parts. Transfer the points to the vertical line and with radii C 1, C 2, C 3, ...C 12 swing round the arcs and intersect the radial lines of the circle as shown. Draw the arcs of the roller at the points of intersection and tangential to these arcs draw the outline of the cam as shown shaded. Intersection points are shown in small circles.

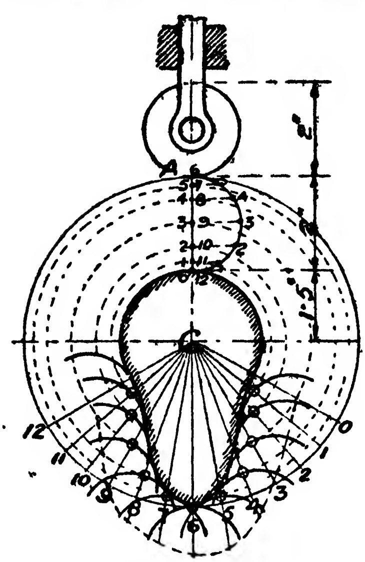


FIG. 271.

EXAMPLE 15:—A cam rotating in the direction shown in fig. 272, has to move the centre of the tappet roller from A to B and back again to A while it rotates through an angle of 110 degrees. The diagram shows by its ordinates the displacement of A along A B in inches while the abscissæ represent the corresponding angles turned through by the cam. Draw, full size, the outline of the cam, its minimum radius being $1\frac{1}{4}$ inches and the roller diameter $1\frac{1}{4}$ inches.

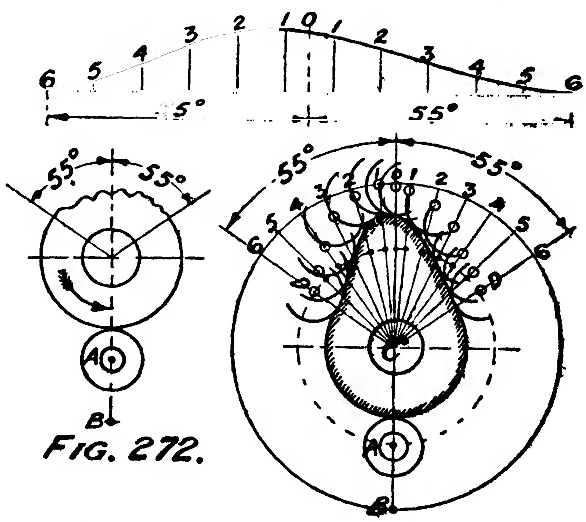


FIG. 272.

(I. Sc. Eng; Part II, 1930.)

SOLUTION:—Mark the point C at the centre of rotation of the cam. From the centre C and radius CB describe a circle, and divide the arc

which subtends an angle of 110° into eleven equal divisions as shown, because in the displacement diagram there are 11 equal divisions. Number these divisions from O to 6 from the mid point of the arc as shown. The centre of the tappet is to be moved from A to B and back again to A, therefore on the radial line C 6 take the centre of the roller D and from centre C and radius CD describe an arc DD. Then the given ordinates of the displacement diagram are to be plotted from this arc of the circle. Plot these ordinates as shown in the diagram and strike arcs of the roller at all these points. Tangential to these arcs draw the outline of the cam as shown shaded.

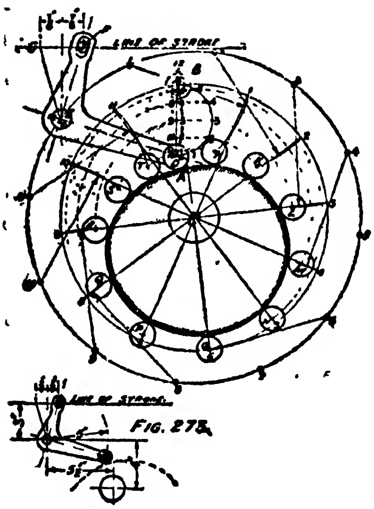


FIG. 273.

EXAMPLE 16:—A valve spindle is actuated by a bell-crank lever the arms of which are at right angles. The line of motion of the valve spindle is 3 inches from the centre of oscillation of the bell-crank lever, and the centre of the camshaft is in the position given in figure 273.

Design the cam to give the valve spindle simple harmonic motion. Diameter of roller, 1 inch.

(I. Sc. Eng: Part II, 1931.)

SOLUTION:—From the figure you can understand that C is the centre of rotation and C A the least radius of the cam. D the centre of the roller, DB the maximum lift of the lever arm ED and FG the greatest motion of the valve spindle. Since the motion of the valve spindle is simple harmonic, draw the semicircle on DB and divide that into at least 6 equal parts and tarnsfer those points on to the line DB. From centre C and radius CB describe a circle and divide this circle into 12 equal parts commencing from the point B as shown. In one half the revolution of the cam, the centre of the roller D is to be raised to the point B and in another half of its revolution it is to come down from B to D. The valve spindle also in one revolution of the cam travels from F to G and from G back again to F.

In this example it is to be assumed that the cam remaining stationary and the axis of the lever arm going round the cam centre in its path and obtain as many points as there are points of division on DB. Therefore from centre C and radius CE describe a circle as shown and

this circle represents the path of the axis of the lever. Now from centre C swing round arcs with radii C 1, C 2, C 3......C 12 as shown. The points of intersection of these arcs with the lever arm are to be obtained as follows.—Take the length of the lever arm DE with a compass and adjust the pencil point of it on the divisions of the inner circle and the needle point on the path of the axis of the lever, that is in the outer circle. Strike arcs 1—1, 2—2, 3—3,......12—12 or 0—0, and obtain points of intersection H, K, L.........T as shown. Lastly draw the roller circles at those points, and tangential to these circles draw the outline of the cam as shown.

CHAPTER XVI.

MISCELLANEOUS EXAMPLES.

In this chapter we will try to work out some examples which require some skill, imagination and construction in arriving at their proper solutions

Equilibrium of a bicycle.—

EXAMPLE 1:—The frame diagram of an ordinary B. S. A. bicycle is shown in figure 274. (*a*) Draw the stress diagram for a load of 500 lbs on its saddle, and determine the stresses in all the members. (*b*) Discuss the equilibrium of this bicycle. (*c*) Determine the maximum bending moment and shear on the steering pillar. (*d*) Why the forked end of the steering pillar is curved? (*e*) Draw the stress diagram and determine the stresses in the spokes and rim of the driving wheel of this bicycle. What is the nature of the stress in the spoke of the wheel?

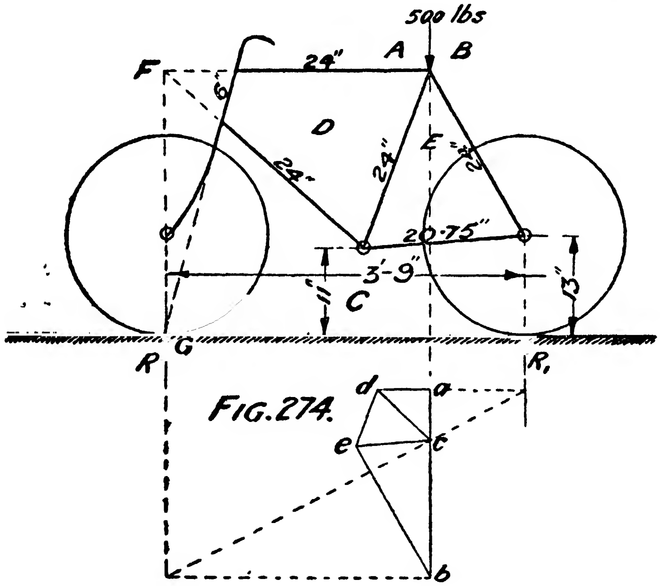


FIG. 274.

SOLUTION:—(*a*) The load on the seat of the bicycle is 500 lbs, the reaction will be under the two wheels and the directions of reactions are vertical as the wheels are free to move. Determine the magnitudes of reactions as per figure 53 page 41. Draw the force triangle *b c e* by going round the hub joint of the driving wheel, then solve the joint at the seat and the polygon of forces for this joint is *b e d a* and lastly work out the joint at the peddle and get the force triangle *c d e*. Now you have determined the stresses in all the members except the

steering pillar and the tube that receives it. The stresses in these two members cannot be determined statically as the frame of this bicycle is an imperfect one. See the descriptions of figures 80, 81 & 82 of part I.

To determine the stress in the 6 inch length of the bar see the enlarged figure 275. HM is the 6 inch length of the bar, HK is the compressive stress in the bar AD. Resolve the force HK parallel and perpendicular to the bar HM, then the parallel component KL is the stress in the bar HM.

To determine the stress in the staight portion MP of the steering pillar, take the stress in the bar CD and resolve it parallel and perpendicular to the bar MP. Now then MN is the tensile stress in the bar CD, and ON the parallel component, is the stress in the bar MP. Similarly determine the stress in the curved portion PQ of the forked end by resolving the reaction acting on the hub of the front wheel parallel and perpendicular to the curved portion PQ. Then SQ is the reaction and ST the parallel component is the strees in the bar PQ.

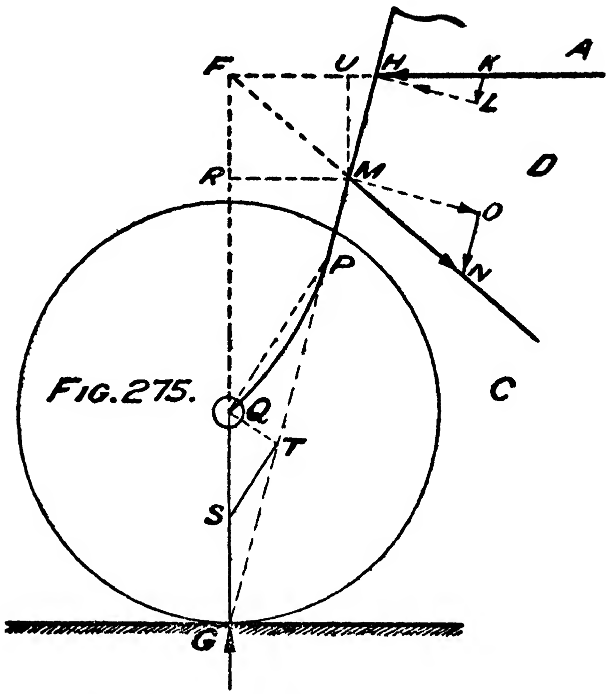


FIG. 275.

(c) The bending moment in the steering pillar is greatest at M and this is equal to KH × UM, viz. the stress in the member AD multiplied by the perpendicular distance UM. The bending moment at M is also equal to the reaction SQ × RM. For equilibrium of the bicycle these two must be equal. Shearing force from H to M is equal to LH the normal component of the force HK. Shear from M to P the straight portion of the steering pillar is equal to MO − LH.

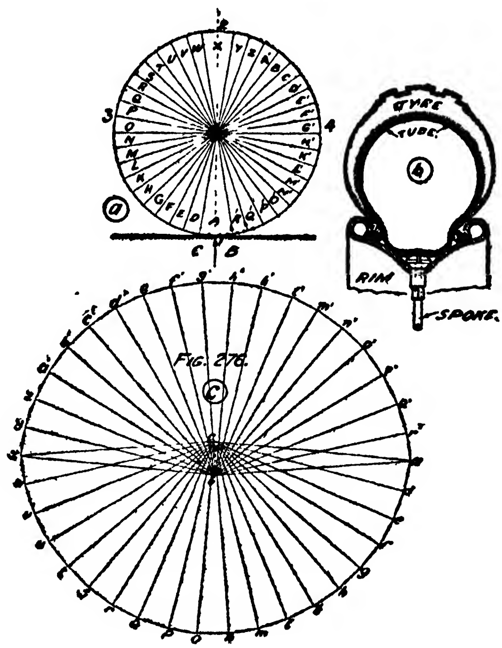
(b) For equilibrium the thrust in the steering pillar must meet the ground at G perpendicularly below the centre of the wheel where

the upward reaction acts, in order to maintain the equilibrium of the bicycle. *If the thrust* meets a *little distance* say 1″ or 1½″ beyond that point *still the* bicycle will be *going* on *moving* for ever, even if the rider leaves his hands from the handle bar, but if the thrust meets behind it, the bicycle will not be in equilibrium, and the rider is to be very careful about his grip on the handle bar and the natural tendency of the bicycle is to move in opposite direction.

For equilibrium and safety of the structure the three forces must meet at a point.

(*d*) The reason of curving the forked end of the steering pillar is to enable the thrust of the steering pillar if produced to meet the vertical reaction of the front wheel on the ground at G. Then only the equilibrium of the bicycle is maintained.

(*e*) There are 40 and 32 spokes in the driving and leading wheels respectively of this bicycle. The maximum reaction is at the driving wheel which contains 40 spokes at 2″ apart. In fig. 276 (*a*) you see the frame diagram of the driving wheel, the direction of reaction is vertical and acts upwards as shown. This is the load acting on the wheel and presses the wheel at the point of contact of the rim with the ground, the spokes that come directly over the reaction point, do not receive any stress. Every time only two spokes come directly over the load and the parts of the rim at 1 and 2 are pressed inwards towards the centre of the wheel. The spokes shown dotted shoot inwards of the tube and tire and do not receive any stress The



end of the spoke that shoots inside the tube has a screw end and fitted with curved shaped nut as shown in the diagram (*b*). Consequently the tube is not damaged. The other parts of the rim at that instant try to move themselves away from the centre of the wheel, but the

spokes connected to those parts pull them towards the centre of the wheel and consequently the stresses in the spokes are in tension. The stresses in the parts of the rim are in compression. In one revolution of the wheel, spokes come in pairs directly over the load 40 times and will be relieved of thier tensile stresses 40 times.

Knowing this fact fully well at first, you can then draw the stress diagram as follows. Take the load *b c* to any suitable scale fig. (c) acting upwards and obtain the force triangle *b c a*, by drawing lines parallel to the parts of the rim CA and AB. Similarly proceed on to every joint and draw force triangles as shown. Note that the right half of the wheel belongs to the letter B or the field B, and the left half belongs to the letter C or the field C.

EXAMPLE 2:—The projections of a tripod and its load are given in figure 277. Determine the thrusts in its legs and the magnitude of the load.

FIRST METHOD.

SOLUTION:—The projections of the load and the tripod are given here. Since the load is inclined you will have to take imaginary members *b f* and *g b* in a line with the plan of the load. Take X′ Y′ parallel to *a g b f* and draw the elevations of the two imaginary members as *b″ g′*, *b″ f′* and the given load *a″ b″*, you must know that the members *g b* and *b f* are in the same planes with *b e*, *b d* and *b d*, *b c*, respectively, and in this new elevation drawn on *x′ y′* the true lengths and inclinations of these members and the given load are obtained. Since the given load and the two imaginary members are in one plane, resolve the load 1—2 parallel to two imaginary members and obtain the force triangle 1—2—3. The stress in the member *g b* is in tension.

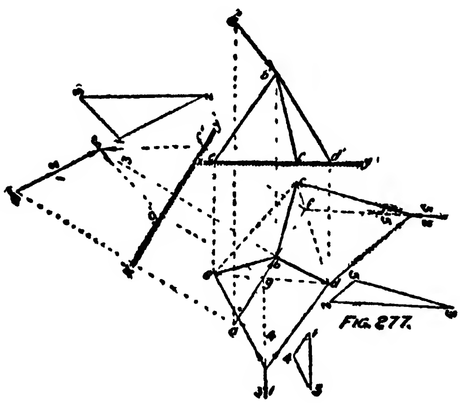


FIG. 277.

Now open out the members *b e*, *b d* and *b g* and lay down on the ground as shown and take the load 3—1 in a line with the imaginary leg and resolve it parallel to the real legs. 3—4 and 4—1 represent

the stresses in $b\ e$ and $b\ d$ respectively. Both are in tension. Similarly open out the members $b\ d$, bf and $b\ c$ and lay them down on the ground as shown. Take the load 3—2 in a line with the imaginary member $b\ f$ and resolve it parallel to the existing members $b\ d$ and $b\ c$, as shown separately. The stresses in these two members are in compression. Now notice the member $b\ d$ is subjected to a tensile stress of 1—4 during the first resolution and a compressive stress of 2—5 in the second resolution, then the stress in the member $b\ d$ is equal to (2—5) minus (1—4) $=\cdot25$ ton. Scale of load one inch equals 8 tons.

The stresses in the members are as follows:—BC $=(3-5)=4\cdot3$ tons compression, BD $=[(2-5)-(1-4)]=\cdot25$ ton compression, and BE $=(3-4)=1\cdot6$ tons ~~compression~~. Load AB $=4$ tons. Ans.
[illegible]

SECOND METHOD.

In this second method there is no necessity of taking imaginary members, but the given force can be resolved parallel to the given members in the planes of projections. Take an auxiliary plane at right angles to $d\ e$ of fig. 278 and draw the elevation of the tripod, and also the elevation of the given load. In this view two lines represent the elevations of three legs of the tripod and thus facilitates the resolution of the given force parallel to two members. Resolve $a''\ b''$ parallel to $b''\ e''\ d''$ and $b''\ c''$ as shown. Mark $b''\ f'$ on $b''\ c''$ equal to b'' G and project f' on to the line $b\ c$ in the plan.

Now proceed to resolve the given load parallel to the given members in the original plan and elevation as follows.—Take $a\ b$ equal and parallel to the plan of the given load and $b\ c$ equal and parallel to $b\ f$. Then draw $c\ d$ and $a\ d$ parallel to $b\ d$, and $b\ e$. This finishes the resolution in plan. Project all these lines in elevation taking care that the resolved parts in the elevation are to be parallel to the legs of the tripod and the load in elevation. The next thing is to get the true lengths of these forces by turning the lines in plan parallel to the vertical plane as shewn in the right side

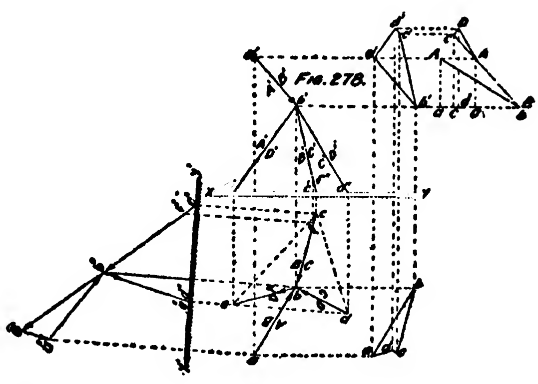


FIG. 278.

of figure 278. These results will exactly tally with those obtained in the first method.

EXAMPLE 3 :—The plan and elevation of a three-dimensioned braced frame are given in figure 279. It is to carry a load of 2 tons at its free end. Determine the forces in all the members of this frame.

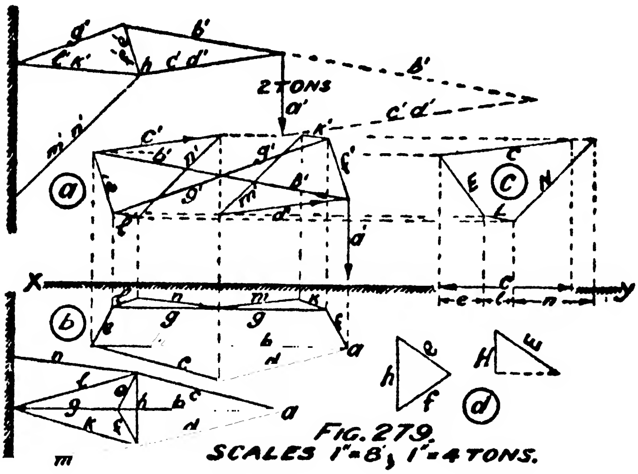


FIG. 279.
SCALES 1″=8′, 1″=4 TONS.

SOLUTION :— Name the members and the given load by letters as shown. Resolve the load 2 tons in elevation parallel to b' and c' d' as this can be easily done without any difficulty. In this force triangle the side b' is the resolved part of the load parallel to the member b', and therefore plot the force b' in elevation parallel to the member b'. Project the ends of the force b' in the horizontal plane and draw its corresponding plan b parallel to the member b of the frame in plan. Resolve this force b parallel to the given members c and d in plan and obtain the corresponding elevations c' and d' in the vertical plane drawing lines from the ends of the force b' parallel to $c'd'$ of the frame in elevation. The next two members are e and f. From the two ends of the force b' in elevation draw lines e' and f' parallel to the members e' f' of the frame to any length. Now you should know the member g' is connected to the top joints of e' f' and so the force in this member must have a connection with the forces e' & f' in the order. Again you will have to consider that the stresses or forces in the members e' & f' are equal to each other since the frame is a symmetrical one with one load at its free end. Therefore draw a line g' parallel to g' of the frame in elevation and adjust it in such a way that the forces e′ & f' should be equal to each other. In that case, the only position is the mid point of the force b' and therefore the force g' must pass through it as shown in the stress diagram (a). In no other position you

get equal forces of e' & f'. Project these lines in the horizontal plane and draw e, f & g as shown.

From the ends of the force g' in elevation draw lines l' & k' parallel to l' and k' of the frame and also from the right end of the force c' and from the left end of the force d' draw lines n' & m' parallel to n' & m' of the frame in elevation. Corresponding lines are to be drawn in plan as shown. The next step is to determine the true lengths of these force lines by turning them parallel to any one of the planes of projections as done in figure 278. Figure (c) shows the stresses in the members c, e, l & n. In figure (b) the lines b and g are parallel to the vertical plane and the stresses in the members b and g of the frame are represented by lines b' & g' in elevation The stress in the member h is to be determined as shown in fig. (d). Here the true shape of the triangle formed by the three sides e f h is drawn. The horizontal component of the stress in e or f is the stress in the member h and in the figure (d) the line H is the stress in the member h.

Note :—A fair knowledge of Descriptive Geometry is necessary in solving the three-dimensioned frame like this

EXAMPLE 4 :- Given a simple roof truss with the distributed load over it is shown in figure 280. Draw the diagrams of bending

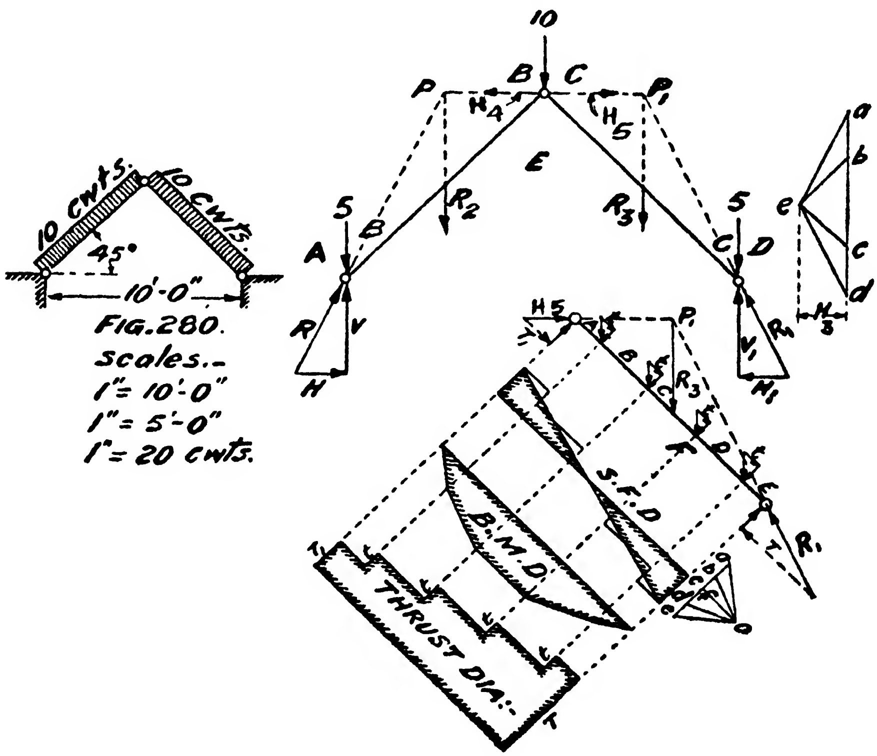


FIG. 280.
Scales.-
1" = 10'-0"
1" = 5'-0"
1" = 20 cwts.

moment shearing force and thrust for one of the rafters. Determine also the horizontal thrust against the supports.

SOLUTION :—A load of 10 cwts. is uniformly distributed over each rafter, and this roof truss contains only three joints. This is really a three hinged arch. Transfer these distributed loads to the adjacent joints of the rafter as concentrated loads as shown in the enlarged figure. Total load is equal to 20 cwts. Here AB= 5 cwts. BC=10 cwts CD=5 cwts. Now go round the ridge joint and draw the force triangle *b c e* and then to the joints at the supports. You get the stress diagram as shown and from this you determine the horizontal stress at each support by resolving the reactions R and R_1 into their horizontal and vertical components V, V_1. H, H_1. Then H or H_1 is the horizontal thrust against the wall. The resultant loads of the left and right rafter act exactly at the centre of each rafter then R and R_1 meet them at P and P_1 consequently the thrust at the crown hinge is horizontal as shown.

Next take one of the rafters separately as shown in the figure with its distributed load and reaction R_1 and the horizontal thrust at crown. This becomes an inclined beam hinged at the ends with a uniformly distributed load throughout its length. Divide this load into 4 equal parts and the magnitude of each part is equal to 2·5 cwts. acting vetically at its middle point. Resolve these four loads parallel and perpendicular to the beam as shown. Take only the perpendicular component into the load line and connect them with a pole O as shown. Draw from this the bending moment and shearing force diagrams as shown. Lastly draw the thrust diagram as follows.—Resolve the reaction R_1 and H_6 perpendicular and parallel to the beam as usual, and the parallel component T is the greatest thrust at bottom of the beam acting along the length of the beam upwards. Plot this T perpendicular to the beam acting upwards and then deduct the parallel components of AB, BC, CD and DE as shown $T-(t+t+t+t)=T_1$.

EXAMPLE 5 :—In fig. 281, LL represents a horizontal beam and the diagram gives the moment of inertia I of any cross section about the neutral axis. You are required to divide LL into eight parts such that the length of each part shall be approximately proportional to the mean moment of inertia of that part ; that is, l/I is to be the same for all the segments. Give the value of the common ratio, l and I being measured on the same scale.

(I. Sc., Eng: Part II, 1929.)

SOLUTION :—This is a question on plane geometry. Take MN parallel to LL and divide MN into eight equal parts as shown.

From the mid points of these divisions draw vertical dotted lines and mark the intersecting points on the curved line as shown in thick dots. From these points draw lines parallel to MN and let them intersect the left end vertical line LP at 1, 2, 3, 4, 5, 6, 7 and 8. Next draw from

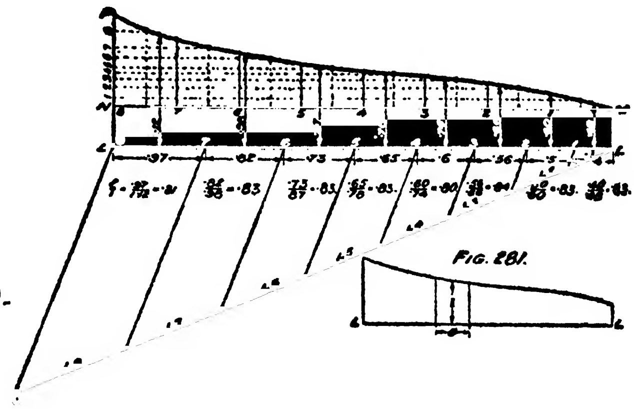


FIG. 281.

L any straight line LO at any inclination and mark on it L 1. L 2, L 3, L 4, L 5, L 6, L 7 & L 8 as shown. Join O to the left side L and draw parallels and divide the given LL proportionately. The figure is now divided into eight parts. Mean heights are marked and the value of l/I for each division are calculated and entered in the figure. The value of the common ratio is equal to ·81+·83+·83+·83+·80+·84+ 83+·83÷8=·825. Ans.

EXAMPLE 6 :—Calculate mathematically the point where the maximum bending moment occurs when two loads W_1 and W_2 of different magnitudes at a constant distance c apart roll over tue span S.

SOLUTION :—

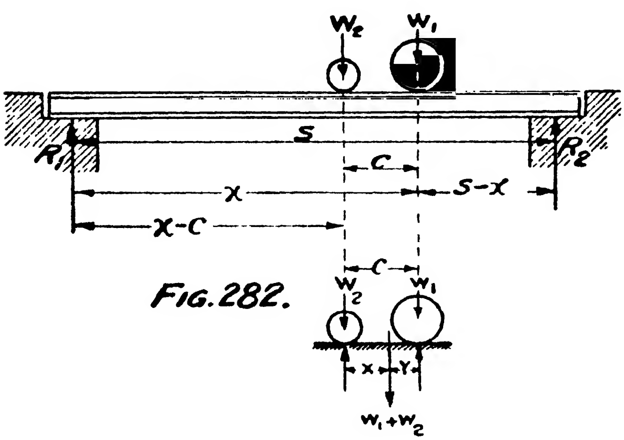


FIG. 282.

The maximum bending moment always occurs under the larger load. In this example the larger load is W_1. Refer to fig. 282. Let M be equal to the bending moment under the load W_1, X the distance from the left support to the load W_1. C the fixed distance between the loads W_1 & W_2, S the span, $S-x$ the distance from W_1 to the right support, and $x-c$ the

distance from W to the left snpport. R_1 & R_2 are the reactions at the left and right supports.

Then $M=R_2\ (S-x)$, but $R_2\times S=W_1\ x+W_2\ (x-C)$, $\therefore\ R_2=\frac{W_1\ x+W_2\ (x-C)}{S}$. $M=R_2\ (S-x)=\left\{\frac{W_1 x+W_2\ (x-C)}{S}\right\}\ (S-x)$.

This is the same as $\frac{W_1 x}{S}(S-x)+\frac{W_2}{S}(x-C)\ (S-x)=\frac{W_1 x}{S}\ (S-x)+\frac{W_2}{S}\ (x S-x^2-C S+C x)=\frac{W_1 x S}{S}-\frac{W_1 x^2}{S}+\frac{W_2 x S}{S}-\frac{W_2 x^2}{S}-\frac{W_2 c S}{S}+\frac{W_2 c x}{S}$. This is equal to $W_1 x-\frac{W_1 x^2}{S}+W_2\ x-\frac{W_2 x^2}{S}-W_2 C+\frac{W_2 c x}{S}$. Differentiating this term we have $\frac{d\ m}{d\ x}=W_1 x^{1-1}-\frac{2\ W_1 x^{2-1}}{S}+W_2\ x^{1-1}-\frac{2\ W_2 x^{2-1}}{S}+\frac{W_2 C\ x}{S}=0$.

$$\frac{d\ m}{d\ x}=W_1-\frac{2\ W_1 x}{S}+W_2-\frac{2\ W_2 x}{S}+\frac{W_2 C}{S}=0,$$

$$\frac{d\ m}{d\ x}=W_1+W_2-\frac{2 x}{S}\ (W_1+W_2)+\frac{W_2 C}{S}=0.$$

Note:—$\frac{d\ m}{d\ x}$ is a symbol to denote the rate of increase of m with respect to x and also W_2 C is a constant quantity and this will disa·ppear during the process of differentiation even if it is connected with plus or minus sign.

Dividing the above term by W_1+W_2 we have.—

$$\frac{W_1+W_2}{W_1+W_2}-\frac{2x}{S}\frac{(W_1+W_2)}{(W_1+W_2)}+\frac{W_2 C}{S\ (W_1+W_2)}=0.$$

$$1-\frac{2x}{S}+\frac{W_2 C}{S\ (W_1+W_2)}=0.$$

$$-\frac{2\ x}{S}=-1-\frac{W_2 C}{S\ (W_1+W_2)}.$$

$$\therefore\ \frac{2\ x}{S}=1+\frac{W_2\ C}{S\ (W_1+W_2)}.$$

Now the resultant of W_1 and W_2 is acting at x and y distances from W_1 W_2. Taking the moment centre under the load W_1 we have $W_2\times C=(W_1+W_2)\ y$. $\therefore\ y=\frac{W_2\ C}{W_1+W_2}$. Substituting this value of y in the above equation we have $\frac{2 x}{S}=1+\frac{1}{S}\times y$.

$$\therefore\ x=\left(1+\frac{1}{S}y\right)\frac{S}{2}=\frac{S}{2}+\frac{S}{2}\times\frac{y}{S}=\frac{S}{2}+\frac{y}{2}.$$

$$\therefore\ x=\frac{S}{2}+\frac{y}{2}.\quad \text{Ans.}$$

Hence the maximum bending moment occurs at a distance of $\frac{S}{2}+\frac{y}{2}$, where S=the span and y is the distance from the larger rolling load to the resultant line of these two rolling loads. The same result we got graphically in figure 55 page 44 of this book, without calculation.

EXAMPLE 7:—A projecting floor is carried on beams which are pin jointed at the inner end and are supported by a strut as shown in fig. 283. Find for the given loading the tension in the beam and the compression in the strut. Draw the bending moment and shearing force diagrams for the beam. The distributed load over the beam is 1 cwt per foot run. Determine the magnitudes and directions of reactions at the supports in the wall.

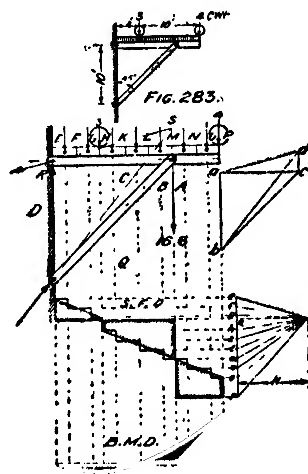


FIG. 283.

SOLUTION:—The loads over the beam are to be transferred on two supports of it, to know the direct stress in the beam. Take moments about the top end of the strut, then $R\times10=-3\times6-14\times3+4\times4$,

$$\therefore\ R=\frac{-18-42+16}{10}=\frac{44}{10}=4{\cdot}4 \text{ cwts.}$$ The vertical reaction at the top end of the strut is equal to $21-4{\cdot}4=16{\cdot}6$ cwts.

In the enlarged figure take AB=16·6 cwts at the right support, and draw the force triangle *a b c* as shown. Then solve the left supporting joint of this beam; at this joint there are three forces AC, CD, and DA are acting; out of these three, two forces AC and CD are already known and only one force DA is to be determined. Therefore plot *c d* equal to 4·4 cwts to the scale and join *a d*. Then *d a* is the magnitude and direction of the reaction of this frame at one support in the wall. At bottom end of the strut there are three forces CB, BD and DC, out of these CB and DC are known; the only force BD is to be determined. Join *bd* in the stress dirgram, then *bd* is the magnitude and direction of

the other reaction at this support in the wall. These two reactions and the load AB meet at S.

Bending moment and shearing force diagrams.—The horizontal beam is loaded with a uniformly distributed load of 1 cwt. per foot run together with concentrated loads of 3 and 4 tons. There are two supports for this, one at the wall and another at the top end of the strut. Magnitudes of reactions also are known, then there is no difficulty in drawing shearing force and bending moment diagrams as shown in the diagram.

EXAMPLE 8:—The frame CDE is attached to a member AB as shown in figure 284, and carries a load of 5 tons at E. AB is pin jointed at both ends and A can move vertically but not horizontally. Draw the bending moment and shearing force diagrams for the member AB.

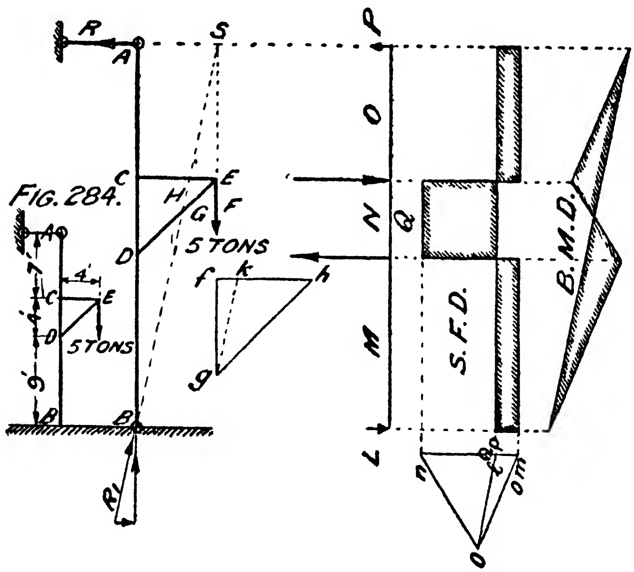


FIG. 284.

SOLUTION:—The figure is enlarged to double of its size. Draw the force triangle $f\ g\ h$ for the joint E, the stresses in the members EC and ED are $h\ f$ and $h\ g$ respectively, and $h\ f$ is in tension and $h\ g$ is in compression. These are the loads in the member AB at C and D. The whole frame is in equilibrium by three forces, viz. load AB and two reactions at A and B. The reaction at A is to be taken as horizontal because the end A can move vertically and not horizontally. This means that the end A is freely supported by the horizontal bar, therefore the reaction is horizontal. The load FG and the reaction at A meet at S, consequently the reaction at B also must meet at S. Draw the force triangle $f\ g\ k$ for the joint S, then $k\ f$ and $g\ k$ are the magnitudes of reactions at A and B respectively.

Bending moment and shearing force diagrams.—Draw the member AB separately as shown and take on this bar the vertical components of

the stresses in the bars ED, EC and of the reaction R_1. The reaction at A is to be taken as it is and name all these forces as shown. This bar AB is acted on by two unlike couples and hence the bar is balanced and no external reactions are required to keep this in equilibrium. Draw the polar diagram and its corresponding equilibrium polygon and this equilibrium polygon is the bending moment diagram. Shearing force diagram is drawn as usual.

EXAMPLE 9:—The figure 285 shows the plan of a ship which is moored by three ropes A, B and C in the positions indicated. The starboard engine is undergoing trial under these conditions, and the propellor is giving a forward thrust of 5000 lbs. Determine the pulls in the ropes.

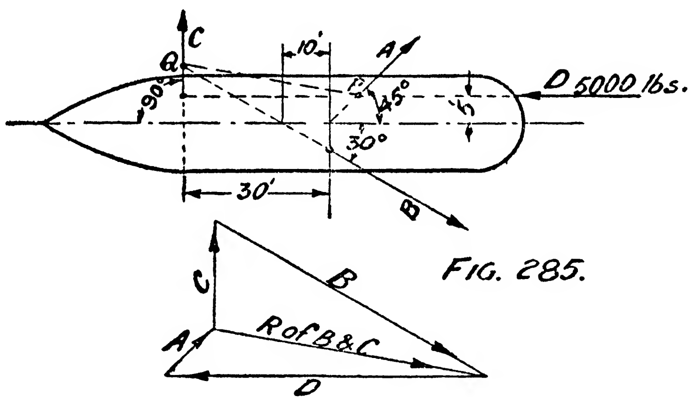


FIG. 285.

SOLUTION:—This is a question of three unknown forces, out of four forces only one force D is known and the pulls in the ropes A, B and C are unknown. Produce the force D to meet the rope A at P. Produce the rope B backwards and get it intersected the rope C at Q. Then join QP and this line QP is the resultant of the forces acting in the ropes B and C. In this way the three unknowns are converted into two unknowns. Next take the load D to scale and resolve this parallel to A and PQ; resolve again this PQ which is equal to the resultant of B and C and you get the magnitudes of the pulls in the ropes as shown in the stress diagram. A=100 lbs, B=485 lbs, C=170 lbs. Answers approximately.

EXAMPLE 10:—A pontoon carrying a crane is moored to a dock wall as indicated in figure 286. The pontoon weighs 140 tons. The load on the crane is 3 tons inclined at 30° to the vertical. Find the force in the mooring rope, the reaction at the wall, and the upward

force due to displeacement, assuming that the centre of buoyancy is 0·55 foot to the right of the C. G. of pontoon. Allow 2·5 tons for the back balance weight, but neglect the weight of the crane, and the wall reaction as horizontal and through the centre of gravity.

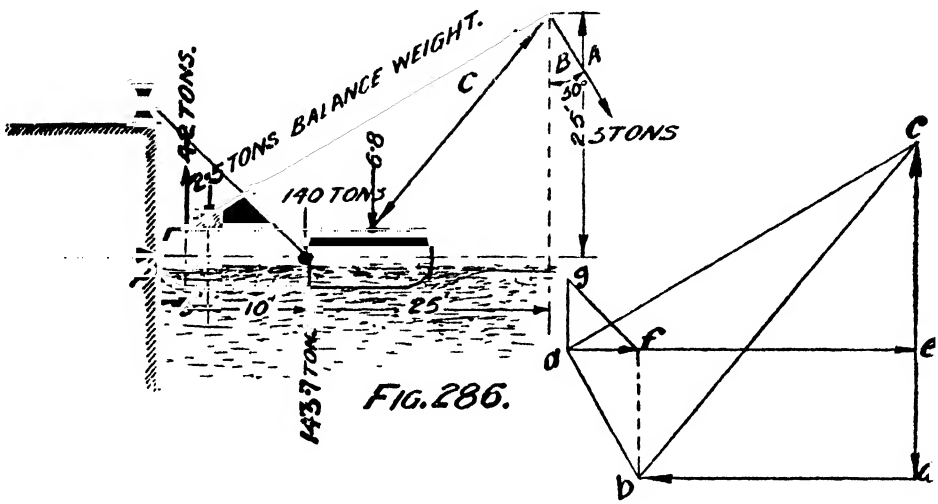


FIG. 286.

SOULTION :—Resolve the load AB on the crane parallel to the jib and backstay and draw the force triangle $a\ b\ c$. Then $b\ c$ is the compressive stress in the jib and $c\ a$ the tensile stress in the backstay. Draw the horizontal and vertical components $b\ d$, $d\ c$, ae, $e\ c$ of these two forces as shown. Out of these the horizontal component $d\ b$ is pushing the pontoon towards the dock wall and the horizontal component $a\ e$ is pulling the pontoon away from the wall, and the algebraical sum of these two is equal to $(d\ b - a\ e) = -a\ f$. Hence the horizontal component of the stress in the mooring rope must be equal to $a\ f$. Then $f\ g$ is the stress in the mooring rope and $a\ g$ and $a\ f$ are its vertical and horizontal components. The reaction at the wall at P is equal to $a\ f$.

Determination of the upward force due to displacement.— The downward forces in the pontoon are the balance weight 2·5, the weight of the pontoon 140 and the vertical component of the stress in the jib of the crane 6·8 tons, and the upward forces are the vertical components of the stress in the backstay of the crane 4·2 and the mooring rope 1·5 tons. Algebraical sum of the above is equal to the upward force due to the displacement of the pontoon. Therefore $(2{\cdot}5+140+6{\cdot}8)-(4{\cdot}2+1{\cdot}4)=(149{\cdot}3-5{\cdot}6)=143{\cdot}7$ tons. Ans.

EXAMPLE 11 :—A card is pinned to a vertical board by means

of two pins P and Q. Forces AB, BC are applied as shown in fig. 287 (*a*). Find the magnitude and direction of a third force CA which causes the force tringle to close. Show if that third force CA were applied at P the system of forces AB, BC, CA will be reduced to a couple. Show how this couple could be balanced by applying vertical forces at P and Q, so that if the pins were removed, the card would remain at rest. Determine the moment of this couple.

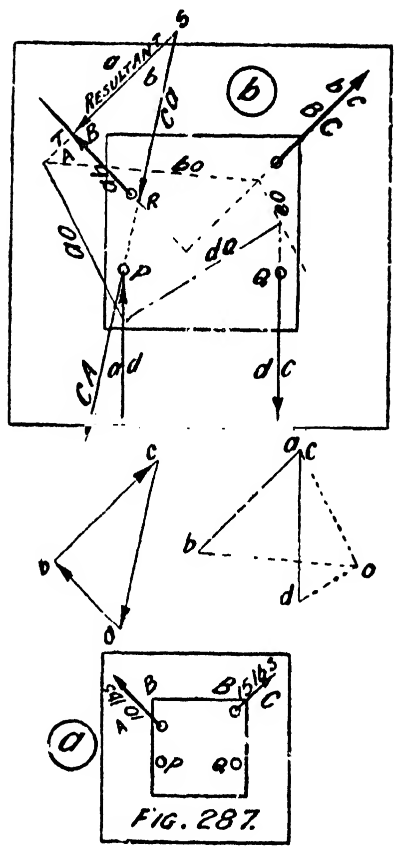


FIG. 287.

SOLUTION :—The given figure (*a*) is enlarged in fig. *b*). The given forces AB, BC are plotted down separately as *a b*, *b c* with their resultant *a c* in the triangle *a b c*. Then *c a* is the magnitude and direction of the third force CA which causes the force triangle to close.

This third force CA is applied at point P as required by the question, and when this is produced it will intersect the given force AB produced at R, At this point these two forces AB and CA are combined and their resultant is ST. This resultant ST is equal and opposite to the force BC. For proof see fig. 10 part I. Hence the system of forces AB, BC, CA have been reduced to a couple. Therefore if the pins at P and Q are removed the card will rotate anticlockwise way, but this rotation is to be stopped by applying another couple at points P and Q to keep the card board in equilibrium. You know that a couple is to be balanced by another couple only. Applying vertical forces at P and Q we can determine the magnitude of these by polar diagram and equilibrium polygon. See figure 25 part I.

These four forces are named in small letters *a b*, *b c*, *c d* and *d a* as per Bow's Notation and the polar diagram the equilibrium polygon have been drawn and the magnitude of the balancing couple *c d* or *d a*

is determined. The moment of this couple can now be calculated as the magnitude of the force and the arm are known.

EXAMPLE 12:—Five forces of 3, $1\frac{1}{2}$, 5, 2 and 4 lbs. act along the sides AB, BC, CD, DE and AE of a regular pentagon of 2″ side. Show that this system of forces is equal to a single force acting at the centre of the pentagon together with a couple. Calculate the moment of this couple.

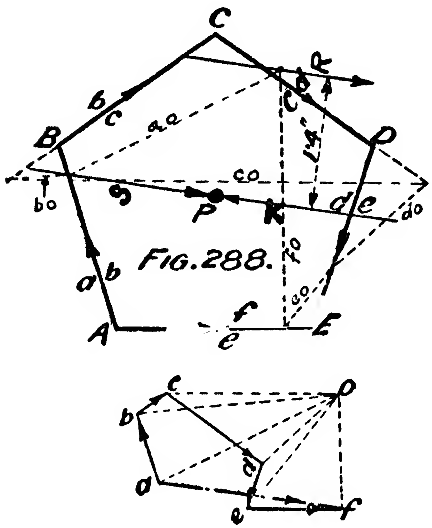


FIG. 288.

SOLUTION:—The required pentagon is drawn in the figure 288 and the given five forces are named as per Bow's Notation. These forces are taken in the order and a polar diagram is drawn by selecting a pole O anywhere. Then the corresponding equilibrium polygon is drawn and at the intersection of the first and the last ray of this polygon the resultant R is drawn parallel to the resultant *a f* of the polar diagram.

This resultant R is equivalent to all the given five forces. At P the centre of the pentagon if two equal and opposite forces S and K of the same magnitude as that of R were applied, the effect of the resultant force R on the pentagon is not at all altered. The forces R and K form one couple with an arm of 1·4″ and the force S is equal in magnitude to the force R and in the same direction.

The magnitude of the resultant measures to the scale 7·6 lbs and the moment of the couple is equal to $7{\cdot}6 \times 1{\cdot}4 = 10{\cdot}64$ pounds inches. By this method any system of co-planer forces may be resolved into a force acting at a given point and a couple; and note also that a single force and a couple acting in the same plane on a body cannot produce equilibrium. Scales for this figure are 6″ = 1 foot and 1″ = 8lbs.

EXAMPLE 13:—Four parallel forces of 90, 80, 30, and 40 lbs act at regular intervals of 5 feet on a bar of 20′ long hinged at F as shown in the figure 289 (*a*). Determine the resultant of these forces.

SOLUTION:—Name these forces in the order and draw the polar diagram as shown with any pole distance as usual. Draw the equili-

brium polygon and in this polygon you observe that the first and the last ray do not intersect, but parallel and in this case the resultant is a couple whose magnitude is equal to the ray *a o* or *o e* with its arm equal to the perpendicular distance between these two parallel rays. Its moment is equal to in this figure *a o* or *o e*$=62 \cdot 20$ lbs. $\times 11 \cdot 25 = 700$ lbs feet anticlockwise. Determination of the sense of the couple is as follows.—Take the moment about the hinge F of all the forces; then $+90 \times 5 + 30 \times 15 - 80 \times 10 - 40 \times 20 = -700$ lbs feet, therefore the sense of the couple is anticlockwise.

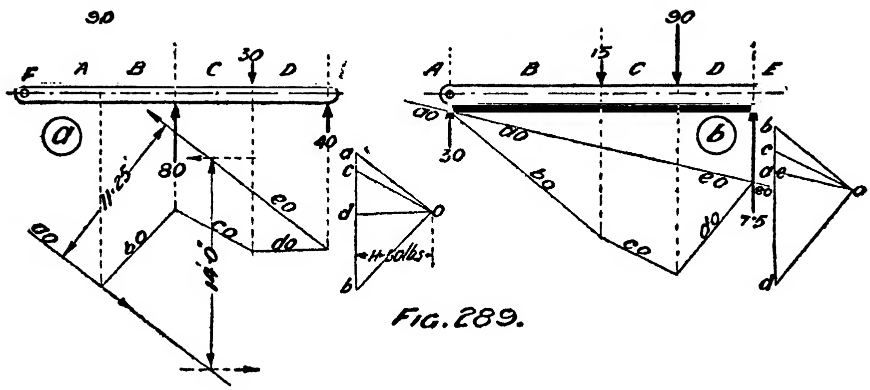


FIG. 289.

Magnitude of the couple and its moment may also be calculated by taking vertical ordinate in linear scale and pole distance in load scale. In this example vertical ordinate is equal to 14′ and pole distance= 50 lbs., then $14 \times 50 = 700$ lbs. feet as obtained above, but the magnitude of the couple is equal to 50 lbs. The forces of the couple are shown in dotted lines acting horizontally as the pole distance is the horizontal component of all the polar rays. Again in figure 289 (*b*) another system of four parallel forces are acting on the rod as shown and it is required to determine the magnitude of the resultant of these forces

In this both the force and the equilibrium polygons close and the ray *a o* and *o e* balance, then the magnitude of the resultant of the given system of forces is equal to zero.

THE THEORY OF THE MIDDLE THIRD.

In masonry structures tension is not allowed in any joint for the reason that the resisting power of the cementing material in the above structure such as mortar, in tension is very weak and hence negligible. Therefore no part of the brick or stone masonry should be allowed to come in tension.

Let AB in fig. 290 (a) represent the top edge elevation of the foundation wall of rectangular in shape and let it be loaded with a wall ABCD. Then the pressure is uniform on the surface of the joint BA and the resultant pressure acts exactly at the centre of the surface AB. If AB measures $10' \times 1'$ and the magnitude of the resultant R = 100 lbs, then the pressure per square foot throughout the surface is equal to $\frac{100}{10 \times 1} = 10$ lbs.

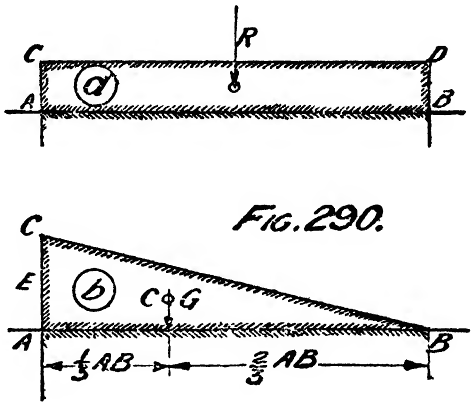


FIG. 290.

If a triangular shaped wall is built on the surface AB as shown in the figure 290 (b) the resultant pressure R acts through the centre of gravity of the triangular wall ABC and touches the surface AB at ⅓rd the distance of AB from A or at ⅔rd the distance from B. If this triangular wall is reversed that is, the greatest height AC were to come at B and zero point at A, then the resultant pressure will be at ⅓rd the distance from B or at ⅔rd the distance from A. In these two cases the surface AB is subjected to compression only, but the pressure is not uniform and the pressure at B fig (b), is zero and at A is maximum. The maximum pressure must not exceed the safe resistance of the masonry, but should be within the safety limit. Hence the action line of the resultant pressure must fall within the middle third to satisfy the condition of no tension on the surface AB. Reinforced concrete structures are capable of resisting tension and the line of action of the resultant pressure may fall beyond the middle third of the joint.

We can determine the distribution of pressure on any joint graphically with reference to the line of action of the resultant pressure by the following figures 291 and 292.—AB is the surface of the joint, measures as before $10' \times 1'$ and R = 100 lbs. see fig. 291 (a). Let the resultant pressure R act at C the centre of AB. Now the ordinate CD is equal to $\frac{R}{\text{Area}} = \frac{100}{10 \times 1} = 10$ lbs and this may be plotted to any desired load scale. The points E and F are at the extreme points of the middle third of AB and are joined to the point D. The resultant R is intersected by the lines ED and FD at D and from the intersecting point D

a straight line HG is drawn parallel to AB, then ABGH is the pressure figure. The ordinate at any point may be measured to the scale in which the ordinate CD is drawn and the pressure at any point may be known, in this case the pressure at any point is equal to 10 lbs. Students must remember this well.

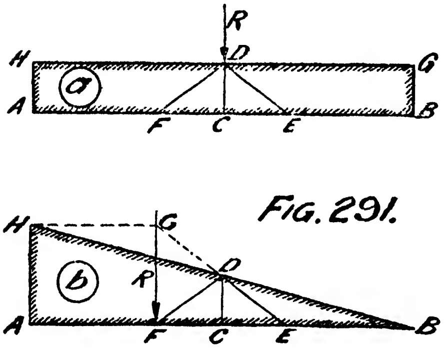


FIG. 291.

Next suppose the resultant pressure R falls exactly at $\frac{1}{3}$rd of the joint AB, as shown in fig. 291 (*b*). Here you must observe that $CD = \frac{R}{\text{Area}} = 10$ lbs., points E and F are at the extreme points of the middle third and are joined to D as before. The resultant R is intersected at two points F and G and from these two points horizontal lines FB and GH are drawn as before. The pressure figure is ABDHA and the ordinate at any point is the pressure at that point. The pressure at B is zero and at A = 20 lbs per square foot. Similarly the pressure figure (*a*) and (*b*) fig. 292 are drawn for different positions of the resultant R. In fig. (*a*) the resultant R falls within the middle third, and the maximum and minimum pressures in compression are 14·2 and 5·8 lbs. respectively from the ordinates at A and B. In fig. (*b*) the resultant R falls outside the middle third and hence the sufrace AB is subjected to compression and tension, then the pressures are 25·6 lbs. compression at A and 5·6 lbs tension at B.

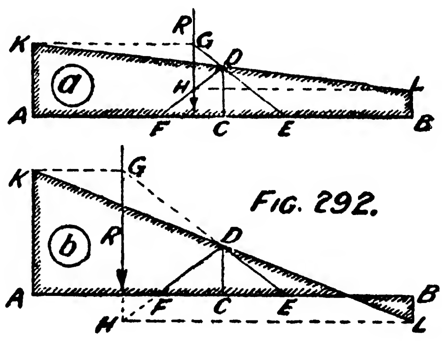


FIG. 292.

The above results can also be verified by the usual formula as follows:—The maximum and minimum pressures $= \frac{W}{A} \pm \frac{M}{Z}$, where W = the resultant load R or the vertical component of R, A = sectional area of base, M = bending moment, that is, the product of the load into its distance from the centre of gravity of the base, Z = Section modulus of the base and for rectangular section $\frac{1}{6}\, b\, d^2$.

In fig. 291 (*b*) the maximum and minimum pressures are equal to $\frac{W}{A} \pm \frac{M}{Z}$, W=R=100 lbs., A=10′×1′=10 □′, M = 100× $\frac{10}{6}$ = 166·66 pounds feet, Z = $\frac{1}{6}$ *b* $d^2 = \frac{1}{6} \times 1 \times 10^2 = \frac{100}{6} = 16·66$ units. Then $\frac{W}{A} \pm \frac{M}{Z} = \frac{100}{10} \pm \frac{166·66}{16·66} = 20$ lbs and 0 lbs. Ans. The same result we got by the pressure figure. Similarly for (*a*) and (*b*) of fig. 292 we get $\frac{W}{A} \pm \frac{M}{Z} = \frac{100}{10} \pm \frac{100 \times ·7}{16·66} = 14·2$ lbs and 5·80 lbs Ans. $\frac{W}{A} \pm \frac{M}{Z} = \frac{100}{10} \pm \frac{100 \times 2·1}{16·66} = 25·6$ lbs. compression and 5·6 lbs. tension.

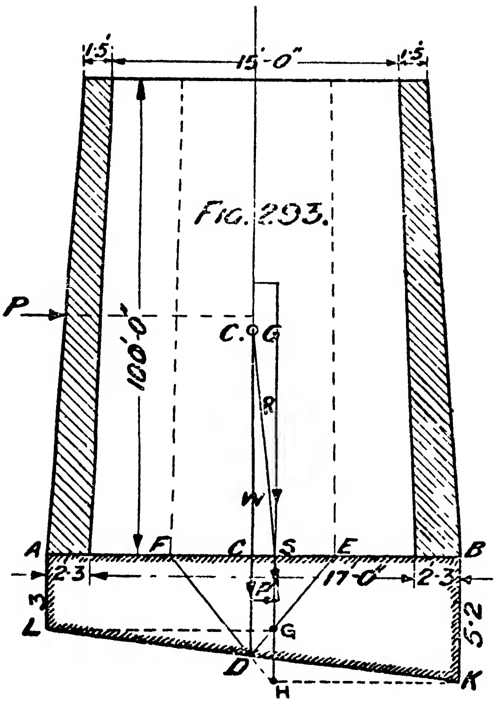
EXAMPLE 14:—The top of a tall circular brick chimney has an internal diameter of 15 feet and a thickness of 1′−6″; at 100 feet below the top its internal diameter is 17 feet and its thickness 2′−3″. Find the amount and position of the resultant pressure at this plane assuming that wind pressure is concentrated 50′ below the top and is equivalent to 30 lbs per square foot of projected area and that brickwork weighs 120 lbs per cubic foot. State whether there is any tention on the joints of the brickwork and express your opinion as to the sufficiency of the design.

(I. Sc. Eng. part II, 1921.)

SOLUTION:—The total weight of this chimney may be calculated as follows.—Internal diameters at top and bottom are 15′ and 17′ respectively and external ones are 18′ and 21′−6″. See figure 293. Then the mean internal and external diameters are $\frac{15+17}{2} = 16'$ and $\frac{18+21·5'}{2} = 19·75'$ respectively. The mean of these two is equal to 17·875′. The total volume of the masonry is equal to π D × width of the ring × height × weight of masonry $= \frac{22}{7} \times 17·875' \times 1·875' \times 100 \times 120$ lbs $= \frac{8848125}{7 \times 2240} = 564·29$ tons.

Total wind pressure is equal to as per formula P=·5pd, where P=total wind pressure, p=wind pressure per square foot of vertical surface=30 lbs, *d*=projected width of the section at right angles to the direction of the wind $= \frac{21·5+18}{2} \times 100 = 1975$ squre feet. Then $P = ·5 \times 30 \times 1975 = \frac{15 \times 1975}{2240} = 13·22$ or say 13 tons. Let us call the

total weight of the chimney as W and the wind pressure as P. The total weight W acts at the centre of gravity of the section downwards as shown in the diagram and wind pressure P at 50′ from the ground. By combining these two loads W and P for their resultant R we see that the resultant R cuts the base at S. This point S is within the middle third or the core of the section of this chimney. The middle third or the core of a section for this chimney is calcutated from the formula $C = \frac{D^2 + d^2}{8\,D}$, where C = the width from the centre line to the middle third or the core, D = external diameter at the section, d = internal diameter at the section. Then $C = \frac{21{\cdot}5^2 + 17^2}{8 \times 21{\cdot}5} = 4{\cdot}36$ at bottom, and at top $= \frac{18^2 + 15^2}{8 \times 18} = 3{\cdot}81$. These widths are shown in dotted vertical lines in the diagram. The



vertical component of the resultant R to be taken to act at S and this divided by the annular or the supporting area of the chimney will give you the ordinate CD = 564.29 ÷ π 19·25 × 2·25 = 4·14 tons. The pressure figure ABKLA is then drawn as usual and the whole base is subjected to compression only. The intensity of pressure at B is 5·2 tons and at A 3 tons per square foot. These pressures are within the safe limit and the design is very satisfactory. The vertical scale for this figure is 1″ = 40′ and the horizontal scale is 1″ = 10′. The reason of taking these two different scales in this figure is to show clearly to the students the line of action of the resultant, the pressure figure, and its distribution over the base clearly. A different scale of 1″ = 8 tons is taken to plot the ordinate CD and the pressure figure, and the scales for W and P are 1″ = 400 tons and 1″ = 100 tons respectively in the same ratio as the linear scales.

EXAMPLE 15:—A timber beam 8′ long and 12″ square floats in

water. The weight of the timber is 40 lbs per cubic foot and of the water 62·5 lbs per cubic foot. An additional weight of 300 lbs is placed at one of the extreme ends of the beam. Draw the bending moment and shearing force diagrams, assuming the weight of the beam acting at its centre of gravity. Determine the portion of the beam that is outside the surface of the water.

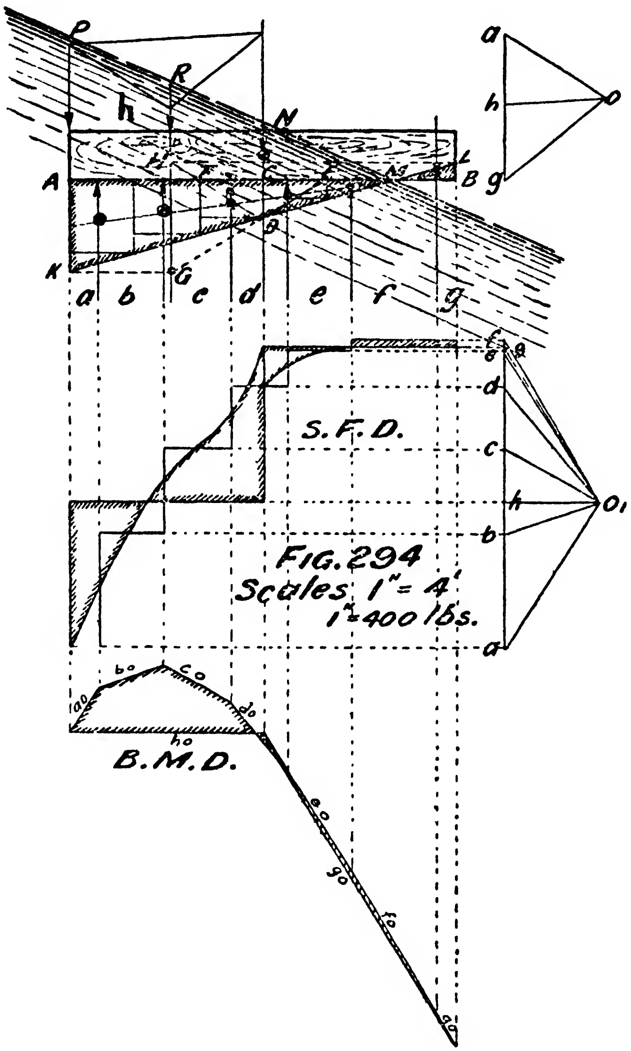


FIG. 294
Scales 1" = 4'
1" = 400 lbs.

SOLUTION:— The weight of the beam is equal to $1 \times 1 \times 8 \times 40 =$ 320 lbs acting at its centre of gravity and an additional weight of 300 lbs is placed at the left end of the beam. The resultant R of these two loads is determined by the polar diagram and the equilibrium polygon as shown in the figure 294. The pressure figure ABLKA is drawn as per fig. 292 (b), for the position of the resultant pressure R. The area of the compressive figure $= \frac{188}{2} \times 6{\cdot}8 =$ 640 lbs, and the area of negative figure $= \frac{33}{2} \times 1{\cdot}2 = 20$ lbs nearly. The net area $= 640 - 20 = 620$ lbs $=$ the external load. The pressure figure is equally divided and the loads are named as per Bow's notation. The magnitudes of the forces are

$ab=230\cdot4$, $bc=179\cdot2$, $cd=128\cdot0$, $de=76\cdot8$, $ef=25\cdot6$ and $fg=20$ lbs, *this last force fg is acting downwards and the rest of the loads are* acting upwards as shown in the figure. In the load line the external loads also are included and mamed as fh and ha. The bending moment and shearing force diagrams are drawn as usual as shown in the figure.

The pressure is zero at the point M in the beam and the pressure at A is equal to 188 lbs measured from the pressure figure, therefore the depth of water at the point $A=\frac{188}{62\cdot5}=3$ feet. From A a height $AP=3'$ is taken and PM is joined and this represents the surface of the water. The water mark intersects the top surface of the beam at N, and the ordinate in the pressure figure at this point is exactly equal to 62·5 lbs. This shows the water level is drawn accurately.

THE RESISTANCE FIGURE OR THE MODULUS FIGURE.

In part I we gave a short description of the "Resistance Figure" or "Modulus Figure" by referring to the figure 48 (*b*). Now the method of drawing the resistance or the modulus figure graphically will be shown for different forms of sections. To start with take a rectangular section shown in fig. 295. First find the neutral axis, this axis exactly passes through the centre of gravity of the section, since the section is symmetrical. Draw base lines 1—1 and 2—2 to touch the upper and lower extremities of the figure respectively. Then draw some more horizontal lines 3—3, 4—4 as shown and project the intersected points at the sides of the rectangle to the base line 1—1. From 1—1 draw lines to meet the centre of gravity of the section. These two lines intersect the horizontal lines 3—3, 4—4, 5—5 at 3′—3′, 4′—4′ and 5′—5′. The shaded triangular figure is known as the Modulus Figure for the upper half of the section, and for the lower half similarly you can draw as shown.

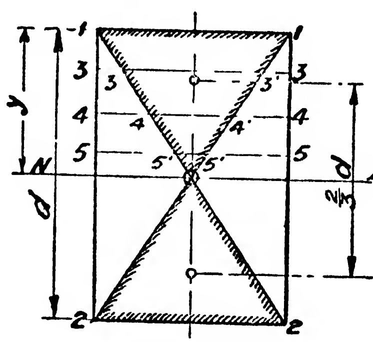


FIG. 295.

Section Modulus = Area of either half of the resistance figure multiplied by the length to the centres of gravity of the resistance figure.

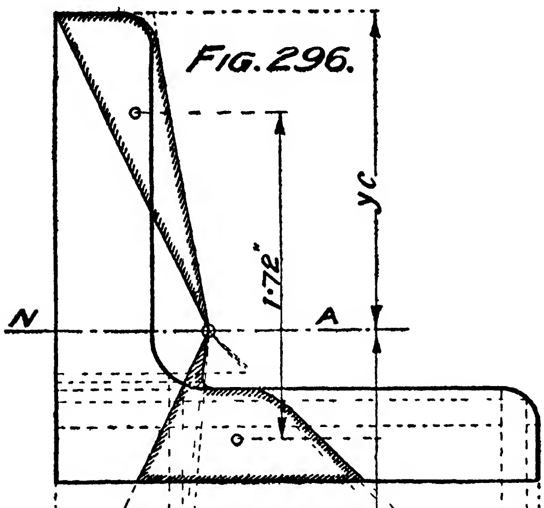
$$\text{Section Modulus or } Z=\frac{b\,d}{4}\times\tfrac{2}{3}\,d=\frac{b\,d^2}{6}.$$

Moment of Inertia or I=Section modulus multiplied by the distance y of the extreme fibre from the neutral axis.

$$I=\frac{b\,d^2}{6}\times y=\frac{b\,d^2}{6}\times\frac{d}{2}=\frac{b\,d^3}{12}.$$

EXAMPLE 16:—Draw the modulus figure for the new British Standard equal angle $2\frac{1}{2}''\times 2\frac{1}{2}''\times\frac{1}{2}''$ and from this calculate the section modulus and moment of inertia of the section. The neutral axis passes at ·80″ from both the sides of the angle iron.

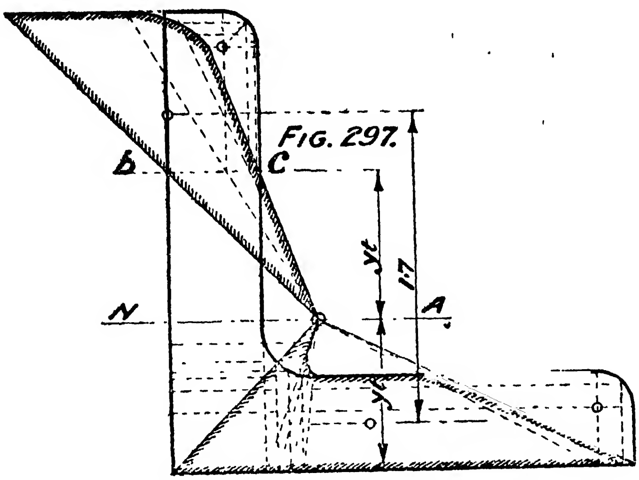
SOLUTION—Here the neutral axis is not at the centre of the section and it is nearer to the outermost tension fibres. Therefore two modulus figures one for compression and another for tension can be drawn. Modulus figure for compression.—Draw the base line through the uppermost fibre parallel to the neutral axis at a distance $y\,c$ from the neutral axis and draw another base line below the neutral axis and parallel to it at the same distance $y\,c$ from the axis. If you draw the base line below, through the outermost tension fibre, i. e. at a distace $y\,t$ from the axis, the extreme fibre stress $f\,t$ will be less than the extreme fibre stress $f\,c$, because $y\,t$ is less than $y\,c$. The areas of the modulus figures above and below the neutral axis must be equal to each other, therefore the base line below the neutral axis is to be taken at $y\,c$ distance to satisfy the above condition.



The modulus figure is drawn similar to the figure 295. Section modulus=area of the modulus figure multiplied by the distance of centres of gravity of the resistance figure=·4225 × 1·72=·72.

Moment of inertia=Section modulus multiplied by the distance $y\,c$ to the extreme fibre from the neutral axis=·72 × 1·7=1·2.

The modulus figure for tension is drawn in figure 297 thus. The base line is drawn below the neutral axis through the outermost tension fibres at $y\,t$ distance and another base line above the axis is drawn at the same distance $y\,t$. Here you observe the intensity of stress above the base line $b\,c$ is greater than the stress in the fibres at $b\,c$ hence the outline of the modulus figure at $b\,c$ must be continued upwards to meet a line drawn through the extreme fibres parallel to the axis. Then only the areas of the modulus figures will be equal to each other.



Section modulus $Z = \cdot 88 \times 1\,7 = 1{\cdot}50$

Moment of inertia $I = Z \times y = 1{\cdot}5 \times 1{\cdot}7 = 2\,55$

Similarly you can draw the modulus figure for the flange-footed rail section shown in figure 298. The construction is clearly shown for upper half of the figure above the neutral axis and can be followed without difficulty.

EXAMPLE 17:—Determine and measure the forces in the members A, B, C, D, E and F of the given truss loaded as shown in the figure 299.

The scale of the figure being $\frac{1}{4}$ inch to 1 foot, draw the diagram of bending moment for the horizontal beam, and measure the maximum bending moment.

(I. Sc. Eng. part II, 1931.)

SOLUTION:—This truss as a whole is a difficient one for want of two members as per following formula.—Number of bars in a perfect frame $= 2\,J - 3$, where J = the joint. There are 7 joints in this truss $2 \times 7 - 3 = 11$. There must be 11 members to make this frame perfect. In this frame there are only 9 members, and hence the stress diagram

will never close. The members 5 – 8, and 6 – 8 cannot have direct stresses but they are subjected to bending moments.

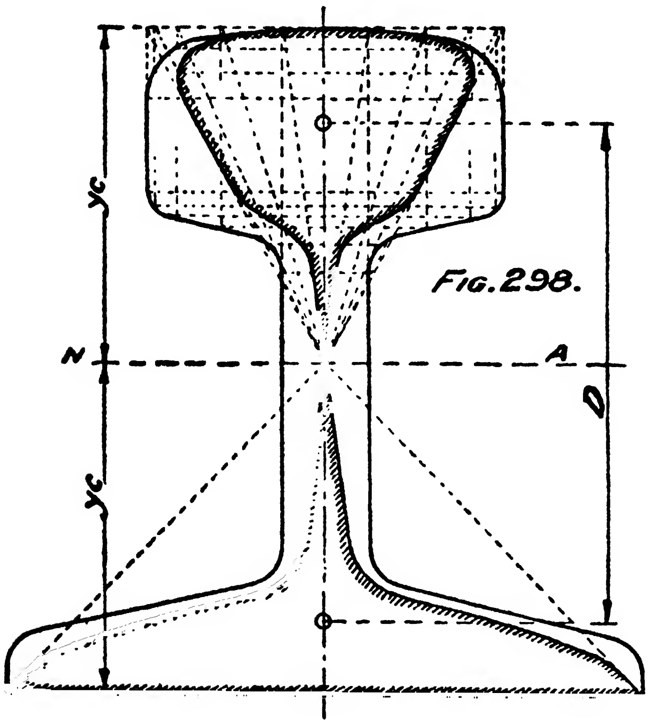


FIG. 298.

Plot all the given loads in a load line as 1 – 2 – 3 – 4, and draw the force triangle 3 – 4 – 5 for the right hand top chord joint. Next draw the polygon of forces for the remaining two joints on the same chord as shown and by these you have determined the stresses in all the required members A, B, C, D, E & F.

Now the members A, F, E and D are directly connected to the

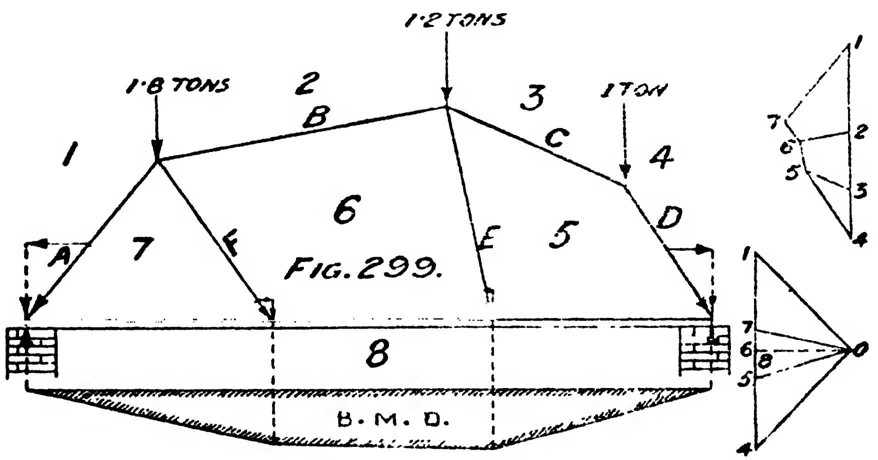


FIG. 299.

horizontal beam and you should consider the stresses in them are

the loads on the beam, then these loads are to be resolved parallel and perpendicular to the beam as usual and draw the bending moment diagram for the vertical components as shown in the diagram. The maximum bending moment is under the load 6—5 and measures 2·6 tons feet.

DISTRIBUTION OF SHEAR STRESS IN A BEAM SECTION.

EXAMPLE 18 :—Draw the intensity of shearing stress diagram for a rectangular section and also for the new British Standard steel beam section of 8"×4" shown in the figures 300 (*a*) and (*b*). The vertical shear for the rectangular section is 10 tons and for the steel beam 6 tons.

SOLUTION :—Construct first the resistance figure of the section and divide that into any number of equal parts parallel to the neutral axis. The intensity of shearing stress on any horizontal plane is given by the formula $f\,s = \frac{AYS}{w\,I}$, where $f\,s$=shear stress, A=area of the modulus figure above the section, S=vertical shear, w=width of the section at the plane, and I= moment of intertia of the section.

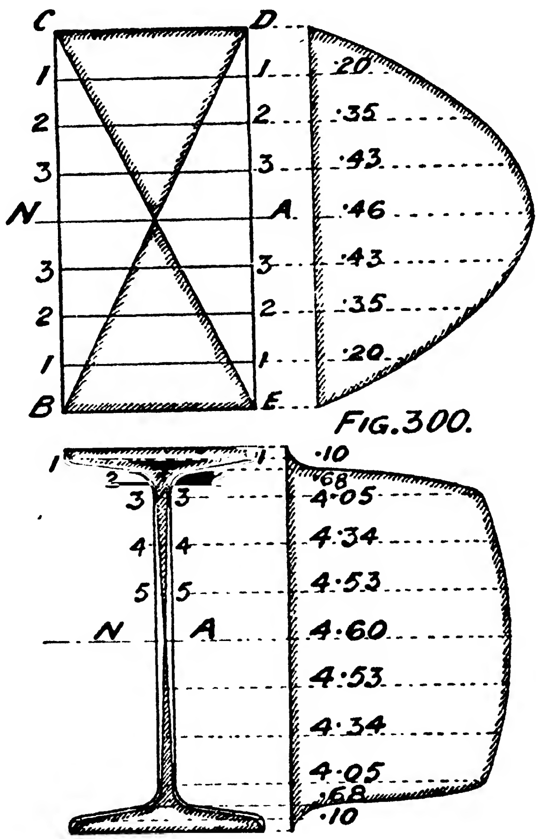


FIG. 300.

The given rectangular section measures 4"×8" and the area of the modulus figure above the section 1—1 is equal to $\frac{4+3}{2}\times 1 = \frac{7}{2} =$ 3·5 □", Y=4" w=4", S= 10 tons, $I = \frac{BD^3}{12} = \frac{4\times 8^3}{12} = \frac{512}{3}$. Here $\frac{YS}{I}$ is constant for any section $=$

$$\frac{4\times 10\times 3}{512} = \frac{120}{512} = \frac{15}{64}.$$

$$f\,s=\frac{AYS}{w\,I}=\frac{8\cdot5}{4}\times\frac{15}{64}=\cdot20 \text{ ton per } \square''.$$

At section 2—2, the area of the modulus figure $=\frac{4+2}{2}\times2=6\ \square''$,

$$\therefore\ f\,s=\frac{6}{4}\times\frac{15}{64}=\frac{90}{256}=\cdot35 \text{ ton per } \square''.$$

At section 3—3, the area $A=\frac{4+1}{2}\times3=\frac{15}{2}=7\cdot5$,

$$\therefore\ f\,s=\frac{7\cdot5}{4}\times\frac{15}{64}=\cdot43 \text{ ton per } \square''.$$

A the neutral axis NA the area of the modulus figure above this line is equal to $\frac{4}{2}\times4=8\ \square''$, then $f\,s=\frac{8}{4}\times\frac{15}{64}=\frac{15}{32}=\cdot46$ ton per $\square''$. Lastly plot all these values at these sections and draw a fair curve as shown.

Again for the steel beam draw the modulus figure and divide the section into any number of parts parallel to the neutral axis. The moment of inertia of the beam from the tables about the axis $XX=55\cdot63$, The given vertical shear $=6$ tons, then the constant $\frac{YS}{I}=\frac{4\times6}{55\cdot63}=\cdot43$. At section 1—1, the area of the modulus figure above that section $=\frac{4+3\ 7}{2}\times\cdot25=\cdot96\ \square''$.

$$f\,s=\frac{AYS}{w\,I}=\frac{\cdot96}{4}\times\cdot43=\cdot103 \text{ ton per } \square''.$$

At section 2—2, $A=\cdot96+\left(\frac{3\cdot7+1\cdot4}{2}\times\cdot5\right)=2\cdot23\ \square''$, $w=1\cdot4''$,

$$f\,s=\frac{2\cdot23}{1\cdot4}\times\cdot43=\cdot68 \text{ ton per } \square''.$$

At section 3—3, $A=2\cdot23+\left(\frac{2\cdot4+1\cdot4}{2}\times\cdot5\right)=2\cdot64\ \square''$, $w=\cdot28''$,

$$f\,s=\frac{2\cdot64}{\cdot28}\times\cdot43=4\cdot05 \text{ tons per } \square''.$$

As section 4—4, $A=2\cdot64+\left(\frac{\cdot24+\cdot15}{2}\times1\right)=2\cdot83\ \square''$,

$$f\,s=\frac{2\ 83}{\cdot28}\times\cdot43=4\cdot34 \text{ tons per } \square''.$$

At section 5—5, $A=2\cdot83+\left(\frac{\cdot15+\cdot1}{2}\times1\right)=2\cdot95\ \square''$,

$$f\,s=\frac{2\cdot95}{\cdot28}+\cdot43=4\cdot53 \text{ tons per } \square''.$$

At the neutral axis N·A, area $A = 2{\cdot}95 + \left(\frac{\cdot 1}{2} \times 1\right) = 3{\cdot}00$ □″,

$$f\,s = \frac{3}{\cdot 28} \times \cdot 43 = 4{\cdot}6 \text{ tons per □″}.$$

From these two shearing stress diagrams we observe that there are small intensity of shear stresses in the flanges and almost even distribution of shear stress over the web area. Therefore the web is to be designed to resist the whole shearing action.

TRUSSED BEAM, BOLLMAN TRUSS AND FINK TRUSS.

In the trussed beam shown in figure 301 (*a*) the load AB is exactly in the centre and in the stress diagram we observe that the stress in the central strut DE is exactly equal to the external load AB. If there are more than one struts in the length of the beam such as we observe in the Ballman Truss in the figure (*b*) the stresses in these struts are equal to the external loads that are acting directly over them as observed in the figure (*a*). This fact enables us to draw the stress diagram very quickly as the usual method of drawing the stress diagram is very labourious for Ballman and Fink Trusses shown in the figures (*b*) and (*c*)

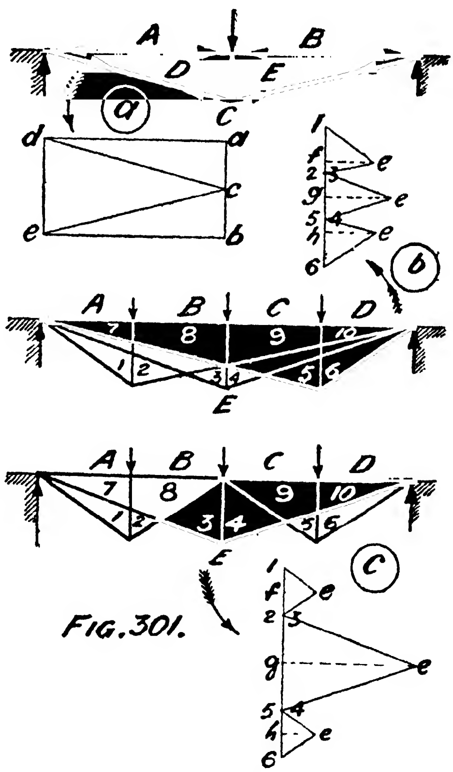


FIG. 301.

Ballman Truss.—The stress in the strut 1—2 is equal to the load AB and in the load line 1—2 is taken equal to the load AB and from 1 and 2 lines are drawn parallel to the ties 1 E and 2 E. In this force triangle 1 *e* and 2 *e* represent the stresses in these two ties. Similarly the force triangles 3—4—*e* & 5—6—*e* are drawn for the other two struts. The stresses in the horizontal members A—7, B—8, C—9 and D—10, are equal to one another; the magnitude of the stress is equal to sum of

the horizontal components of the stresses of the ties. These components are shown in dotted in the force triangles. Hence each horizontal member is stressed to the amount of $ef+eg+eh$. Reactions are equal to $\frac{W}{2}$ where W = sum of all the loads as the loads are symmetrical.

Fink Truss:—Figure (c) shows the Fink Truss. In this the central strut is to bear the froces equal to the vertical components of the stresses in the ties 2 E and 5 E in addition to the load BC. Since the ties are equally inclined to the struts, the vertical components of the ties are equal to half of the loads AB and CD. Hence the stress in the central strut is equal to twice the load BC as the loads are symmetrical.

As usual three force triangles are drawn and the stress in each horizontal member is equal to sum of all the horizontal components of the stresses of the ties $= ef+eg+eh$.

Note:—If the loads are unsymmetrical, the same method of procedure should be followed.

CHAPTER XVII.

VELOCITY AND ACCELERATION DIAGRAMS.

Angular velocity.—The velocity of a rotating body is usually measured in revolutions per minute, or in radians per second. A radian is the angle subtended at the centre of a circle by an arc of that circle equal in length to the radius, and also the radius is contained in the circumference 2π times. Therefore in a circle there are 2π radians. One circle contains 360°, one radian is equal to 360° divided by $2\pi = \frac{360}{2\pi} = 57{\cdot}3°$ nearly. Suppose a body makes N revolutions per minute, then its angular velocity which is generally denoted by the Greek letter omega ω is—

$$\omega = \frac{2\pi N}{60}$$

The relation between the linear velocity and the angular velocity is obtained from the following equations.—

$v = \omega r$, then $\omega = \frac{v}{r}$, where v = linear velocity in feet rer second, r = radius in feet of the circle round which the point moves. If the velocity is given in N revolutions per minute,

$$v = \frac{2\pi r N}{60}$$

$$\omega r = \frac{2\pi r N}{60}$$

$$\therefore \omega = \frac{2\pi N}{60}.$$

If n is the velocity in revolutions per second $v = 2\pi r n$.

EXAMPLE 1:—A rod AB is 8 feet long and it rotates at 90 revolutions per minute about A. What is its angular velocity in radians per second ?

SOLUTION:—Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = \frac{2 \times 3{\cdot}1416 \times 90}{60} = 9{\cdot}42$ radians per second. Ans.

Angular Acceleration—A rotating body may have a constant or variable angular velocity; if it has a variable velocity, the rate of increase of angular velocity is known as the Angular Acceleration.

METHOD OF DRAWING THE VELOCITY & ACCELERATION DIAGRAMS.

The following brief notes are to be known by the students in drawing the velocity and acceleration diagrams.—

(1) Usually the velocity and acceleration of one point in a mechanism of a particular configuration are given and from these the diagrams are to be drawn.

(2) To the known velocity or acceleration of a point the relative velocity or acceleration of a second point in the mechanism is to be combined, but this relative motion may not be completely known, still we can draw a locus of the image of the second point.

(3) The direction of the relative velocity of two points in a rigid body will be perpendicular to the straight line which joins them. From this fact we can understand that one point describes a circle relative to the other.

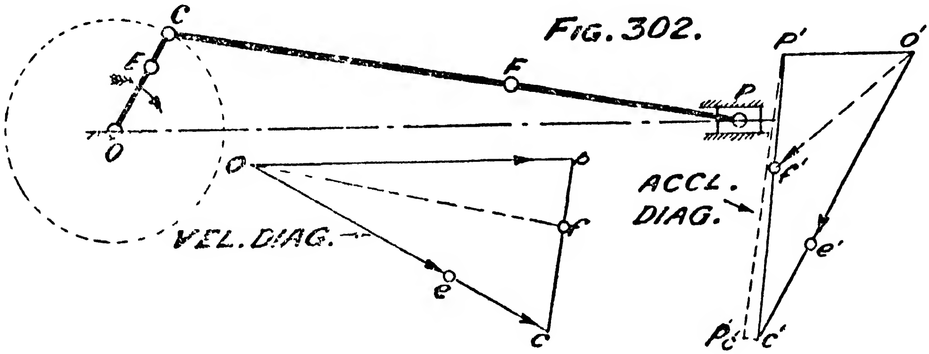
(4) There are two components in the relative acceleration of two points. One is the magnitude and the other the direction. The magnitude $\frac{v^2}{r} = \omega^2 r$ is along the line joining the two points. Here v = the relative velocity, r = the distance between the points. The direction will be along the perpendicular to the line joining the points $= a\,r$, where a = angular acceleration.

(5) The velocity diagram is to be drawn first and from this the magnitude of the relative acceleration is to be calculated from $\frac{v^2}{r}$ and when this is done, the direction only of the other component will be known, and in this case we can draw the locus of the image of the second point.

EXAMPLE 2:—Figure 302 shows the simple engine mechanism whose crank is 9″ long and rotates in the clockwise direction at 90 revolutions per minute. Its connecting rod is 4 feet long. Determine the velocities and acclerations of points E, F and P.

SOLUTION:—Velocity Diagram.—The crank rotates at 90 revolutions per minute and by using the formula we have $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 9{\cdot}42$ radians per second. The velocity of the end of the crank $o\,c = \omega\,r = 9{\cdot}42 \times \frac{9}{12} = 7{\cdot}06$ feet per second. Select any origin O and draw a line $o\,c$ perpendicular to OC to any convenient scale 7·05 feet,

The point P moves along OP and therefore from o draw a line $o\,p$ indefinitely parallel to OP. Again the point P moves relative to C along the perpendicular to CP. Hence draw a line $c\,p$ perpendicular to CP. This is a second locus of P and the point p is the intersection of the two loci. The complete velocity diagram is $o\,c\,p$. Scale for this diagram $1'' = 4$ feet per second.



To find the velocity of any point E on OC, divide $o\,c$ in the same ratio as E divides OC, then the length $o\,e$ gives the velocity of E relative to O. Since O is fixed $o\,e$ is the absolute velocity of E. Similarly the velocity of any other point F on PC is represented by $o\,f$ in the diagram. Here f divides $p\,c$ in the same ratio as F divides PC.

Acceleration Diagram.—There are two bars in this mechanism and we know their velocities from the velocity diagram. The acceleration along the bar $OC = \frac{v^2}{r} = \frac{7{\cdot}06^2}{{\cdot}75} = 66{\cdot}45$ feet per second per second and that of the bar $CP = \frac{v^2}{r} = \frac{3{\cdot}2^2}{4} = 2{\cdot}56$ feet per second per second.

Draw $o'\,c'$ to some suitable scale $66{\cdot}45$ feet sec^2. parallel to CO. In this case ω is constant the point C has no component acceleration perpendicular to OC, therefore $o'\,c'$ is the acceleration of C.

The resultant acceleration of P is along PO, therefore draw a line through o' parallel to OP indefinitely and the acceleration of P relative to C has two components, one of magnitude and another of direction. Draw $C' - p'_{c'}$ equal to $2{\cdot}56$ feet sec^2 parallel to PC and through $p'_{c'}$ draw a line perpendicular to PC. Then p' is the point of intersection of the two loci. Join p' to c' and $p'\,c'\,o'$ is the complete acceleration diagram. Scale for this diagram $1'' = 40$ feet per sec. per sec.

To find the acceleration of any point E in OC, divide $o'\,c'$ in

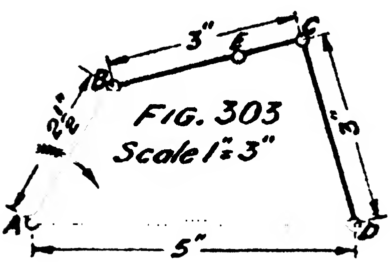
the same ratio as E divides OC, then $o'e'$ is the acceleration of the point E. Similarly the acceleration of any other point F in PC is represented by the line $o'f'$.

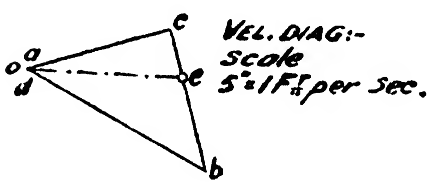
Note :—$p'c'$ is only a point used in construction and has not got any value.

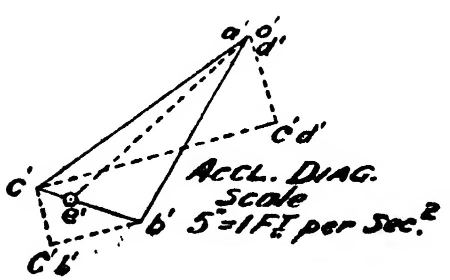
Velocity of point E=o e=4·7 feet per sec.
,, ,, ,, F=o f=6·6 ,, ,, ,,
,, ,, ,, P=o p=6·7 ,, ,, ,,
Acceleration of E=$o'e'$=45 feet per sec. per sec.
,, ,, F=$o'f'$=38 ,, ,, ,, ,, ,,
,, ,, F=$o'p'$=27 ,, ,, ,, ,, ,,

EXAMPLE 3 :—The bar AB of the four bar mechanism shown in the figure 303, rotates at 10 revolutions per minute in the direction shown. When the mechanism is in the configuration indicated in the figure, find (*a*) the angular velocity of BC and CD; (*b*) the angular acceleration of BC and CD. The link AD is fixed.

(I. Sc. Eng. part. II, 1930.)







Solution :—Velocity Diagram.—From the formula $\omega = \frac{2\pi N}{60} = \frac{2 \times 3{\cdot}1416 \times 10}{60} = 1{\cdot}05$ radians per second. Velocity of the bar AB = $\omega r = 1{\cdot}05 \times \frac{2{\cdot}5}{12} = {\cdot}218$ foot per sec.

Select any orgin O draw o b perpendicular to AB to represent this magnitude to any suitable scale in the direction of the rotation. From b and o draw b c and o c perpendicular to BC and CD, then these two lines are the loci of C and hence their point of intersection. A and D are fixed points and therefore their images coincide with pole O. Then o c b is the complete velocity diagram, and the velocity of any point E in BC is represented by o e if e divides b c in the same ratio as E divides BC. Scale for this fig. is 5"=1 foot. per sec.

Angular vel. $\omega = v/r$. ω of BC and CD are each $= \frac{\cdot155}{\cdot25} = \cdot62$ radian per sec.

Acceleration Diagram—Tabulate the relative velocities of ends of bars and radial accelerations of bars as shown here.

Name of bar.	Rel: Vel: of ends of bar in feet per sec. $v = \omega r$	Length of bar in feet r	Radial acceleration of bar $= \frac{v^2}{r}$ in feet sec².
AB	$o\,b = \cdot218$	·208	$\frac{\cdot218^2}{\cdot208} = \cdot228$
BC	$b\,c = \cdot155$	·25	$\frac{\cdot155^2}{\cdot25} = \cdot096$
CD	$c\,d = \cdot155$	·25	$\frac{\cdot155^2}{\cdot25} = \cdot096$

Draw $o'\,b'$ parallel to AB of length equal te ·228 ft. sec². towards BA and from b' and o' draw $b' - c'b'$ and $o' - c'd'$ each equal to ·096 ft. sec². towards the directions CB and CD respectively. Now you should draw the first locus of C' through $C'\,b'$ perpendicular to BC and a second locus of C' through $C'\,d'$ perpendicular to CD. C' is the point of intersection of the loci, then $o'b'c'$ is the complete acceleration diagram. Scale $- 5'' = 1$ foot per sec².

The acceleration of any other point such as E on the link BC is represented by the line $o'\,e'$ and its sense being from the pole to the point as shown in the diagram. On measuring we obtain the following result :—

Acceleration of the bar AB = ·228 foot per sec. per sec.
" " BC = ·112 " " " " "
" " CD = ·275 " " " " "
" of the point E in BC = ·250 " " " " "

Angular accelerations of the bars BC and CD are $\alpha = \frac{a}{r} = \cdot24$ and 1·02 radians sec². respectively, where a = tangential component of acceleration of the bar.

Note :—The line $b' - C'\,b'$ drawn parallel to BC indicates the radial component of acceleration of C relative to B and the line $C'b' - C'$ is the tangential component of acceleration of C relative to B. Similarly the line $O' - C'\,d'$ drawn parallel to CD is the radial component of acceleration of C relative to D, and $C'\,d' - C'$ drawn perpendicular to CD is the tangential component of acceleration of C relative te D. These

points C' b' and C' d' do not count in the acceleration, but they are only points used in construction. Scale.—1″=40 feet per second.

Angular acceleration α=tangential acceleration÷r.

EXAMPLE 4:—Draw the velocity and acceleration diagrams of the given linkage which is a combination of 4 bar chain and slider crank chain for the position shown in the figure 304. The bar AB turns at 30 radians per second, A and D are fixed and the slider is constrained to move between the bars at F horizontally.

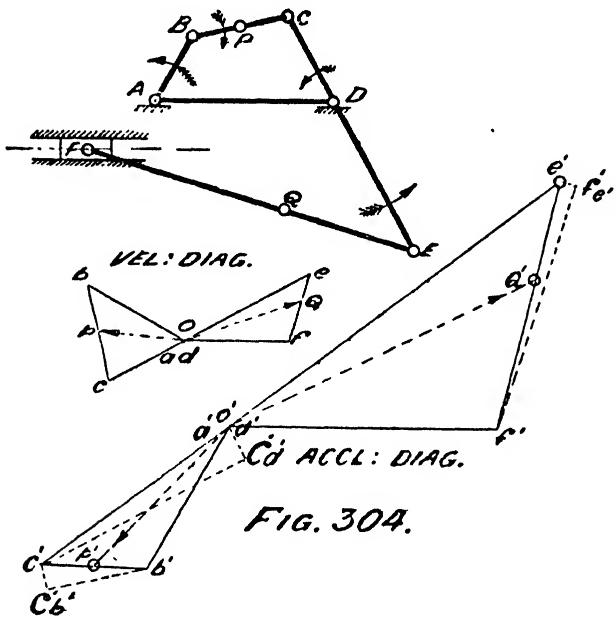


FIG. 304.

SOLUTION:—Velocity Diagram.—The bar AB turns at 30 radians per second and the velocity of B relative to A is $\omega r=30\times\frac{9}{12}=22{\cdot}5$ feet per second perpendicular to the radius in the direction of rotation of AB. Select a pole o and from it draw o b perpendicular to AB to represent this magnitude to any suitable scale. The points A and D being fixed have zero velocities, therefore their positions are at O. The velocity of C relative B is perpendicular to BC, therefore draw b c perpendicular to BC, also the velocity of C relative to D is perpendicular to CD, hence draw d c perpendicular to CD. The intersection of these two rays gives the point C. Now produce c d to e so that d e to be in the same ratio to c e as DE is to CE, next the velocity of F relative to E is perpendicular to EF. Hence draw e f perpendicular to EF and the point F is constrained to slide horizontally

in its guides and therefore draw $o\,f$ horizontal, then the intersection of these two rays gives you the point f. The velocity diagram is completed. Scale 1″=40 feet per second.

Acceleration Diagram.—Tabulate the velocities and radial accelerations of the bars as shown below.—

Name of bars	Rel: vel: of ends of bars in ft: per sec. $v=\omega\, r$.	Length of bar in feet r	Radial acceleration of the bars$=\frac{v^2}{r}$ in ft. sec².
AB	$o\,b=22{\cdot}5$	·75	$\frac{22{\cdot}5^2}{{\cdot}75}=675$
BC	$b\,c=20\ 0$	1	$\frac{20}{1}=400$
CD	$c\,d=17$	1	$\frac{17^2}{1}=144{\cdot}5$
EF	$e\,f=15{\cdot}5$	3·5	$\frac{15{\cdot}5}{3\ 5}=68{\cdot}64$

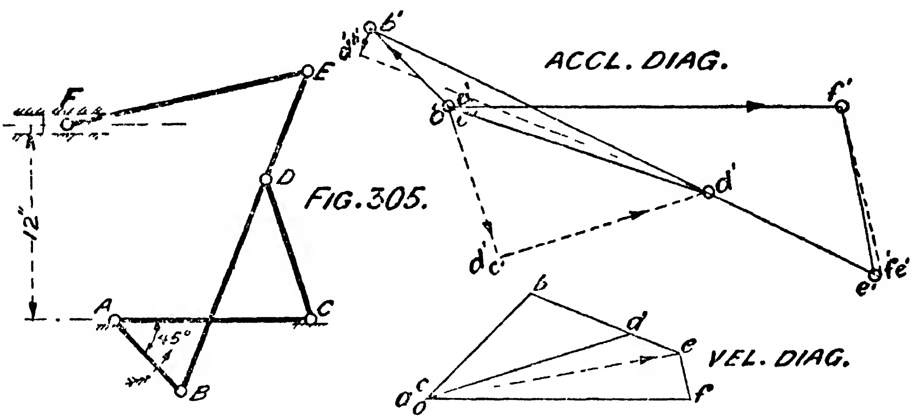
Draw $o'\,b'=675$ feet sec². parallel to AB, here ω for AB is assumed to be constant and hence no tangential component. Point C relative to B has radial component of acceleration 400 ft. sec², draw $b'-C'\,b'$ equal to this magnitude and from the point $C'\,b'$ draw a line indefinitely at right angles to $b'-C'\,b'$. Again the point C relative to D has 144·5 ft. sec² as for its radial acceleration, draw $o'-C'\,d'$ to this magnitude and draw another line for its tangential acceleration indefinitely. These two tangential acceleration lines meet at C'. Now C' is the point of intersection of the loci. Join $b'\,c'$, and the points A and D being fixed have zero velocities and hence $a'\,d'$ coincide with the origin o'. The line $d'\,c'$ is the acceleration of the bar CD. The point e' is to be fixed in a line with $c'\,d'$ in the same ratio as E stands in the line CDE. Then CD : DE :: $c'\,d'$: $d'\,e'$, substituting the values we have 12 : 21 :: 960 : $d'\,e'$. $\therefore\ d'\,e'\times 12=960\times 21$, $\therefore\ d'\,e'=1680$ ft. sec².

Point F is constrained to move horizontally, therefore draw $o'\,f'$ horizontal. The radial acceleration of F relative to E is 68·64 ft. sec², draw $e'-f'\,e'$ parallel to FE and draw its tangential component as

shown and it will intersect the line $o'\ f'$ in f'. Join $e'\ f'$ and the complete acceleration figure is $o'\ b'\ c'\ e'\ f'$. Scale.—1″=800 feet per sec. per sec.

The accelerations of any other points such as P and Q in the links BC & EF are represented by the lines $o'\ p'$ and $o'\ q'$ respectively and their sense being from the pole to the points as shown.

EXAMPLE 5:—The crank AB of the mechanism shown in the figure 305 turns at 30 radians per second. The slider F is constrained to move on the guides horizontally. Determine the velocity and acceleration of the point E for the given position of the mechanism. AB= 6″, BE=1′−9″ and EF=15″.



SOLUTION:—Velocity Diagram.—The velocity of the bar AB relative to A= $\omega\ r=30\times\cdot5=15$ ft. per sec. Draw $o\ b$ perpendicular to AB equal to this magnitude. Since the points A and C are fixed they have zero velocities and coincide with the origin O. The velocity of D relative to C is perpendicular to CD, also the velocity of D relative to B is perpendicular to BD Therefore draw the loci of the point D by drawing $o\ d$ and $b\ d$ perpendicular to CD and BD respectively. Then d is the point of intersection of the loci.

The point e is to be fixed in the velocity diagram in the same ratio as E is in the line BDE. Therefore BD : DE : : $b\ d : d\ e$, then BD$\times d\ e$=DE$\times b\ d$. Substituting the values we have $14\ d\ e=7\times11$ $\therefore\ d\ e=77\div14=5\cdot5$. Next the point F is constrained to move horizontally in its guides, draw $o\ f$ horizontal indefinitely. The velocity of the point E relative to F is perpendicular to EF, therefore from e drawn a line perpendicular to EF and it will intersect the horizontal line $o\ f$ at f. The figure $o\ b\ d\ e\ f$ is the complete velocity diagram. The line $o\ e$

represents the velocity of the point E=26 feet per second. Scale.—1″= 20 feet per second.

Acceleration Diagram.—Calculate the velocity and radial acceleration of each bar and tabulate them as shown below.—

Name of bar	Rel: Vel: of ends of bar in ft. per sec. $v=\omega r$	Length of bar in feet $=r$	Radial acceleration of the bar $=\frac{v^2}{r}$ ft. sec².
AB	$o\ b=15$	·5	$\frac{15^2}{\cdot 5}=450$
BD	$b\ d=11$	1·04	$\frac{11^2}{1\cdot 04}=116\cdot 34$
CD	$c\ d=22\cdot 5$	·79	$\frac{22\cdot 5^2}{\cdot 79}=640\cdot 82$
EF	$e\ f=5$	1·25	$\frac{5^2}{1\cdot 25}=20$

Take any origin o' and draw $o'\ b'=450$ ft. sec². equal to the radial acceleration of AB. Then get the first locus of the point D relative to B by drawing $b'-d'_{b}'=116\cdot 34$ ft sec² which is the radial acceleration of BD, and its perpendicular component as shown. The second locus of the point D relative C can be determined by drawing $o'-d'_{c}'=640\cdot 82$ ft. sec². which is the radial acceleration of CD and its perpendicular component. These two perpendicular components meet at d', join $b'\ d'$ and produce $b'\ d'$ to e' in the same ratio as E stands in the bar BDE, viz: BD : DE : : $b'\ d' : d'\ e'$, BD $\times\ d'\ e'=$ DE $\times\ b'\ d'$. Substituting the numerical values we have 14 $d'\ e'=7\times 1520$ $\therefore\ d\ e=760$ ft. sec².

The point F is constrained to move horizontally in its guide therefore draw $o'\ f'$ to an indefinite length. The point E relative to F has for its radial acceleration from the table above 20 ft. sec²., and therefore draw $e'-f'_{e}'$ equal to this magnitude with its perpendicular component. Then this perpendicular component intersects $o'\ f'$ in f'. The complete acceleration diagram is $o'\ b'\ d'\ e'\ f'$. The line $o'\ e'$ is the acceleration of the point E. Scale.—1″=800 feet per sec. per sec.

EXAMPLE: 6—In the mechanism in figure 306, AB is 4″ long, CD

8 inches, BD 16 inches and CE 20 inches long. AB rotates at 150 revolution per minute and the point D is guided to move in straight line GH. Find the velocity of the point E when the mechanism is in the configuration shown in the figure.

(I. Sc. Eng. Part II, 1929.)

SOLUTION :—Velocity Diagram:—AB rotates at 150 revolutions per minute and the angular velocity in radians per second is equal to $\omega = \frac{2\pi N}{60} = \frac{2\pi 150}{60} = 15{\cdot}7$ radians per second. Then velocity $= \omega r =$ $15{\cdot}7 \times \frac{4}{12} = 5{\cdot}23$ feet per second. Draw o $b = 5{\cdot}23$ ft. per second perpendicular to AB and the velocity of D relative to B is perpendicular to BD, therefore draw a line from b perpendicular to BD indefinitely. Now the point D is constrained to move along the line GH, therefore from O draw a line parallel to GH. This will intersect the line drawn from b perpendicular to BD at d, hence the point d is fixed. The point C is to be fixed in the line b d in the same ratio as C in BD, the point C is exactly in the centre of the bar BD, therefore fix the point C in the centre of the line b d. Next the velocity of the point E relative to C is perpendicular to CE therefore draw a line from C perpendicular to CE indefinitely. The point E moves in a straight line EK, therefore draw a line from O the origin, a line parallel to EK, you find that this will intersect the line drawn from C perpendicular to CE at e, then o e is the velocity of the point E. This measures to the scale 2·2 feet and the velocity of E is therefore 2·2 feet per second or $2{\cdot}2 \times 60 = 132$ feet per minute. Ans. Scale $1'' = 5$ feet per second.

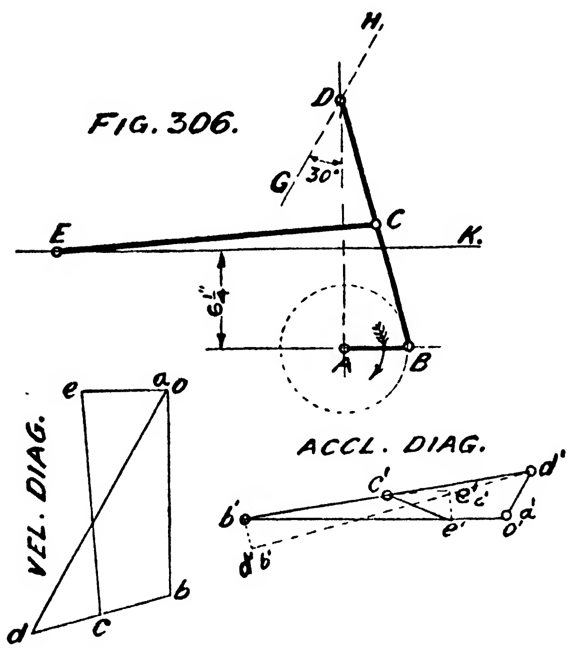


FIG. 306.

Acceleration Diagram.—From the velocity diagram tabulate the velocity and radial acceleration of each bar in the order as shown below.

Name of bar	Rel : vel : of ends of bar in ft: per sec. $v=\omega r$	Length of bar $=r$ in feet	Radial acceleration $=\frac{v^2}{r}$ ft. sec.^{2}
AB	ob - 5·23	$\frac{1}{3}$	$\frac{5·23^2}{\frac{1}{3}}=82·05$
BD	$bd=3·6$	$1\frac{1}{3}$	$\frac{3·6^2}{1\frac{1}{3}}=9·72$
CE	$ce=5·7$	$1\frac{2}{3}$	$\frac{5·7^2}{1\frac{2}{3}}=19·49$

Draw $o'\,b'$ equal to the radial acceleration of 82·05 ft sec.^{2} parallel to BA, then from b' draw the radial acceleration of D relative to B = 9 72 ft. sec^2. and draw its perpendicular component indefinitely. From the origin o' draw a line parallel to GH, as the point D moves in this line. This line which is drawn parallel to GH meets the tangential acceleration line of the point B in d'. Join b' d' and in this line the point C' is to be fixed exactly at the centre as the point C is at the centre of the bar BD. From C' draw the radial acceleration of the point E relative to C = 19·49 ft. sec.^{2} parallel to EC and its perpendicular component indefinitely. The point E slides along the line EK and therefore draw a line from o' parallel to EK and this line will intersect the tangential ecceleration of the point E at e'. The line o' e' gives you the acceleration of the point E and it measures to the scale 18 ft. sec.^{2} Ans Scale 1" = 60 feet per sec. per sec.

EXAMPLE 7:—A mechanism shown in the figure 307 has cranks BC and EF rotating about fixed centres C and F respectively, and A slides in the line AC. AB = ED = FE = 3 feet. BC = BD = 1 foot. The point B has a velocity of 50 ft. per second. When θ is 30° determine (1) the velocities of the points D and E. (2) the accelerations of D and E.

SOLUTION :—Velocity Diagram.—Draw $o\,b$ equal to the velocity of 50 ft. per sec. to some suitable scale at right angles to BC. The vilocity of A relative to B is perpenlicular to AB, therefore from the point b draw a line perpendicular to AB indefinitely but the point A is constrained to move along the line AC and from the point o draw another line parallel to AC to meet the line drawn from b perpendicular

to AB at *a*. Produce *a b* to *d* such that *b d* to be in the same ratio in *a d* as BD is in AD. Next the velocity of E relative to D is perpendicular to ED and also the velocity of E relative to F is perpendicular to EF, therefore draw from *d* and *f* lines *d e* and *f e* perpendicular to DE and FE respectively, then the point *e* is fixed in the diagram. The velocity diagram is completely drawn. The velocity of D is *o d* and it measures 62 feet per second, and that of E is *o e*=59 ft. per second. Scale 1″=50 feet per second.

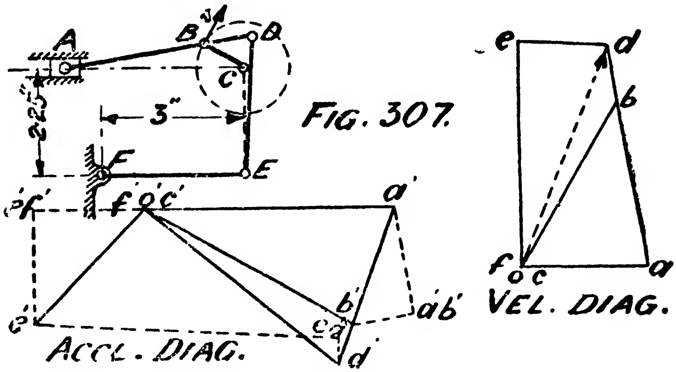


FIG. 307.

Acceleration Diagram.—Tabulate the velocity and radial acceleration of each bar in the order as usual.—

Name of bar	Rel: vel: of ends of bar in ft. per sec. $v=\omega r$.	Length of bar r in feet.	Radial acceleration $=\frac{v^2}{r}$ in ft. sec.^{2}
CB	$o\,b=50$	1	$\frac{50^2}{1}=2500$
AB	$a\,b=44$	3	$\frac{44^2}{3}=645$
DE	$d\,e=24$	3	$\frac{24^2}{3}=192$
EF	$e\,f=59$	3	$\frac{59^2}{3}=1100$

From the origin o' draw $o'\,b'$ equal to 2500 ft. sec.^{2} to any scale and draw b'—$a'\,b'=645$ ft. sec.^{2} the radial acceleration of the bar AB

and its perpendicular component, but the point A moves horizontally and this line meets the tangential acceleration of the bar AB in *a'*. Hence *a'* is fixed. Next produce *a' b'* to *d'* and fix *d'* in the line *a' d'* in the same ratio as D is in the line AD. From *d'* draw *d'—e' d'* = 192 ft sec² the radial acceleration of the bar DE and its tangential component of acceleration indefinitely. From the origin *o'* draw *o'—e'f'* = 1160 ft. sec² the radial acceleration of EF and its tangential. acceleration indefinitly. These two tangential accelerations of the bars DE and FE intersect at *e'* as shown. Join *o' d'* and, *o' c'*, these two lines represent the accelerations of the points D and E. Acceleration of D = 2620 ft sec² and that of E = 1800 ft. sec ² Ans. Scale 1″ = 2000 ft. sec².

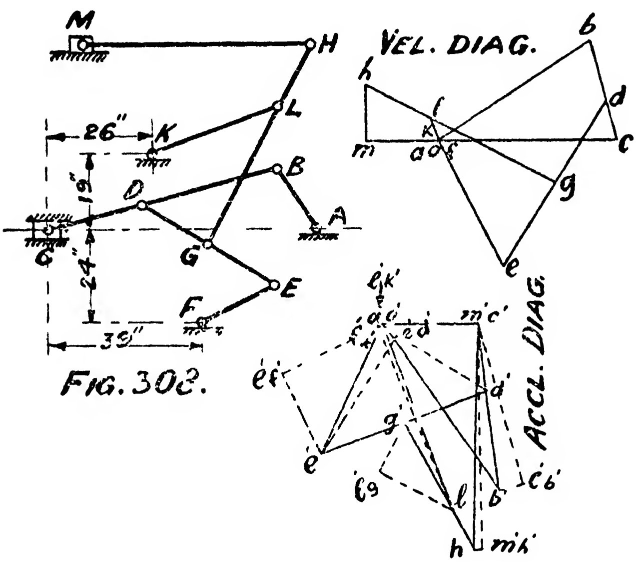


FIG. 308.

EXAMPLE 8:—Figure 308 is an outline sketch of a Joy valve gear. You are required to construct the velocity and acceleration diagrams for the position of the crank whose length is 18″ and makes 180 revolutions per minute.

SOLUTION:—Velocity Diagram.—Crank makes 180 revolutions per minute and $\omega = \frac{2\pi N}{60} = 18{\cdot}85$ radians per second. Velocity = ωr = $18{\cdot}85 \times 1{\cdot}5 = 28{\cdot}26$ ft. per second. Draw *o b* equal to this magnitude to any convenient scale at right angles to AB along the direction of rotation. The velocity of B relativo to C is perpendicular to BC, so from *b* draw a line at right angles to BC indefinitely and the slider C is constrained to move horizontally, therefore from the origin *o* draw an horizontal line to intersect the line drawn from *b* at right angles to BC at C. Fix the point *d* in *b c* in the same ratio as D stands in the line BC. The velocity of E relative to D is perpendicular to DE, and the velocity of

the same relative to F is perpendicular to EF. Therefore draw from d and f lines $d\ e$ and $f\ e$ perpendicular to DE and EF to intersect in e as shown. The point g is to be in the centre of $d\ e$ as G exactly bisects the line DE. The velocities of the point L relative to G and K are perpendicular to the lines LG and LK. These two perpendiculars meet at l. The line $g\ l$ is to be produced to h proportionately as H stands in the line GH. Lastly the slider M is constrained to move along the line MH, therefore from the origin draw a line $o\ m$ horizontal; the velocity of H relative to M is perpendicular to MH. Draw therefore from h a line perpendicular to MH to meet om at m. The velocity diagram is completed. Scale 1″ = 30 feet per second.

Acceleration Diagram.—As usual determine the velocities at the ends of the bar from the velocity diagram and calculate the radial acceleration of each bar by the formula $\frac{v^2}{r}$ and tabulate them.—

Name of bar	Rel: Vel. at ends of bar $v = \omega r$ in ft. sec.	Length of bar r in feet.	Radial acceleration $\frac{v^2}{r}$ in ft. sec.^{2}
AB	$o\ b = 28{\cdot}26$	1·5	$\frac{28{\cdot}26^2}{1{\cdot}5} = 532{\cdot}35$
BC	$b\ c = 16{\cdot}60$	5	$\frac{16\ 6^2}{5} = 55{\cdot}11$
DE	$d\ e = 30{\cdot}00$	3·25	$\frac{30^2}{3{\cdot}25} = 280{\cdot}00$
GL	$g\ l = 22{\cdot}00$	3·25	$\frac{22^2}{3{\cdot}22} = 148{\cdot}95$
HM	$h\ m = 9{\cdot}00$	4·83	$\frac{9^2}{4{\cdot}83} = 16{\cdot}77$
EF	$e\ f = 22{\cdot}00$	1·66	$\frac{22^2}{1\ 66} = 291{\cdot}56$
LK	$l\ k = 4{\cdot}25$	2·83	$\frac{4{\cdot}25^2}{2{\cdot}83} = 6\ 38$

From the origin o' draw $o'\ b'$ equal to the radial acceleration of 532·35 ft. sec.^{2} the radial component of acceleration of C relative to B =

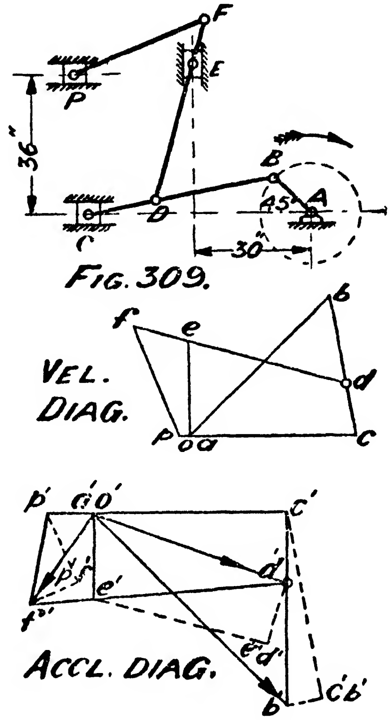
55·11 is next to be drawn from b'. Then $b'-C'b'$ is drawn in dotted line to represent this magnitude and from $C'b'$ its tangential component is drawn at right angles to this indefinitely. The joint C is constrained to move horizontally in its guides, therefore draw from the origin a line $o'\ c'$ horizontal and this will intersect the tangential component drawn from $C'b'$ at c' as shown; join $b'\ c'$. Fix the point d' in $b'\ c'$ in the same ratio as D is in the line BC. The radial component of acceleration of E relative to D is 280 ft. sec.^{2} and also its radial component of acceleration relative to F is 291·56 ft. sec.^{2} therefore draw $d'-e'\ d'=280$ parallel to DE and $f'-e'f'=291·56$ parallel to EF from the origin o'. Next draw their corresponding tangential accelerations from $e'd'$ and $e'f'$ then these two lines will intersect at e'; join $d'\ e'$ and $o'\ e'$. Fix the point g' exactly at the centre of the line $d'\ e'$ as G is at the centre of the line DE. The joint L similarly has two radial components of acceleration 148·95 and 6·38 ft. sec.^{2} relative to G and K respectively. Drawing the corresponding tangential components of acceleration as usual you get the point l' as shown. Join $g'\ l'$ and produce $g'\ l'$ to h' so that $l'\ h'$ to be in the same ratio in the line $g'\ h'$ as LH is in the line GH. Lastly the point M moves horizontally and its radial component of acceleration relative to H is equal to 16·77 ft. sec.^{2} From the point h' draw $h'-m'h'$ parallel to MH and its tangential component of acceleration indefinitely. From the origin draw $o'\ m'$ horizontal, these two lines meet at m'. Join $h'\ m'$. The acceleration diagram is now completed. Scale 1″=500 feet sec.^{2}

EXAMPLE 9:—In the mechanism shown in figure 309 the crank AB rotates at 200 revolutions per minute. Find the velocities and accelerations of C, D, E, F, and P. Also find the velocity of F relative to E, of D relative to E and of P relative to F. [B. Sc. Eng. Part I, 1932.]

Use the following scales:—Linear scale 2″= 1′. Velocity scale 1″= 5 feet per second. Acceleration scale 1″=50 feet per sec. per sec.

SOLUTION:—Velocity Diagram. The crank AB rotates at 200 revolutions per minute, then $\omega=\frac{2\pi N}{60}=\frac{2\times 3·1416\times 200}{60}=20·94$ radians per second. $v=\omega\,r=20·94\times 1=20·94$ feet per sec. Draw $o\,b$ to the scale specified above equal to 20·94 feet per sec. at right angles to AB. The velocity of B relative to C is perpendicular to BC, therefore draw from b a line perpendicular to BC indefinitely and the point C is constrained to move along the line CA, so from the origin o draw a line parallel to CA and you observe that this line intersects the line drawn perpendicular to BC at C. The point d is to be fixed in the line $b\ c$ in

the same ratio as D stands in the line BC. The velocity of D relative to E is perpendicular to DE Draw from *d* a line perpendicular to DE indefinitely and the point E is constrained to move vertically, therefore from the origin draw a line vertically down to intersect the line drawn from *o* perpendicular to DE at *e*. Produce *d e* to *f* so that *e f* is $\frac{1}{3}$ of *d e* as EF is $\frac{1}{3}$ DE. The velocity of F relative to P is perpendicular to FP, draw from *f* a line perpendicular to FP indefinitely and the point P slides horizontally, therefore draw from the origin a straight line horizontally to intersect the line drawn perpendicular to FP at *p* The figure *o b c f p* is the complete velocity diagram. Connect *o d* and *o f*.



The velocities of C, D, E, F and P are 18, 18, 10, 13, and 1·2 feet per second respectively. (approximate). The velocity of F relative to E is 6, of D relative to E is 17·5 and of P relative to F is 13 feet per second approximately. Scale used 1″ = 20 ft. per sec.

Acceleration Diagram.—First tabulate the velocities and radial accelerations of these links in the order as shown here to enable you to draw the acceleration diagram.—

Name of bar	Rel: Vel: of ends of bar in feet per sec. $v = \omega r$.	Length of bar in feet r.	Radial acceleration of the bar $\frac{v^2}{r}$ in ft. sec.^{2}
AB	20·94	1	$\frac{20{\cdot}94^2}{1} = 438{\cdot}48$
BC	15	4	$\frac{15^2}{4} = 56{\cdot}25$
DE	17·5	3	$\frac{17{\cdot}5^2}{3} = 102{\cdot}08$
EP	13	3	$\frac{13^2}{3} = 56{\cdot}33$

From the origin o' draw $o'\,b'$ parallel to AB to represent an

acceleration of 438·48 ft. sec². Since ω is constant B has no tangential acceleration, and therefore o' b' is the acceleration of B, The radial acceleration of BC is 56·25 ft. sec². as per table above, draw b'—$C'b'$ equal to 56·25 ft. sec². parallel to BC and from the point $C'b'$ draw its perpendicular component of acceleration indefinitely. You know the resultant acceleration of the point C is along AC. Hence draw a line through o' parallel to AC and this line will intersect the perpendicular component of acceleration of the point C in c'. Join b' c' and locate the point d′ in the line $b'c'$ in the same ratio as D is in the line BC. The radial component of acceleration of the bar DE is 102·08 ft. sec²., draw d'—e' d' equal to this magnitude and its tangential component of acceleration. The point E is constrained to move vertically therefore from o' draw a line vertically downwards to intersect the tangential component of acceleration of the point E in e'. Join d' e' and produce d' e' to f' so that e' f' to be in the same ratio in d' f' as EF is in the bar DE. Again the radial acceleration of P relative to F is along PF, therefore draw f'—p' f' equal to 56·33 ft. sec.² parallel to PF and draw its tangential component of acceleration indefinitely as shown and the point P is guided to move horizontally. From the origin o' draw o' p' to intersect the tangential component of acceleration of the point P at p'. Join f' p' and the acceleration diagram is now completed.

The acceleration of the points C, D, E, F and P are approximately 310, 325, 140, 180 and 80 feet per sec. per sec. respectively. Scale used in this diagram is 1″=300 ft. sec².

Note:—Relative velocity of ends of bar $v=\omega r$. Angular velocity $\omega=\frac{v}{r}$. Radial component of acceleration of ends of bar$=\omega\, v$. Tangential component of acceleration of ends of bar$=a$. Angular acceleration $\alpha=\frac{a}{r}$.

See fig. 309. The acceleration of B relative to A is towards BA, when b' is fixed, the next point is C, then the acceleration of C rel: to B is towards CB and not in the direction of BC. In the line b' c' the point d' is to be fixed up, when d' is fixed, the acceleration of E rel: to D is towards ED. The point f' is to be fixed by producing the line d' e' to f', when f' is fixed, the acceleration of P relative to F is towards PF. Students should be very careful in the directions of these radial accelerations specially while drawing acceleration diagrams.

EXERCISES

CHAPTER IV.

(1) A teak cantilever $9'' \times 6''$ projects 5 feet from the wall. Determine the greatest safe load that can be placed at its free end, allowing a safe stress of 1200 lbs. per □". *[Ans. 0·72 ton.]*

Draw the bending moment and shearing force diagrams for the following examples graphically.

(2) Cantilever figure 310. Total load = 10 tons.

(3) Cantilever figure 311, has two supports in the wall as shown. Determine the magnitudes and directions of reactions.

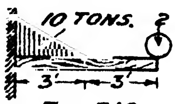


Fig. 310.

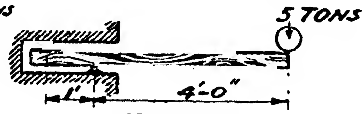


Fig. 311.

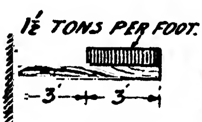


Fig. 312.

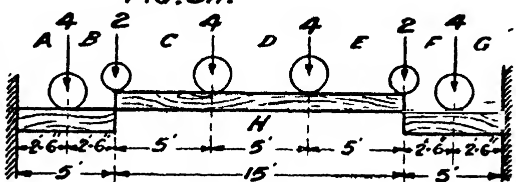


Fig. 313.

(4) Cantilever fig. 312, distributed load of 1½ tons for 3 feet from the free end.

(5) Two cantilevers 5' long carry a beam of 15' in the centre with the loads as shown in the figure 313. Bending moment diagram is to be drawn in one common figure and also shearing force diagram.

Hint.—The funicular polygon is to be drawn as usual for the given loads, the intersecting points of the load lines BC and EF with the funicular polygon are to be connected and this represents the closing line of the funicular polygon for the loads on the central beam, and then the same closing line is to be produced both ways to meet the fixed end supporting lines of the two cantilevers. This represents the common bending moment diagram for both the cantilevers and the supported beam.

(6) A balcony projects 6 feet and is carried by a number of beams, spaced 8' apart, and built into the wall at right angles to it;

the outer end of each beam being supported by a pillar whose reaction is equal to one-third of the whole load on the beam. The balcony is loaded with 100 lbs. per square foot.

(*a*) Draw the shearing force diagram for one of the beams.

(*b*) Draw the bending moment diagram for one of the beams.

(c) Calculate the maximum bending moment and state where it occurs.

(*d*) State where the points of zero bending moment are situated.

(I Sc. Eng: Part I, 1931. App: Mech.)

[Ans. (c) 4,800 lbs. inches, at the fixed end of the beam.

(d) At 2′ from the fixed end & at the right support.]

(7) A beam of 16′ clear span carries two separate loads 2 and 4 tons at 4 feet from each support, and also 8 tons uniformly distributed Draw the shearing force and bending moment diagrams to scales of 1″=4 feet, and 1″=4 tons. (City & Guilds Exam. 1915. Mech Eng.)

(8) A beam of 15 feet span is supported at each end and carries loads of 40 and 100 lbs, respectively, at distances of 8 and 11 feet from the left hand end. Draw a shearing force diagram for the beam and find the shearing force at points 6 and 10 feet from the left hand end. *[Ans. 68 and 28 lbs]* (Civil Eng: Drg: 1931.)

(9) A beam is 30 long and carries an uniform load, over its whole length, of 500 lbs. per foot run. There is a downward reaction at one end, and an upward reaction 10 feet from that end. These are the only supporting forces. Draw carefully the diagrams of bending moment and shearing force. (B. Sc. Eng. Part I, 1922 Theo: of Struct.)

(10) A rectangular beam, 25 feet long, is supported 3 feet from the left end and 2 feet from the right end. It carries 200 pounds per foot uniformly distributed, 600 pounds 1 foot from the left end, 800 pounds 7 feet from the left end and 1200 pounds 6 feet from the right end. Draw to scale the bending moment and shearing force diagrams, stating clearly the maximum values in each case.

If the allowable stress is 1000 lbs per square inch and the depth of the beam is three times its breadth, find the necessary dimensions for the cross section of the beam.

(B. Sc. Eng: Part I, 1931. Strength of Mat:)

[Ans. Max: Bending moment = 12,600 lbs. feet. Shearing force 3000 lbs. Size of the beam $4\frac{5}{8}'' \times 14''$. *]*

(11) Prove that the algebraic sum of the moments of any two

forces about any point in their plane is equal to the moment of their resultant about the same point.

A beam 20 feet long rests on two supports 16 feet apart, and overhangs the left-hand support 3 feet, and the right hand support by 1 foot. It carries a load of 5 tons at the left-end of the beam, and one of 7 tons midway between the supports. The weight of the beam, which may be looked upon as a load at its centre, is 1 ton. Find the reactions at the supports. What upward force at the right-hand end of the beam would be necessary to tilt the beam ?

(I. Sc. Eng: Part I, 1930 Math:)

[Ans. 10 and 3 tons reactions. 2·82 tons.]

(12) A simply supported beam has a span of 20 feet and supports concentrated loads of 5, 10 and 8 tons at 6, 11 and 13 feet, respectively from the left hand end and also a uniformly distributed load of 2 tons per foot run beginning at 4 feet from the left hand end and extending 11 feet along the beam. Draw the shearing force and bending moment diagrams. (B. Sc. Eng: Part I, 1926.)

(13) Two concentrated loads of 8 tons each and 7 feet apart move across a bridge of 60 feet span. Draw the curves of maxima bending moments and shearing forces. (B. Sc. Eng: Part II, 1926.)

(14) A cantilever bridge consists of anchor arms 500 feet, cantilever arms 300 feet, and suspended truss 400 feet long. The bridge carries a uniformly distributed load of 2 tons per foot run. Draw the bending moment and shearing force diagrams. Discuss the advantages of this type of bridge construction. (B. Sc. Eng: 1924.)

(15) A beam AB is 40 feet long and is supported at two points C and D, the distance AC being 10 feet and the distance BD being 8 feet.

There is a load of 5 tons at A and a load of 4 tons at B. There is also a load 8 tons in the middle of the length CD. Draw diagrams of bending moment and shearing force for the beam, and give values of the bending moment and shearing force at the two supports and at a section halfway between the two supports. (B. Sc. Eng: Part I, 1924)

[Ans 50 and 32 tonsfeet at the supports and 3 tonsfeet at the centre. 4·81, –3·19 and 4 tons shear.]

(16) Two loads 10 and 15 tons respectively at 6 feet apart roll over a girder of 60 feet span. Draw to scale the maximum 'bending moment' and 'shearing force' curves for the girder.

(B. Sc Eng: Part I, 1924)

(17) A rolling load, consisting of two wheels of 5 tons each and 12 feet centre to centre, moves across a beam 20′ span. Find the maximum bending moment, and also find the position of the load to give the maximum bending moment when the load is confined to the span, and write down the value of this maixmum. (City and Guilds 1921.)

[Ans. 25 tons feet. At 13 feet from one of the supports and its value is 24·5 tons feet.]

(18) Draw the diagrams of bending moment for a braced girder of 6 bays or panels, span 120 feet, if the bridge is uniformly loaded with a load of 1 ton per lineal foot:—

(1) When the floor consists of trough flooring laid crosswise and carried on the lower flange of the girder.

(2) When the trough flooring is laid longitudtially and carried on cross girders at the panel points. (C. E. Sub-ordinates 1925)

(19) Draw the shearing force diagram for a girder of 80 feet span freely supported at each end and carrying a uniform load of 2 tons per foot run on a length of 60 feet from the left hand end. Determine the value of the maximum bending moment, and mark on your drawing the section at which this occurs. (City and Guilds 1920.)

[Ans. 1406·25 tons feet at 37·5 from the left support.]

(20) Determine the maximum value of the live load shear in the bay B of the girder shown in figure 314 due to a locomotive of the following dimensions passing over the bridge from R_2 to R_1 :—

Axle load in tons 127, 168, 167, 10. Distances between axle centres in feet 7, 8, 7. (The inclined members are inclined at 45 degrees to the vertical). (City and Guilds 1915.)

[Ans. 36·5 tons. (approx)]

(21) A girder of I section 8″ deep is supported at its ends and has a clear span of 12 feet. It carries a uniformly distributed load of ½ ton per foot run over its whole span, and a concentrated load of 5 tons placed 4 feet 6 ins. from one end.

Draw the shearing force and bending moment diagrams for the girder and find the deflection produced by the loads at the point in the girder immediately beneath the concentrated load.

Moment of inertia of section about neutral axis = 20·2 $inch^4$ units. E = 13, 250 tons per square inch. (City and Guilds 1915)

[Ans. Deflection under the load = 1·92 inches.]

(22) A foundation 60 feet long (See fig. 315) is subjected to two upward forces of 50 tons each at two points 10 feet from each end.

Between these points there is a uniform downward load of 5 tons per foot. The supported force along the lower surface of the foundation is uniform over the whole length. Determine the values of the bending moment and shearing force at the points of application of the 50 ton loads, and at the middle of the span. Draw the diagrams.

[Ans 83'33 tons feet and 750 tons feet at the centre.]

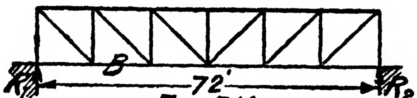


FIG. 314.

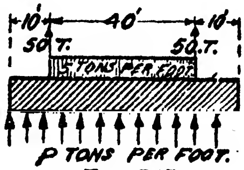


FIG. 315.

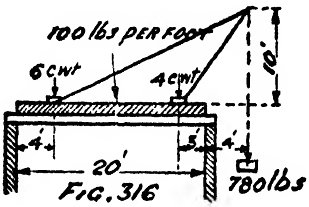


FIG. 316

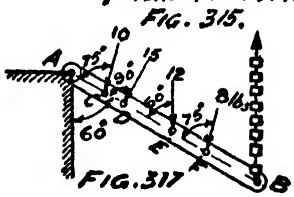


FIG. 317

(23) A pair of shear legs are mounted on the floor of a building, and project 4 feet beyond the inner edge of the wall, which forms a simple support for the floor beams as shown in the fig. 316. The floor also carries a uniformly distributed load of 100 lbs. per foot over the whole span of 20 feet.

Find the position and magnitude of the maximum bending moment on the floor, and sketch the bending moment and shearing force diagrams.

[Ans. Max: bending moment=8167'6 lbs. feet at 12' from the left support.]

(24) A beam AB is hinged at A and supported by a vertical chain at B. It is inclined at 60° to the vertical and is acted upon by four forces as shown. Find the tension in the chain and the direction and magnitude of the reaction at the hinge. Use the funicular polygon, and prove that the method is correct. Draw the bending moment and shearing force diagrams. AC=EF=4 feet. DE=FB=6 feet. CD=2 feet.

[Ans. Tension in the chain=20 lbs. reaction at the hinge=26'25 lbs. acts at 62° to the horizontal. (Approx.)]

(25) A vertical post ABCD freely supported at A and D is loaded through the framework BCE. The loads carried, 200 lbs. horizontal and 200 lbs. vertical, act at E. See fig. 318. For these conditions draw the bending moment shearing force and thrust diagrams for the post.

(26) A beam 30′ long rests on two supports, each support being 5′ from the end. A load of 2 tons hangs from one end of the beam, one ton from the other, and 4 tons rest on the mid point of the beam.

(*a*) Calculate the reactions at the supports.

(*b*) Draw to scale the diagram of bending moment, state clearly the scale employed.

(*c*) State the value of the maximum bending moment in tons feet.

(*d*) Determine the points of zero bending moment.

(I Sc. Eng: Part I, App: Mech: 1930)

[Ans (a) 4·25 and 2·75 tons; (c) 12·5 tons feet; (d) 4·44′ and 2·857′ from the left and right support.]

(27) A beam 30′ long, rests on two supports, each support being 5′ from one end. A weight of 2 tons hangs from one end of the beam, 1 ton from the other end and 4 tons from the mid point of the beam. Draw the bending moment and shearing force diagrams and give the magnitude of the maximum bending moment and state where it acts.

(I. Sc. Eng: Part I, 1929. App. Mech.)

[Ans. Max: Bending moment = 12·5 tons feet, at the point of 4 ton load]

(28) A beam 25′ long and freely supported at the ends carries a uniformly distributed load of 50 tons.

Draw to a scale, the bending moment and shearing force diagrams, and find, by measurement to scale, the shearing force and bending moment and at a distance of 3 feet from one end of the beam.

(Dip: Eng: 1931.)

[Ans. 66 tons feet and 19 tons.]

(29) A train of 3 wheels 10, 20 and 5 tons at 10 feet centres crosses a span of 50 feet In what position must this train stand to produce the maximum bending moment at the middle of the span? Calculate this bending moment. Is this maximum necessarily the greatest bending moment that can occur at any section whatever? If not, show how to obtain the section at which the obsolute maximum occurs, and find its value. (B. Sc. Eng. Part II, 1391 Theo. Struct.)

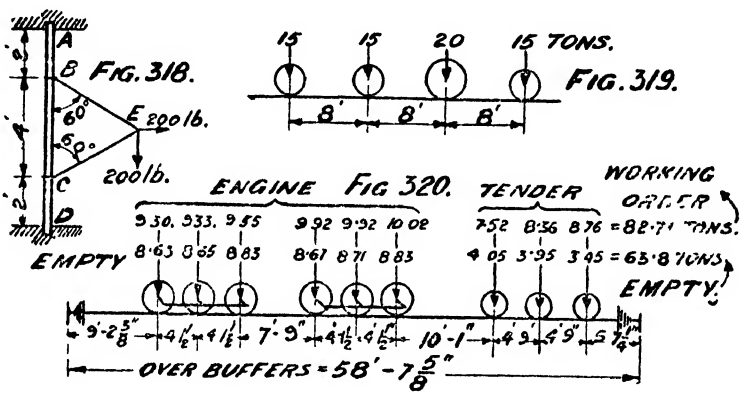
[Ans. When the 20 ton axle load is exactly at the middle of the span and its value = 625 tons feet Yes, this is the greatest.]

(30) One of the girders of a travelling crane of 50 feet span has to carry two rolling loads of 5 tons each spaced 6 feet apart. The wheels rest directly on the plate girder, and there is, therefore, no platform effect. Draw the bending moment and shearing force diagrams and get an envelope of all possible bending moment diagrams.

(B Sc. Eng. Part I, 1922 Theo. of Struct.)

(31) System of wheel loads shown in fig. 319 moves along a girder across a span of 60 feet. Determine the maximum bending moment and shear force at a section 20 feet from the right hand abutment. Find also the position and the magnitude of the greatest bending moment which occurs at any section of the girder due to this system of loading. (City and Guilds 1920.)

[*Ans.* 673·85 *tons feet and* 29·91 *tons. The greatest maxm: bending moment occurs at the middle of the beam when* 20 *ton axle load is over that point and its value is* 735 *tons feet.*]



(32) The diagram shown in fig. 320, represents the locomotive axle loads of the heaviest type of engine known as Mallet Articulated compound Locomotive (I. M. Class) of (M and S. M. Railway Co., Ltd. India) metre gauge section in working order. Draw the bending moment and shearing force diagrams for a truss bridge of 120 feet span.

CHAPTER V.

(1) By means of the method of sections, describe how you would calculate the forces in the members of an N girder loaded at the pin joints of the lower boom. (B. Sc. Eng: Part I, 1926.)

[*Ans. See part I figs* 95, 96 *and* 97.]

(2) Prepare a stress-diagram for the unequally loaded girder shown in figure 321. (C. E Subordinates 1925.)

(3) Draw the bending moment for the deck type lattice girder when loaded as indicated in the figure 322. Determine with the aid of this diagram the loads in the members AB, CE and BD.

(City and Guilds 1920.)

[*Ans.* $AB = 78{\cdot}25$, $BD = 55{\cdot}5$, $CE = 91$ *tons.*]

(4) The diagonal members of the girder shown in fig 323 make an angle of 45° to the vertical, and the bays are all of equal length. Draw the stress diagram for this girder when each of the five panel points within the span of the lower boom carries a dead load of 10 tons. What is the load in the bar MN ? (City and Guilds 1915.)

[Ans. MN = 5 tons.]

FIG. 321.

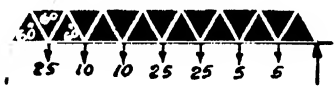
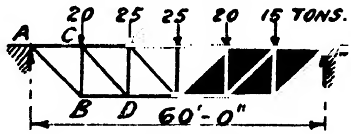


FIG. 322.



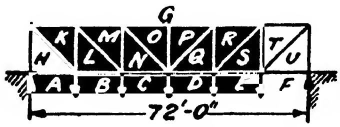
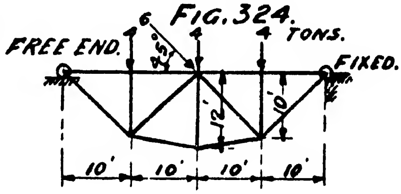


FIG. 323.

FIG. 324.



(5) A girder 40 feet span and supported at the ends A and B is free to slide over the support at A but fixed in position at B. The top chord is divided into 4 equal parts by three verticals, the centre one being 12 feet long and the others each 10′ long. Determine the forces in the members of the girder for the loading indicated in the figure 324. (L. E. Exam. 1931. App. Mechanics.)

[Ans. Verticals = 4 tons. Centre diagonals = 3 tons. End bottom chord members 11·4 tons, central bottom chord members 10·4 tons, top chorp members from left support 8 tons, right top chord members 12·4 tons.]

(6) A roof truss fig. 325 is hinged at A and supported on a roller bearing at B. The spacing of the trusses is 16 feet, the uniformly distributed load is 16 lbs. per square foot of the inclined roof surface, and the normal wind pressure is 20 lbs. per square foot. Draw the three force diagrams for (*a*) the dead load, (*b*) the wind load only acting on the left side, and (c) the wind load only acting on the right side. Tabulate the maximum forces in the members of the truss for the combined action of dead and wind load. State the maximum reactions at A and at B and show their directions.

(B. Sc. Eng. Part I, 1932. Theo. Struct.)

[Ans. See the chapter on roof trusses Part I.]

(7) A Warren girder composed of equal equilateral triangles has six bays and is 120 feet long. A concentrated load of 60 tons hangs from each joint in the lower boom. The ends of the girder are simply supported. Determine the forces in the three members cut by a vertical section 35 feet from the left support. Determine a suitable section for the inclined member. (B Sc. Eng: Part II, 1923. Theo. of Struct.)

[Ans. Top chord + 311 tons, bottom chord — 294 tons. Inclined members — 34 tons. Two angles $4'' \times 3'' \times \frac{3}{8}''$ may be used.]

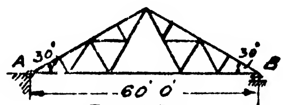


FIG. 325.

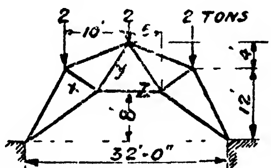


FIG. 326.

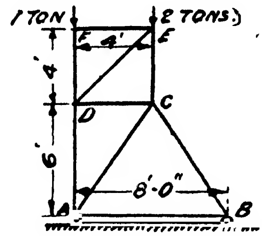


FIG. 327.

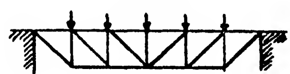


FIG. 328.

(8) Deduce the criterion which serves to show on inspection whether a plane frame containing N joints is perfect, deficient, or redundant. A roof truss shown in figure 326 is loaded at the joints as indicated. Determine, by any method, the forces in the members marked X, Y, Z. (B. Sc. Eng: Part I, 1930.)

[Ans. X = 1·9 tons, Y = ·5 ton and Z = 3·5 tons.]

(9) State the three guiding principles of statics which are applied to determine the forces in the individual members of a perfect frame.

A built-up framework is loaded as shown in fig. 327. It rests on rollers at A and B so that it can be easily moved. Calculate the nature and magnitude of the forces in the various members.

(B. Sc. Eng. Part I, 1930. Theo. of Struct.)

[Ans. AB = — ·7, BC = + 1·2, CD = 0, DE = 0, EF = 0, FD = + 1, DA = + 1, EC = + 2, and AC = 1·2 tons.]

(10) Each of the loads on the frame fig. 328 is 10,000 lbs. The panels are square, other dimensions are deleberately omitted. Draw the stress diagram, and tabulate all stresses. Check four of them by the method of stress coefficients and one other by the method of sections.

Ans. *Top chord three members–35,000; 40,000; & 44,000 lbs respectively.*
Bottom chord two hori-members–25,000 & 40,000 lbs. respectively.
Three diagonal web members–35,000; 21,000; & 7000 lbs. respectively.
Three verticles–25,000; 15,000; & 10,000 lbs. respectively.

CHAPTER VI.

(1) Draw AB horizontally from left to right and 5 inches long. Draw AC vertically 6 inches long. Join D the middle of AB, to C. These are the centre lines of a Foundry crane, carrying a load of 2 tons at B, to a scale of one inch=2 feet. The reaction at A is along BA. Find the reaction at C. Find the stresses in AC and CD. Draw diagrams of bending moment, shearing force, and direct tension on AB. Neglecting the last named, design a suitable section for AB at D. Calculate the neglected tensile stress. Design and draw the joint at D.

(B. Sc. Part I, 1923)

[*Ans. Reaction at* $C=2{\cdot}6$ *tons,* $AC=2$ *tons,* $CD=1{\cdot}7$ *tons, angle section* $6''\times6''\times\frac{3}{8}''$ *for CD and* $2\frac{1}{2}''\times2\frac{1}{2}''\times\frac{3}{8}''$ *for AD with usual gusset and rivet connections.*]

(2) A triangle of sides 8, 7 and 9 feet. The angle bisectors form the plan of a derrick 10 feet high, on which is a load of 15 cwt. Find the stresses on each leg [*Ans* 5, 5·6 *and* 5·8 *cwts*]

(3) Figure 329 shows the plan and elevation of a three dimensioned framed crane. Determine the stresses in all the members for a load of 5 tons suspended at the free end, neglecting the weight of the crane.

[*Ans.* ab *and* bc *each* $=-7{\cdot}2$, $ac=+5{\cdot}1$, $ad=-5{\cdot}3$, de, ef *and* fd *each* $=-2{\cdot}1$, $cd=-1{\cdot}7$, $ce=-5{\cdot}9$, $gd=ge=-6{\cdot}8$, $gf=+6$, $gh=-8{\cdot}4$, $fh=+12$, $fb=+7{\cdot}5$, $af=fc=+8{\cdot}5$.]

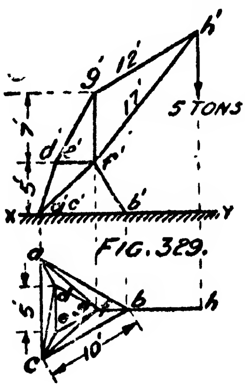


FIG. 329.

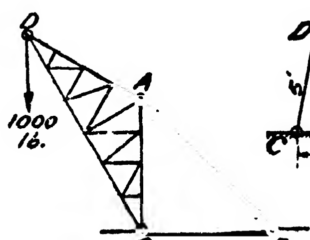


FIG. 330.

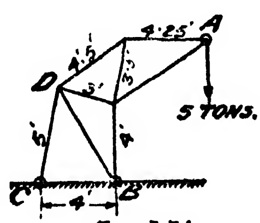


FIG. 331.

(4) State the conditions of equilibrium for any system of forces acting at a point.

A tripod with three equal legs, each 8 feet long, is supported on a horizontal plane with the feet at the vertices of an isosceles triangle whose sides are respectively 8 feet, 8 feet and 5 feet long.

Find the stresses in the legs of the tripod if a load of 800 lbs. is applied at the top. (I. Sc. Eng. Part II, 1930 Math.)

[*Ans.* 270, 270 *and* 410 *lbs*]

(5) Figure 330 represents the line diagram of a pinjointed structure. The vertical AC is divided into four equal parts of 5 feet length, AD is drawn at 30° to the horizontal and is also 20 feet long and divided into four equal parts; CD is divided into six equal parts, and these points are joined by members as shown. The back-guy AB is at 45° to the horizontal Find the tension in AB, and the magnitude and direction of the reaction at C. Draw the force diagram for this structure and insert on each member in your line diagram its tension or compression. (B. Sc. Eng Part I, 1931 Theo. Struc.)

[*Ans. Tension in AB* = *1220 lbs. reaction at C* = *2050 lbs. at 65° to the horizontal.*]

(6) Figure 331 is the line diagram of a crane, and is anchored down at C and B. The load carried at A is 5 tons. The dimensions of the members are as shown in the diagram. Draw the stress diagram, show the nature of the stresses in each member and the magnitudes and directions of the reactions

[*Ans. Reaction at C is along DC acting downwards* = *6·2 tons and at B acts upwarde at 83° to the horzl:* = *11 tons approx:*]

CHAPTER VII.

(1) Calculate the forces in a framed truss for a roof over a railway platform, as per sketch. Trusses are 10′ apart. Assume suitable loading. Angle of roof 30°. See fig. 332.

(Civil Eng. Final, 1929.)

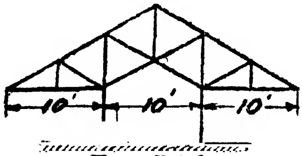


FIG. 332.

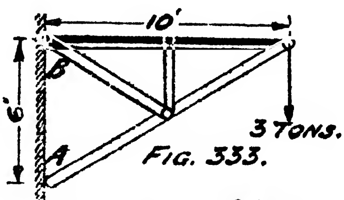


FIG. 333.

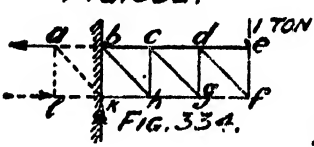


FIG. 334.

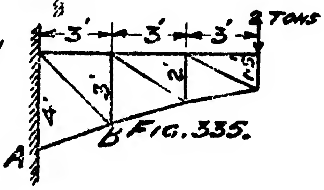


FIG. 335.

(2) The cantilever frame shown in fig. 333 has to carry a load

of 3 tons at the free end. The end B is hinged and the surface at A is smooth. Draw the stress diagram.

(3) The cantilever shown in figure 334 is secured to the wall as shown. Assuming the wall is substantial to withstand the pull and push from the top and bottom chords of this braced cantilever, determine the stresses in all the members due to a load of 1 ton at the free end. Panel length 2 feet, depth 2 feet.

[*Ans.* $ab=3$, $bc=2$, $CD=1$, $de=0$, $ef=1$, $fg=1$, $hg=2$, $hk=3$, $kl=3$, $la=0$, $ak=0$, $bh=cg=df=1{\cdot}4$ *tons.*]

(4) The bracket shown in figure 335 is subjected to a load of 2 tons at the free end. Draw the stress diagram and determine the directions of reactions at the supports.

[*Ans. Reaction at A is along AB and is equal to 4·75 tons.*]

CHAPTER VIII.

(1) Figure 336 shows the arrangement of members of an intermediate transverse bent of a kneebraced mill building. The bent is of 60 feet span, height 18 feet to foot of knee-brace, 24 feet to bottom chord and slope of roof 6 inch to 1 foot. Trusses are 16 feet apart C to C. The wind pressure is 15 lbs. per square foot perpendicular to the sides of the building and 11 lbs per square foot normal to the roof. The columns are assumed partially fixed at the lower end, with the point of contra flexure at one-third the distance between the lower end and the foot of the kreebrace; the upper ends are considered supported. Draw the stress diagram, and also bending moment and shearing force diagrams for the leeward column.

Note—Assume the wind shear to be equally divided between the two columns.

CHAPTER IX.

(1) A portal frame shown in figure 337 stiffens the ends of two main girders. A horizontal load of 10 tons acts at B, A, B, C anb D are pin joints. Determine the reactions at A and D and the forces in the members. Draw the bending moment and shearing force diagrams for DC. Determine suitable sizes for the cross sections of all the members. Note the horizontal 10 ton load may act at C instead of B, depending on the direction of the wind. (B. Sc. Eng: Part II, 1924.)

[*Ans. Reactions at* $A=D=9{\cdot}71$ *tons,* $AE=8\frac{1}{3}$, $EB=4$, $BF=6$, $FG=8{\cdot}5$, $FK=12$. $KG=12$. $BK=21{\cdot}5$ *and* $KL=5$ *tons.*]

(2) The symmetrical frame shown in this figure 338 is pin jointed to rigid supports, and carries a single central vertical load of 50 tons. Determine the stress in the member marked X.

[*Ans.* $X = -50$ *tons.*]

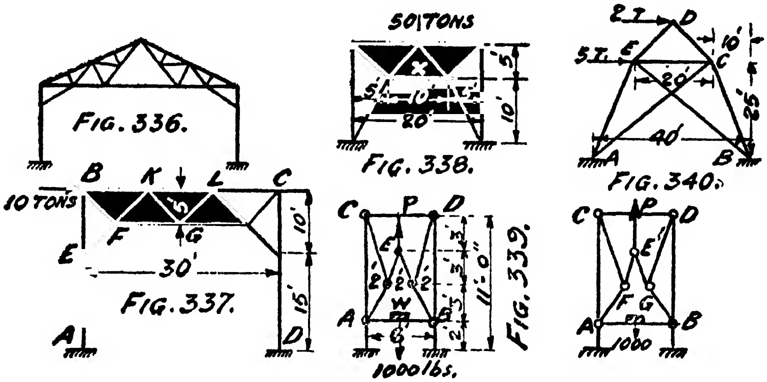


FIG. 336. FIG. 337. FIG. 338. FIG. 339. FIG. 340.

(3) The frame in figure 339 is in equilibrium, and the load carried by the horizontal member AB at its mid point is 1000 lbs. C and D are pin jointed and the joints A and B slide vertically. The whole frame is lifted by the upward force P applied at E and when the frame is in the position shown in the figure; determine the force P and the forces in all the members.

[*Ans.* $P=1500$, $AB=350$, $AF=BG=600$, $CF=GD=270$, $EF=EG=800$ *and* $CD=90$ *lbs.*]

(4) Figure 340 represents a steel redundant framework acted upon by the two forces shown. All the joints are pin-jointed and the footings A and B do not move. The safe stress in the tension members is 6 tons per square inch and in the compression members 5 tons per square inch. (Members AC and BE are not connected at their point of intersection). Determine the cross-sectional area of each member.

(B. Sc. Eng: Part II, Theo. of Struct: 1932.)

[*Ans.* $AE=\cdot 53$, $ED=\cdot 25$, $DC=\cdot 29$, $CB=\cdot 64$, $EC=\cdot 32$, $AC=\cdot 52$ *and* $BE=\cdot 60$ □″.]

CHAPTER X.

(1) A masonry parabolic arch has a span of 60 feet, a width of 16 feet, a thickness of ring of 3 feet and a rise of the centre line of the arch of 20 feet. Determine the maximum stress in the arch due to a uniformly distributed load of 3·5 tons per foot run over the whole span.

[*Ans. Max: Stress*$=8\cdot 3$ *tons.*] (B. Sc. Eng: Part I, 1926.)

(2) A masonry segmental arch has a span of 60 feet, ring thickness of 3 feet, and rise of centre line of arch of 25 feet. Determine the maximum stress in the arch due to a uniformly distributed load of 3 tons per foot run resting on the whole span and a width of 15 feet of arch. (B. Sc. Eng: Part I, 1924.)

[*Ans. Max: Stress = 7 tons*]

(3) A segmental arch has a span of 150 feet, a rise of 40 feet, and is hinged at the ends and at the centre. The arch carries a uniformly distributed load of 2 tons per horizontal foot run on half of its length. Draw the diagram of bending moment and write down the maximum bending moment. (C. E. Subordinates 1925.)

[*Ans. Max: Bending moment = 852 tons feet.*]

(4) Determine the form of a linear arch of 90 feet span which shall exert a horizontal thrust of 20 tons, when carrying vertical loads of 6, 9, 7½, 6, 6 tons at intervals of 15 feet. Determine the rise of the arch at the crown.

[*Ans. Rise at crown = 24·25 feet*]

(5) The span of a three hinged segmental arch rib is 200 feet and its rise 20 feet The horizontal span is divided into eight equal panels, and the uniformly distributed load of 2000 lbs. per lineal foot is supposed to be concentrated at the panel points When the left half of the span only is loaded, draw the bending moment diagram to scale and write down the values of the horizontal thrust, the reactions at the supports and the maximum bending moment.

[*Ans. Horl: Thrust = 253000 lbs. Reactions 294000 and 253000 lbs. Max: Bending moment = 12,65,000 lbs. feet.*]

(6) A 3-hinged circular arched-rib of span 80 feet and 20 feet rise supports a concentrated vertical load of 10 tons at 25 feet measured horizontally from the left hand hinge. Determine the bending moment under the load; and at the point of application of the load the shearing force normal to the rib and the thrust along the rib.

(B. Sc. Eng: Part I, 1931. Theo: of Struct.)

[*Ans. Bending moment = 62 tons feet. Shear 5·2 tons and thrust 13·4 tons.*]

(7) A three hinged circular arch of span 100 feet and rise 20 feet supports a vertical concentrated load of 10 tons at 25 feet from the left abutment. Draw to scale the bending moment diagram for the arch, and state the shearing force on each of the three pin joints.

(B. Sc. Eng: Part I, 1932. Theo. Struct.)

[Ans. Left hinge=1·3 tons. Crown hinge=2·4 tons. Right hinge=2·5 tons.]

(8) A segmental three hinged arch has a span of 120 feet and a rise of 24 feet. It supports 1,000 lbs. per horizontal foot run of the the span. Determine the magnitude and direction of a reaction, the value of the horizontal thrust, and draw the bending moment diagram for the left hand half of this arch.

(B. Sc. Eng: Part I, 1928 Theo. Struct.)

[Ans. Reaction=86000 lbs. Horl: thrust=75000 lbs.]

(9) In a three-hinged circular arched rib the span is 60′ and the rise 12 feet. When the rib is loaded with a single vertical load of 8 tons, acting at a distance of 20 feet (measured horizontally) from the centre pin, find the maximum bending moment on the rib and state on which cross section it occurs. Find also the magnitude of the NORMAL thrust acting on the same cross section.

(B. Sc. Eng: Part I. 1930. Theo. of Struct.)

[Ans. Max: Bending moment=42·9 tons feet at the loaded point. Normal thrust=5·9 tons.]

(10) Show how you would calculate the "bending moment" at various sections of a uniformly loaded circular arched rib with three hinges and the "shearing force" on each of the three hinge pins.

(B. Sc. Eng: Part I, 1927. Streng: of mater.)

(11) A three hinged parabolic arch, 60 feet span and 12 feet rise, is loaded with 20, 20, 15, 10 and 10 tons at horizontal distances 12, 24, 34, 45 and 50 feet respectively from the left hand hinge. Draw the bending moment diagram and find the resultant thrust at each pin joint for the arch.

Determine the bending moment, the shearing force, the resultant thrust, the normal thrust and tangential shear force at a section 15 feet, horizontally, from the left hand abutment.

(Sessional Work B. Sc. Eng: Part I, 1932.)

[Ans. Resultant thrust at the left hinge=69 tons, at crown=57 tons and at the right hinge=68 tons. Bending moment at the point=0. Shear=4 tons Resultant thrust=60 tons. Normal thrust=59 6 tons. (Approx)]

CHAPTER XI.

(1) A suspension bridge has a span of 100 feet, width of roadway 12 feet, and dip of 12 feet. The total uniformly distributed load is

200 lbs. per square foot. Determine the maximum tension in the cables. The cable passes over a pulley and the anchorage is inclined at 35° to the horizontal. Determine the horizontal and vertical loads on the pulley. (B. Sc. Eng: Part II, 1923.)

[Ans. Max: tension in the cables 135,000 lbs. Horl: load = 10,000 lbs. Vert: load: 137,000 lbs.]

(2) A cable 120 feet long is suspended between two points of the same height and 100 feet apart. An uniform load of 1500 lbs. per horizontal foot hangs from it. Find the greatest tension.

(B. Sc. Part I, 1924.)

[Ans. 202,000 *lbs]*

(3) Sketch three different methods of transmitting the pull of the main cable of a suspension bridge over the top of a supporting tower, and analyse the effect of each of method on the tension in the anchor cable and on the force at the top of the tower.

(B. Sc. Part I, 1932. Theo. Struct.)

[Ans. See Pages 155 to 158.]

(4) A suspension bridge 100 feet span and 20 feet wide is carried by two cables. The load is 200 lbs. per square foot. The dip of the cables at the centre of the span is 10 feet. The cable weighs 480 lbs. per cubic foot and the working stress is 4 tons per square inch; determine a suitable "cross-sectional area" for one cable.

(B. Sc. Eng: Part II, 1923. Theo. of Struct.)

[Ans. 8·92 or 9 square inches]

(5) A suspension bridge has a span of 100′ and a depth of 15 feet; the floor weighs 1 ton per foot run. Neglecting the weight of the chain draw the parabola of the bridge to a convenient scale, and find the values of the terminal tensions. If the backstays are fixed at 60° to the vertical, find graphically the tensions in them which will be necessary to prevent horizontal stress on the supports.

[Ans. Terminal tension = 40 tons. Tension on the backstay to prevent horl: stress = 35·4 tons.]

(6) A Suspension bridge has a span of 324 feet and a dip of $\frac{1}{12}$ span. The distributed load is 162 tons. Find the chain tension at lowest and highest points.

[Ans. 120·5 and 127 tons.]

(7) A suspension bridge has a span of 200 feet. The left abutment is 30 feet higher than the right abutment and a dip of chain at the middle of the span is 20 feet measured from the line connecting

the abutments. There are vertical suspension rods at intervals of 20 feet, loaded with 10, 13, 16, 20, 16, 10, 10, 10, 10 in the order from the left abutment. Determine the shape taken by the bridge cable and measure the horizontal pull in the links.

[*Ans. Horl: pull in the links=178 tons.*]

CHAPTER XII.

(1) A girder is 50 feet long and carries a uniform total load of 2 tons per foot run. It is fixed at one end and simply supported at 20 feet from the other end. Draw the bending moment and shearing force diagrams. (B. Sc. Eng: part I, 1924.)

[*Ans. See example 10 page 188.*]

(2) A cantilever is uniformly loaded throughout its length. Find where a prop must be placed in order that it may support one-half of the load, the top of the prop being at the same height as the unbent cantilever. Construct shearing force and bending moment diagrams.

(B. Sc. Eng: Part I, 1929. S. Mat:)

[*Hint:—Use formula* $\frac{P s^2}{2 EI}\left(S-\frac{S}{3}\right)=\frac{WS^3}{8 EI}$. *Determine s. See pages 188 and 192.*]

(3) A girder is 50′ long and carries a uniformly distributed load of 2 tons per foot run. It is fixed at one end and simply supported at the other. Draw the bending moment and shearing force diagrams.

(Diploma Eng: 1927.)

[*Ans. See example 8 page 185.*]

(4) A continuous beam, 80 feet long, rests on level supports at each end and at points 25 feet from each end. It carries a uniformly distributed load of 1 ton per foot run. Draw to scale the bending moment and shearing force diagrams. State the maximum bending moment in an outer and in the middle span, and the force on an end and on an inner support. (B. Sc. Eng. Part II, 1931. Properties of Mat.)

[*Ans. Max: Bending moment in an outer and in the middle span=76·25 tons feet. End supporting force=9·46 tons and inner supporting force=30·53 tons.*

(5) A winch drum is supported on a raised platform carried by the cantilevers A and B which are fixed rigidly in the wall (See fig. 341) The maximum load 10 tons may be assumed equally divided between A and B at the position marked. A and B are supported at the outer ends by a cross girder C, which again is carried by a short

central rigid column. If the moments of inertia of the cantilevers and cross girder are I=280 inches, determine the reactions on the central column and at the wall. Sketch the bending moment diagrams for the cantilevers and cross-girder, stating the maximum values in each.

Find the maximum stress in each cantilever and in the girder if the depths are 12″. (B. Sc. Eng: Part II, 1931. Theo. Struct:)

[*Ans. Reaction at the column = 1·62 tons. Reaction at the wall = 2·19 tons. Max: Bending moment for cantilevers = 7·6 tons feet. Max: Bending moment for the cross girder = 3·24 tons feet, Max: stress in cantilever and girder = 7·6 and 3·24 tons respectively.*]

(6) A beam of 20 feet clear span has its ends firmly built into the supports and carries a load of 10 tons uniformly distributed over the 20 feet between the supports.

Draw the bending moment and shearing force diagrams, and show clearly where the bending moment changes sign and where it is a maximum and a minimum.

If the beam is of reinforced concrete how will the steel be arranged ? Indicate this clearly by a sketch, taking into consideration both bending and shearing forces.

(I. Sc. Eng: Part II, 1931. App. Mech.)

[*Ans. Bending moment changes its sign at 4·22′ from fixed ends and max: at the fixed ends and minimum at the centre.*]

(7) An encastre beam (*i. e.* a beam with fixed ends) has a clear length of 30 feet and supports a uniformly distributed load of 1 ton per foot run (inclusive of its own weight) and also a concentrated load of 10 tons at 10 feet from the left support. Draw to scale the bending moment diagram and state the maximum bending moment and its distance from the left support. (B. Sc. Eng: Part I, 1932 Theo. Struct.)

[*Ans. Max: Bending moment is at the left fixed end = 115·4 tons feet.*

(8) A rolled steel joist, fixed at the ends, has a span of 30 feet and carries a dead load of 1 ton per foot run. Calculate a suitable cross section. (B Sc. Eng: Part I, 1928. Theo. of Struct)

[*Ans. Steel joist 20″ × 6½″ × 65 lbs per foot.*]

CHAPTER XIII.

(1) A Continuous girder has two equal spans, 200 feet each, and is to carry a uniform moving load of 1,000 lbs. per lineal foot,

which can be distributed in any manner over the spans. Find (*a*) the distributions of the load which will give maximum positive and negative shear at the centre of one span, (*b*) the points of inflexion, (*c*) the loading for maximum positive and negative bending moments at a point half-way between an inflection point and the centre support.

(City and Guilds, 1926.)

[*Ans. (a) Loads to be distributed to cover half the span from the end supports. (b) 65′ from the centre support. (c) Loading to cover throughout the spans.*]

(2) Write down the theorem of three moments for a continuous beam loaded uniformly.

A continuous girder in three spans, AB = 200 feet, BC = 50 feet, CD = 200 feet, is loaded uniformly with 1 ton per lin. foot over the first two spans. Find the supporting forces and the bending moments at the centre of each span and at the supports B and C.

(City and Guilds, 1926.)

[*Ans.* $R_A = 79{\cdot}68$, $R_B = 114{\cdot}10$, $R_C = 56{\cdot}22$, $R_D = 0$. *Bending moment at the centre of 1st span = 2050, centre of 2nd span 1700 tons feet at the centre of the 3rd span = 0. At B = 4062·49 tons feet. At C = 0.*]

(3) A horizontal girder 80 feet in length and carrying a uniformly distributed load, including its own weight, of 2 tons per foot, is lifted by three hydraulic rams which are placed one under each end and one under the centre of the girder.

The three rams are all of the same diameter and the cylinders are in hydraulic communication with each other. Draw the bending moment diagram for the girder. (B. Sc. Eng: Part II, 1931. Theo. Struct:)

[*Ans. See example 1 page 193.*]

(4) A beam is continuous over two spans 20′ and 40′ respectively. The three supports are on the same level. There is a 'central load' of 10 tons in the case of the 20 feet span; and one of 4 tons for the 40 feet span. Draw:—(*a*) the shearing force diagram and (*b*) the bending moment diagram for the continuous beam.

(B. Sc. Eng: Part II, 1923. Theo: of Struct.)

[*Ans. See example 4 page 199.*]

(5) A continuous girder of uniform cross section crosses two spans, 40 feet and 30 feet long, and is simply supported on three supports at the same level. The Span of 40 feet carries a uniform load of 1¼ tons per foot run, and there is a single load of 40 tons placed

at the centre of the span of 30 feet. Calculate the reactions on the three supports and draw the bending moment diagram.

(City and Guilds 1920.)

[Ans. $R_A = 23{\cdot}30$, $R_B = 65{\cdot}63$, $R_C = 11{\cdot}07$ tons.]

(6) A uniform girder AD, 120 feet long, is continuous over three supports A, B and C. Figure 342. The span AB = 40 feet and

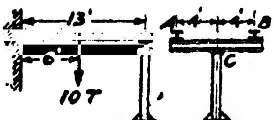


FIG. 341

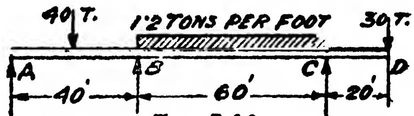


FIG. 342.

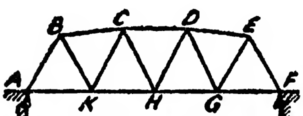


FIG. 343.

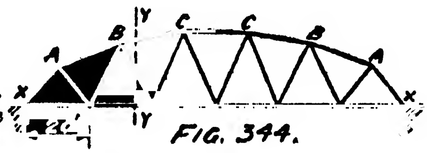


FIG. 344.

BC = 60 feet, and the end D overhangs the support C by 20 feet. It is loaded with an isolated load of 40 tons at the middle of the span AB. carries a uniform load of 1·2 tons per foot on span BC and an isolated load of 30 tons at the end D.

Calculate:—(*a*) the bending moment at each support;

(*b*) reaction at each support;

and sketch the bending moment and shearing force diagrams. Mark on your diagram all points of contraflexture and of maximum bending moments, showing clearly how you fix the different points.

(B. Sc. Eng: Part II, 1932. Prop: of Materials.)

[Ans. (a) $M_B = 264$ tons feet, $M_C = 600$ tons feet.
(b) $R_A = 13{\cdot}4$, $R_B = 57$, and $R_C = 71{\cdot}6$ tons.]

CHAPTER XIV.

(1) Define an "Influence Line".

A girder is 120 feet long simply supported at the ends, draw the 'bending moment influence line' for a point 40 feet from the left support. Use your 'influence line' to determine the maximum 'bending moment' produced by a uniform moving load 30 feet long and of intensity 2 tons per foot run. (B. Sc. Eng: Part I, 1924.)

[Ans. See example 4 page 211.]

(2) Fig. 343 shows the line diagram of a pin-jointed truss. Each member in the lower boom is 20 feet long, the triangles ABK and

GEF are equilateral, and the member CD is 20 feet long and is 20 feet above AF. The truss is symmetrical about the vertical through H. Draw an influence line for each of the members BC, CK, KH. when unit load travels from A to F. (B. Sc. Eng: Part I, 1931. Theo. Struct.)

[Ans. See example 9 pages 224 to 227]

(3) A Warren girder has 6 panels each 10 feet long and carries a rolling load, greater than the span, of 2 tons per foot run.

Find the maximum forces in the members of the second panel from the left end.

Draw the influence lines for bending moment and shear.

(B. Sc. Eng: Part II, 1931. Theo. Struct.)

[Ans. Bottom chord member 76·25 tons tension nearly and for two inclined members = 37·2 tons. See example 6 page 214.]

(4) For unit load draw the bending moment and shearing force influence lines for point 40 feet from the left end of a freely supported girder of 100 feet span. From these diagrams determine the maximum bending moment and shearing force at the given point when a uniformly distributed load of 1 ton per foot run and 40 feet length moves slowly across the girder. (B. Sc. Eng: Part I, 1932. Theo. Struct:)

[Ans. Max: Bending moment at the given point = 760 tons feet. Max: Shearing force = 16 tons.]

(5) Draw the bending moment influence line for a point situated at 20 feet from the left hand end of a simply supported girder of 60 feet length. A rolling load 12 feet long of 1 ton per foot run passes across this girder. From your influence line calculate the maximum bending moment produced at the given point.

(B. Sc. Eng: Part I, 1928. Theo. of Struct.)

[Ans. Max: Bending moment at the point = 144·08 tons feet.]

(6) Define an "Influence line"

Explain the use of 'influence line' for determining the maximum forces produced by the action of moving loads in the members of a Warren girder or any other type of framed girder.

(B. Sc. Eng: Part II, 1923. Theo. of Struct:)

[Ans. See examples 6, 8 and 9, pages 215 to 227.]

(7) What is an influence line ?

Two loads of 2 and 4 tons respectively, at 4 feet apart, roll over a beam of 20′ span. Find the magnitude of the greatest moment and state on which cross section it occurs. Find also the maximum moment at the centre of the beam. (B. Sc. Eng: Part I, 1930. Theo. of Struct.)

[Ans. Greatest bending moment = 26·08 tons feet and occurs at 10·75′ from the support. Max: bending moment at the centre = 26 tons feet.]

(8) The heights of the apices A, B and C of the isosceles triangles in the line diagram of the pin-jointed girder shown in fig. 344 are respectively 12, 20 and 24 feet above XX. This girder is simply supported at the ends, and the six bays are each of equal length.

Draw the scale diagrams of the influence lines for the forces in the three members cut by the section YY, when a unit ton load moves slowly across the track which is supported from the pin-joints in the lower boom. (B. Sc. Eng: Part II, 1932. Theo. of Struct.)

[Ans. See example 9 page 224.]

CHAPTER XV.

(1) The profile of the cam shown in fig. 345 is made up of circular arcs and tangential straight lines. The cam acts on a roller at the end of a tappet moving in a vertical guide, as shown. Draw the tappet displacement diagram on a base line along which 9″ represents 360° rotation of the cam shaft. If there is a clearance of $\frac{1}{16}''$ between the end of the tappet and the valve spindle, during what angular period of the cam is the valve open ?

(2) The inclined cam-plate A, mounted on a uniformly rotating shaft fig. 346 operates a reciprocating tappet B, as shown. The contact

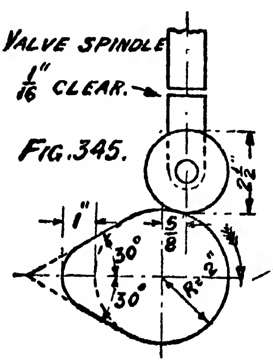


FIG. 345.

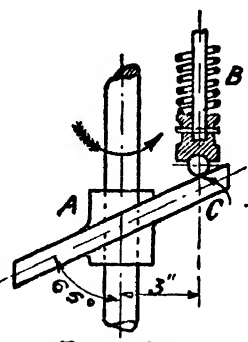


FIG. 346.

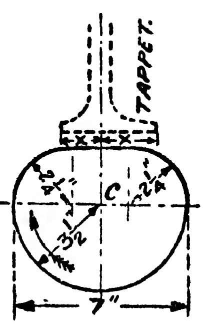


FIG. 347.

between ball and plate is to be considered as taking place at C, on the line of action of the tappet. Draw the displacement diagram for the tappet. Is this any common geometrical curve ?

Take 6″ as 360° for the base.

(3) The outline of a cam is an ellipse major axis 5″ and minor

axis $4\frac{1}{4}''$. The cam rotates uniformly about an axis passing through one of the foci. Draw the displacement diagram for a vertical reciprocating tappet, having a roller $2''$ dia., the line of action of which passes through the centre of rotation of the cam, Take $9''$ as $360°$ for the base. How would the diagram be modified for $\frac{1}{8}''$ clearance between cam and roller at the lowest position of the latter ?

(4) Design a cam profile to raise and lower a tappet with uniform acceleration and retardation during both the lift and fall periods. The periods of lift, fall, and rest at bottom are 120°, 90° and 150° respectively.

The lift is $1\frac{1}{4}$ ins. and the minimum diameter of the cam $4\frac{1}{2}$ ins. The centre line of the tappet's path passes through the centre of rotation of the cam. The tappet is fitted with a roller 2 ins. diameter.

Draw a tappet displacement diagram on a base 6 ins. long representing the angles of rotation of the cam.

(5) Design a cam to give the following motion to a tappet, fitted with a $3''$ diameter roller; the line of action of the tappet passes through the cam centre: lift of $1''$ during 80° rotation, remain at this position during next 80°; and a further lift of $1\frac{1}{4}''$ during next 80°; roller then to fall through $2\frac{1}{4}''$ back to its original position during the remaining 120°. The cam profile is to be made up of circular arcs and tangential straight lines, least radius of cam being $3''$. Determine the complete displacement diagram. Scale $\frac{1}{2}$ size.

(6) The outline of a cam which rotates round an axis passing through the point C, is shown in fig. 347. The cam operates a reciprocating tappet having a flat face perpendicular to its axis, and

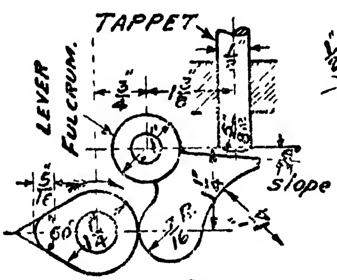
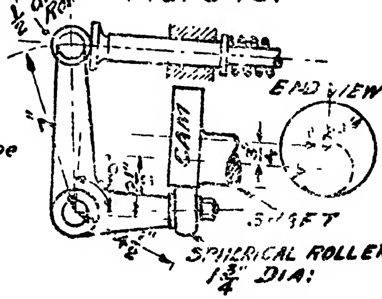


FIG. 348.

FIG. 349.


the line of action produced passes through C. Draw the curve of tappet displacement, finding lifts for each 15° rotation of cam, and

taking $\frac{3}{4}''=30^\circ$ for the base line. What is the *minimum* length of tappet face on each side of the axis ?

(Dimensions X in sketch.)

(7) The fig. 348 shows the exhaust valve gear for a small petrol motor. Draw the tappet displacement diagram.

Scale 3 times full size

Let a distance of 4″ represent 120° rotation of cam shaft.

(8) A cam fig. 349 the profile of which is circular, operates a reciprocating tappet through the medium of a bell crank. Draw a diagram showing the motion of the tappet on a base line along which 6″ represents 360° rotation of the cam,

(9) Determine the profile of a cam, made up of circular arcs and tangent straight lines fig. 350 to operate a reciprocating tappet having a $1\frac{3}{4}''$ diameter roller, and whose line of action passes 1″ from the centre of rotation of the cam. The tappet is to be raised $1\frac{1}{4}''$ during a cam rotation of 60°; to remain at rest at top of stroke for next 30°; and to be lowered during next 60° rotation. Least radius of cam$=2\frac{1}{2}''$. Also, draw the complete displacement diagram, taking $\frac{1}{4}''$ to represent 10° cam rotation on the base line.

(10) A cam is required to give a vertical motion of 6″ to a slider, the centre line of which passes through the axis of the cam shaft. The slider is to rise with uniform velocity and then descend uniformly, but at half the speed of the ascent; the ascent and the descent occuring during a revolution of the cam. The diameter of the camshaft is $2\frac{1}{2}''$, and the least distance from the camshaft centre to the bottom of the slider is $4\frac{1}{2}''$. Set out the profile of the cam.

CHAPTER XVI.

(1) A triangular roof-frame ABC has a horizontal span AC of 40 feet, and the angle at the apex B is 120°, AB and BC being of equal length. The roof is hinged at A and simply supported on rollers at C. The loads on it are as follows: (1) A force of 4,000 lbs. midway along and perpendicular to AB; (2) a vertical load of 1,500 lbs. at B; and (3) a vertical load of 1,400 lbs. midway between B and C. Find the reactions or supporting forces on the roof at A and C.

(I. Sc. Eng: Part I, 1929. Math: Paper II.)

[*Ans. At A=3900 and at B=3000 lbs.*]

(2) The depth of water in a dock is 30 feet. Find (*a*) the pressure on the dockwall in tons per foot of its length; (*b*) the position

of the centre of pressure; (c) the overturning moment about the bottom of the wall in tons-feet per foot length, due to the water pressure. Water weighs 62·4 lbs. per cubic foot.

(I. Sc. Eng: Part I, 1929. Math: Paper II.)

[Ans. (a) 12·53 tons. (b) at 20′. (c) 125·3 tons feet.]

(3) Two rafters each 30 feet long span a space of 52 feet. They are subject to an uniform load of 8 cwt. per horizontal foot. The three joints are pin joints. Find the greatest 'bending moment', 'shearing force' and 'direct thrust' on the rafters. (B. Sc. Eng: Part I, 1924)

[Ans. See example 4 page 256.]

(4) A masonry boundary wall 18 ins. in thickness has a height above the ground level of 14 feet. If the masonry weighs 140 lbs. per cubic foot, calculate what uniformly distributed load acting on the wall face would just nullify the compression in the toe of the wall at the ground level. Why is tension objectionable for brick and masonry work ? (City and Guilds 1915.)

[Ans. A uniformly distributed wind load of 8 lbs. per □′.]

(5) A beam of rectangular cross section, 7 inches wide by 8 inches deep, was made of concrete reinforced with two tension rods $\frac{5}{8}$ inch diameter, 1 inch from the lower face. It was tested on a span of 11 feet, and a central load of 2,500 lbs. was applied. Determine the maximum stress in the coucrete and steel. (City and Guilds, 1926.)

[*Asn. 1129 lbs. and 21295·72 lbs.*]

(6) State the principle of virtual work. A square framework,

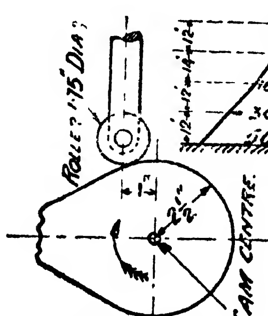


FIG. 350.

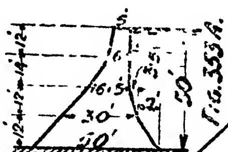


FIG. 353A.

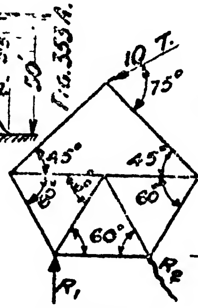


FIG. 351

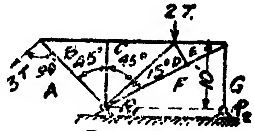


FIG. 352.

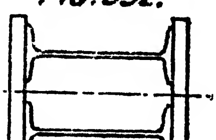


FIG. 353.

formed of uniform heavy rods of equal weights W jointed together, is hung up by one corner. A weight W is suspended from each of the

three lower corners and the shape of the square is preserved by a light rod along the horizontal diagonal. Prove that its compression is 4 W.
(I. Sc Eng: Part II, 1930. Math.)

[*Hint:—Each of the lower corners will have 2 W acting downwards and the suspended corner will have 6 W acting upwards. Draw triangle of forces for each corner.*]

(7) The structure shown in fig. 351 is acted on by a force of 10 tons. The reaction R_1 is vertical. Find the reaction R_2 in magnitude and direction. Draw the force diagram and state the forces in each member.

[*Ans.* $R_2 = 16{\cdot}2$ *tons at* 122° *acting down. Outline members* $+9{\cdot}6, -2{\cdot}6, -2{\cdot}0, -3, +8$ *tons. Horl. members at the centre,* $-10{\cdot}8$, $+2{\cdot}8$ *tons. Web members* $+13{\cdot}5, -13{\cdot}6$ *tons.*]

(8) The frame shown in fig. 352 is hinged at R_1 and is anchored by the tie, FG, at R_2. The loads carried are as indicated. BC is vertical and the top members are horizontal.

Find the reaction at R_1 and R_2. Draw the stress diagram, and determine the stresses in AB, CH and EF, indicating whether tensile or compressive.

[*Ans.* $R_1 = 5{\cdot}8$, $R_2 = 1{\cdot}2$ *tons.* $AB = +3$, $CH = -4{\cdot}2$ *and* $EF = +2{\cdot}5$ *tons.*]

(9) A railway bridge, 80′ span, with two lines of way, is supported by two main girders of I section. The girders are 7′ feet deep between centres of booms. If the dead weight of the structure between the abutments is 200 tons and the weight of each train of carriages $1\frac{1}{2}$ tons per foot, what must be the effective area of the boom at the centre of the span? The mean stress permissible in each flange is 7 tons per square inch. The resistance of the web to bending may be neglected. (I. Sc. Eng: Part I, 1930.)

[*Ans. Area of each boom at the centre* $= 32{\cdot}65$ *or 33* □″]

(10) Define the "modulus of a section".

The cross section of an I beam, which is symmetrical about its neutral axis and has an overall depth of 12 inches, has a moment of inertia of 320 inch^4 units about its neutral axis. Find the maximum intensity of stress due to bending when the bending moment at the section is 25 tons feet. (Diploma Eng: 1931.)

[*Ans.* $5{\cdot}625$ *tons.*]

(11) The composite section of figure 353 consists of two steel joists, each 15″ by 6″., thickness of web 0·5″., flange 0·88″ thick, and centre lines of webs are 8″ apart. Each plate is 16″ by $1\frac{1}{2}$″. Calculate the moment of inertia about the central axis YY.

A column of this section is 30′ high and has fixed ends. calculate the safe central load. (B. Sc. Eng: Part I, 1931 Theo. Struct:)

[*Ans* $I=1935$, *safe central load*$=414$ *tons.*]

(12) The rectangular cross section of a reinforced concrete beam is 8″ wide and 16″ deep. The tensile reinforcement consists of four $\frac{3}{4}$″ diameter steel rods, their centres being 2″ from the bottom of the beam. Take $m=15$ and $c=600$ lbs per square inch. Determine the depth of the neutral axis and the moment of resistance of the section.

(B. Sc. Eng: Part I, 1931. Theo. Struct.)

[*Ans.* $N.\ A=6{\cdot}8''$, $M=192564{\cdot}74.$]

(13) Show that the external effect of a couple on the body remains unaltered if (*a*) it be translated to any other position in its own plane, the arm remaining parallel to the original position, and if (*b*) it be rotated through an angle in its own plane.

Along the sides AB, EF, FG, BC, DC and HG of a regular octagon ABCDEFGH are acting forces of magnitudes 15, 15, 7, 7, 4 and 4 lbs. weights respectively. Find the magnitude and direction of the force P which should be applied along the remaining sides, which keep the whole body in equilibrium.

(I. Sc Eng: Part I, 1931. Math.)

[*Ans.* $P=18$ *lbs. anticlockwise.*]

(14) If any number of forces in one plane acting on a rigid body have a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of their resultant.

ABCD is a rectangle of which the adjacent sides AB and BC are equal to 3 and 4 feet respectively: along AB, BC and CD forces of 30, 40 and 50 lbs. weights act; find the resultant.

(I. Sc. Eng: Part I, 1931, Math.)

[*Ans. 45 lbs. nearly.*]

(15) Sketch free hand, a regular hexagon. Join every angle to the centre. You are to suppose it represents a frame in equilibrium under two equal and opposite forces applied at opposite angles, and are to consider two cases:—

(*a*) In which there is a pin joint at the centre.

(*b*) In which there are only three rods crossing at centre but not attached to each other.

And say whether they are difficient, sufficient, or redundant. A stress diagram must be drawn if possible.

(B Sc. Eng: Part II, 1923. Struct: Design.)

[*Ans. (a) Redundant. (b) Sufficient. Stress diagram is a straight line=given force.*]

(16) A masonry dam has a vertical face and is 32 feet high, the water level is 2 feet below the top of the dam. The dam is 3 feet wide at the top and 15 feet 6 inches at the base. Does the resultant thrust pass through the middle half ? Why is the middle half used by some designers ? (B. Sc. Eng: Part II, 1923. Theo. of Struct.)

[*Ans. No, it passes beyond the middle third.*]

(17) A rolled steel joist 12″ deep has flanges 6″ wide by 0·7 inch thick and the web is 0·4 inch thick. The section of the joist is subjected to a shearing force of 20 tons. Draw to scale a diagram showing the distribution of shear stress in the section and determine the value of the maximum shear stress. What is the ratio of this maximum shear stress to the mean stress calculatod on the assumption that the web takes the whole of the shear ? (City and Guilds 1920.)

[*Ans. See example 18 page 276.*]

(18) The figure 353A shows the section of a masonry dam built on a rock foundation. Draw the lines of pressure when the dam is (1) empty (2) full of water. For each case show also the distribution of pressure on the base.

[*Ans. See example 14 page 269.*]

CHAPTER XVII.

(1) Draw the velocity and acceleration diagrams for a steam engine mechanism when the crank is 30° from the inner dead centre. Length of crank 1 foot; connecting rod 3 feet; speed of crank pin 10 feet per sec. State the velocity and acceleration of the piston.

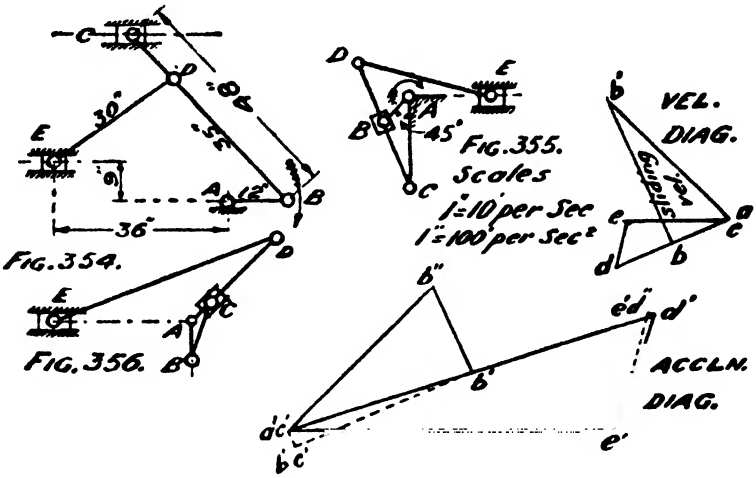
(I. Sc. Eng: Part II, 1931. App. Mech:)

[*Ans. Vel: 6·5 feet per sec. Accl: 92 feet per sec*^{2}.]

(2) Draw the velocity and acceleration diagrams for the mechanism shown in fig. 354 which has two slides at C and E, and find from

the diagrams the velocities and accelerations of the sliders C and E, when the crank AB rotates at 60 revolutions per minute. AB = 12", BC = 48", BD = 33" and DE = 30".

[*Ans. Vel : 17 and 10·8 feet per sec. Accl : 33 and 42 feet sec.²*]



(3) The crank AB fig 355 revolves uniformly with an angular velocity of 12 radians per second about the centre A. The end B is pivoted to a block which slides along CD, and CD revolves about the centre C and the point E slides along AE. Draw the velocity and acceleration diagrams for the mechanism when AB is 45° to AC. Determine the velocities and accelerations of B and E. AB = 9" CD = DE = 36" and AC = 24".

[*Hint :—The sliding velocity of B along the line BC is parallel to BC and its relative vel : to C is perp : to BC. Similarly for accln. Accln. of B rel : to C is towards BC. ∴ c′ – b′ c′ should be parallel to BC.*

(4) BC fig. 356 rotates uniformly about B, the volocity of C being 4 feet per second. C slides along the link AD, which is pivoted at A, and E reciprocates on the line AE. If BC = 6", AB = 4", AD = 12" and DE = 24", draw the velocity and acceleration diagrams and determine the velocities and accelerations of the points C and E, when BAC is 135 degrees.

[*Ans. Vel. 3·5 and 4·8 feet per sec. Accl 62·5 and 220·7 feet sec²*]

(5) The diagram shown in figure 357 shows a radial valve gear. The crank CP turns uniformly at 12 radians per second, and is pinned at P to the rod PR, the point Q in this rod being guided in the circular path SS, centre T. For the position of the mechanism shown in the diagram, determine and measure the velocities and accelerations of the points R and V.

CP = 4", PQ = 12", PR = 14·3", RV = 16·3" and QT = 16·2".

[*Ans. Vel.* $R = 2{\cdot}67$ *feet per sec.* $V = {\cdot}6$ *foot per sec. Accl.* $R = 32{\cdot}9$, $V = 14{\cdot}4$ *feet sec*^{2}.]

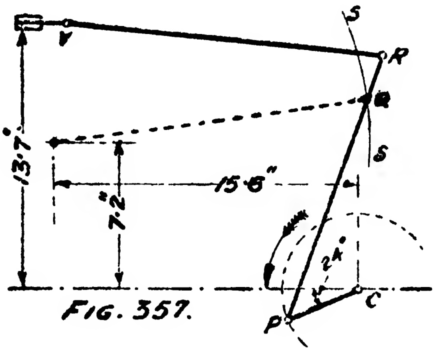


FIG. 357.

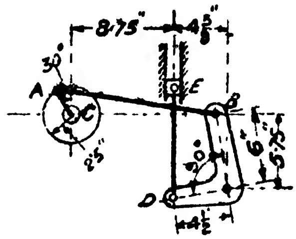


FIG. 358.

(6) Find the velocity of the slider E in the mechanism shown in fig. 358 when the crank rotates at 1 revolution per sec. AB = 13¼" long, DE = 9" long.

[*Ans.* ·78 *foot per sec.*]

[*Hint :—After getting the vel. of the 6" portion of the lever, consider the point D to be an image of D in the lever BD*]

UNIVERSITY QUESTIONS ON PRACTICAL GEOMETRY.

INTERMEDIATE EXAMINATION IN SCIENCE

Engineering Part II.

EACH PAPER IS OF 4 HOURS DURATION

1921.

(1) A bridge consisting of a pair of girders of 40 feet span carries a locomotive, the loading and spacing of the wheels on each girder being as shown in figure 359. Draw to scale the shearing force and bending moment diagrams for the loading when the locomotive is in the position shown.

(2) Find the stresses in the members of the roof frame shown in the fig. 360, and mark the stress beside each member, + representing tension, and – compression. The dead load is 100 lbs, and the normal wind pressure 150 lbs per foot run of slope. The wind is on the right slope and the left reaction is vertical.

[Ans. AB = –7550, BC = –6550, CD = –6550, DE = –7750, EF = –6950 and FG = –8900. Bottom chord members +6400, +4100 and +9150. Web members –900, –900, +2550, +5200, –2200 and –2500 lbs.]

FIG. 359.

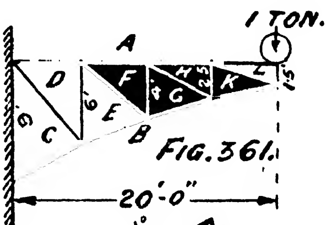


FIG. 361.

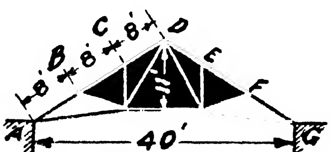


FIG. 360.

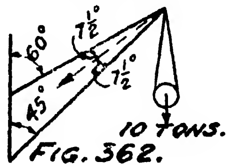


FIG. 362.

(3) The top of a tall circular brick chimney has an internal diameter of 15 feet and a thickness of 1 foot 6 inches; at 100 feet below the top its internal diameter is 17 feet and its thickness 2 feet 3 inches. Find the amount and position of the resultant pressure at this plane assuming that wind pressure is concentrated 50 feet below

the top and is equivalent to 30 lbs. per square foot of projected area and that brickwork weighs 120 lbs. per cubic foot. State whether there is any tension on the joints of the brickwork and expresss your opinion as to the sufficiency of the design.

[Ans. See example 14 page 269.]

(4) A frame secured to a vertical wall has dimensions as shown in the figure 361. The bars AD, AF, AH and AL are each 5 feet in length. Find the forces in all the parts produced by the load of 1 ton.

[Ans. $AD=-2{\cdot}45$, $AF=2{\cdot}45$, $AH=-1{\cdot}98$, $AL=0$, $BL=+1$, $BK=+2$, $BG=+2{\cdot}55$, $BE=+2{\cdot}65$, $BC=+2{\cdot}55$, $CD=+4$, $DE=0$, $EF=0$, $FG=+{\cdot}25$, $GH=-{\cdot}5$, $HK=+{\cdot}6$, $KL=-2{\cdot}05$ *and reactions* $BM=+2{\cdot}4$ *and* $MA=-2{\cdot}2$ *tons]*

Note:—The sign + represents compression and − tension.

(5) The tie and jib of a crane are inclined at 60° and 45° respectively to the vertical post, and the direction of the chain is such that it bisects the angle between them. If the tension in the chain is, by means of tackle, made half the weight carried, find the forces in the tie and jib when the load carried is 10 tons. See fig. 362.

[Ans. Tie $=-24{\cdot}5$ *and jib* $+36$ *tons]*

(6) The legs of a pair of sheer legs are 60 feet long and spaced 20 feet apart at the base ; the backstay is 80 feet long. A load of 20 tons is suspended 15 feet in front of the hinges at the base of the legs; find the stresses in the legs and in the backstay.

[Ans. Legs $+14{\cdot}5$ *and in the backstay* -11 *tons.]*

1922.

(1) The crank of an engine has a radius of 18 inches, the connecting rod is 6 feet long and the number of revolutions made by the engine is 80 per minute. Find graphically the velocity of the piston in feet per second when the crank has passed through an angle of 30° from the dead centre during the forward stroke.

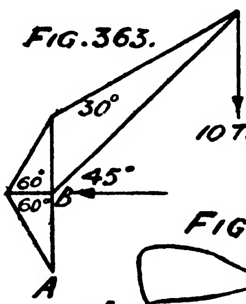
[Ans. 7·7 feet per sec.]

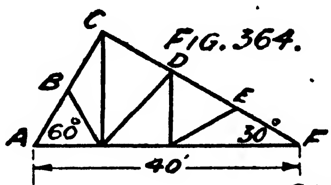
(2) What is a cam ? For what purposes in mechanism are cams generally used ? Sketch and describe the construction and actual form of a cam in use in any machine with which you are acquainted. Draw the outline of a cam which would give a slow forward and quick return motion to a reciprocating piece, with an interval of rest between the two motions.

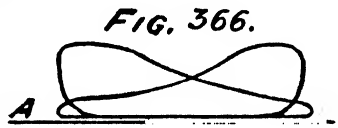
(3) Draw the stress diagram of the crane shown in the figure 363 and distinguish between ties and struts. A is a footstep bearing and the reaction at B may be assumed horizontal.

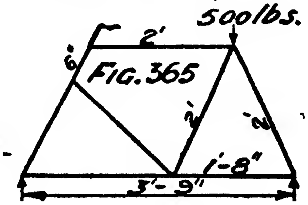
[*Ans. See example 10 page 85.*]

(4) Draw a diagram of stresses for the roof frame shown in the figure 364 when carrying a vertical load of 2, 000 lbs. at each of the joints B, C, D, and E and 1,000 lbs. at A and F, and a uniformly distributed normal wind pressure on the left side 6,000 lbs. assuming all the joints to be flexile. Show for each bar whether the stress is push or pull.









(5) The figure 365 represents the frame of a bicycle the load on the seat is 500 lbs. Find the amount and nature of the stresses on all the members of the frame and also the bending moment on the fork at A.

[*Ans. In the triangular frame 2 feet members +125, +430, and 1—8″ member = —180 lbs. Top horl: member +125 and inclined member —120 lbs. 6 inches tube = +60 and the forked end = +60 lbs. Bending moment at A = 71·04 lbs. feet. Bottom end of the 6″ member is the point A.*]

(6) A girder 100 feet long is supported at each end and in the middle and carries a uniform load of 2 tons per foot run. Draw the bending moment and shearing force diagrams and find the pressure on each support.

[*Ans. See example 1 page 193.*]

1923.

(1) A vertical reciprocating piece moves in guides under the action of a cam attached to a shaft which rotates uniformly, the centre of the cam lies in the line of motion of the reciprocating piece,

Suppose a friction roller used of diameter equal to $\frac{1}{6}$ th stroke and suppose also that the least radius of the cam $\frac{1}{4}$ th stroke. Trace the form of the cam that the piece may slide uniformly and make one complete movement in each revolution.

[*Ans. See example 9 page* 241.]

(2) The indicator diagrams from a simple steam engine taken with $\frac{1}{23}$ spring are shown below in fig. 366. The length of the stroke is 12″. Draw the polar curve of crank effort for one complete revolution of the engine.

(3) A girder 70 feet long carries a uniform load of 2 tons per lineal foot from one end to the middle and two loads of 20 tons at 20 feet from each end. Draw the bending moment and shearing force diagrams.

(4) A load of 7 tons is suspended from a tripod, the legs of which are of equal length and inclined at 60° to the horizontal. Find the thrust on each leg. If an additional horizontal force of 5 tons be applied at the summit of the tripod in such a way as to produce the greatest possible thrust on one leg, find that thrust and determine the stress in the other two legs.

[*Ans. See example 4 page 76 for the first part of the question For the combined load* $+9{\cdot}5$ *tons,* $-{\cdot}7$ *and* $-{\cdot}7$ *ton.*]

(5) A bridge is constructed of a pair of Warren girders with the platform resting on the lower booms, each of which is in 6 divisions. The bridge is loaded with 20 tons in the middle. Find graphically the stresses on each part.

[*Ans. Diagonal members 5·75 tons. Top chord members,* $=5{\cdot}75$, *11·5 and 17·25 tons. Bottom chord members 2·9, 8·5, and 14·5 tons. The other half is the same.*]

(6) A spur wheel and pinion have 48 and 11 teeth respectively and $1\frac{1}{4}$ inch pitch. The generating circles are 3″ and $2\frac{3}{16}$″ respectively. Plot out full size the shape of one of the teeth both of the pinion and spur wheel.

1924.

(1) In a steam engine the cut-off takes place at ·7 of the stroke, the angle of lead is 6° 9′, the width of the steam ports is $1\frac{1}{2}$ inches and the steam port opens $\frac{17}{24}$ of its area. Find by using a Zeuners or Reubaux diagram the travel of the slide, the angle of advance, the outside lap and the outside lead. Assume connecting rod equal to 4 cranks in length.

(2) Two shafts intersecting at right angles are connected by bevel wheels with 22 and 44 teeth respectively, of 1 inch pitch. Draw to a scale of $\frac{1}{4}$ the pitch surfaces of the wheels, and find the development of the conical surfaces on which the shape of the ends of the teeth are set out.

(3) The piston of a reciprocating engine has a stroke of 15 inches and a connecting rod 40 inches long from centre to centre. Find the velocity of the piston at one third of its stroke when the engine is making 240 revolutions per minute.

(4) The truss shown in the figure 367 has principals 10 feet apart; the struts bisect the rafters at right angles; the middle tie is cambered $1\frac{1}{2}$ feet and carries shafting at the centre of weight 400 lbs. Assuming dead load to be 12 lbs. per square foot of ground area, draw the stress diagram.

[Ans. See part I chapter on roof trusses fig. 102]

(5) A pit crane is as sketched, a being a footstep bearing and the reaction at b assumed horizontal. Find graphically the forces in the different members when a load of 10 tons is suspended from the extremity of the jib, and distinguish between ties and struts.

[Ans. See example 10 page 85.]

(6) A girder 100 feet long is supported at each end and in the middle and carries a uniform load of 2 tons per foot run. Draw the bending moment and shearing force diagrams, and find the pressure on each support. State clearly any assumptions you make and don't forget to give the SCALE you use.

[Ans. See example 1 page 193.]

1925.

(1) A cone, base $4\frac{1}{2}''$ diameter and height $4''$ stands on the horizontal plane. The cone is pierced by an elliptical hole, major axis $1.8''$, minor axis $1.3''$. The major axis of the hole is inclined 30° to the V. P. and the axis of the hole is $1.2''$ distant from the axis of the cone. Draw the plan and elevation of the solid.

(2) A pinion wheel is in gear with a rack, the number of teeth in the wheel is 20 and the pitch of teeth $2''$. The height of the teeth above the pitch line is $0.6''$, and the depth of the teeth below the pitch line is $0.8''$. The wheel teeth have involute faces and radial flanks, the rack teeth have cycloidal faces and straight flanks.

Draw the rack and pinion in gear showing three teeth in each, and state briefly why the given curves are suitable.

(3) The piston of a reciprocating engine has a stroke of 13″, and the length of connecting rod centre to centre is 32″. The engine makes 200 revolutions per minute. Construct polar curves of piston velocity, and show the mean piston velocity.

(4) A load of 5 tons is suspended from sheer legs. The lengths of the legs PA, PB and PC being 40′ 0″, 32′ 6″ and 37′ 6″, while the lengths AB, BC and CA measured along the ground are 37′ 6″, 25′ 0″ and 50′ 0″. Determine the height of the apex P above the ground and the force acting along each leg.

[Ans. See example 3 page 74]

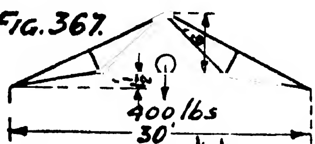


FIG. 367.

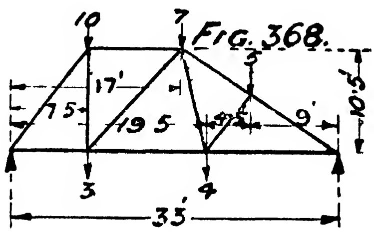


FIG. 368.

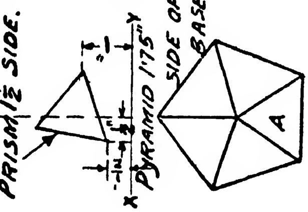


FIG. 370.

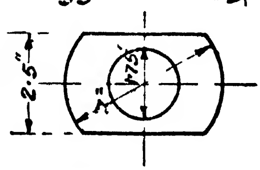


FIG. 369.

(5) A vertical spindle is supplied with a plane horizontal face at its lower end. The face is actuated by a cam keyed to a shaft, the axis of which is in the line of stroke. Design the cam to raise and lower the plate under the following conditions:—

The valve to be raised through the first half of its stroke with uniform acceleration and through the second half with uniform retardation. Simple harmonic motion is given on the return stroke. The least radius of the cam is 2″, and the travel of the spindle is 1½″. The spindle is raised in 2/5, lowered in ⅓, and remains at rest during the remainder of a complete revolution of the cam shaft.

[Ans. See example 10 page 242]

(6) Draw the stress diagram for the unsymmetrically-loaded truss shown in the figure 368. State clearly whether a member is in tension or compression.

[Ans. See fig. 101 part I.]

1926.

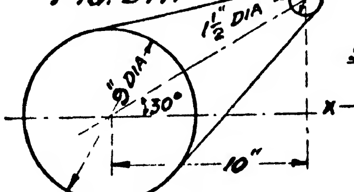
(1) Fig. 369 shows the plan of the flanged end of a $1\frac{3}{4}''$ dia. rod, the minimum thickness of the flange being 1″. The rod is joined to the flange with a turned radius of 1″. Draw an elevation of the flat side of the flange.

(2) The diagram Fig 370 shows the plan of a pentagonal pyramid of 3″ height, whose base lies on the H. P., and also the elevation of an equilateral triangular prism of indefinite length lying perpendicular to the V. P. Draw complete plan and elevation, showing the intersection of the prism with the pyramid.

Find also the true shape of the face A of the pyramid showing the portion cut away by the prism.

(3) Two parallel cylinders whose elevations are as shown in fig. 371 (the axes being horizontal) are connected togther by part of the tangent cone, the axis of the cone being at right-angles to the axes of the cylinders. Draw a plan of the composite solid, and develop the surface of the conical connection.

FIG. 371.



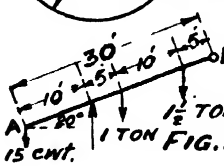


FIG. 372.

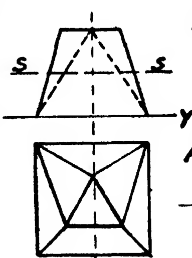


FIG. 373.

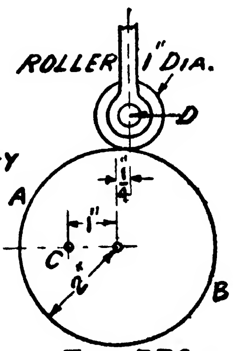


FIG. 374.

(4) Find the profile of a cam to raise a tappet, having a roller 2″ diameter, through 2″ lift, the motion being S. H. M. Rise takes place in 1|4th revolution, then rest for 1|12th revolution, and fall during the next 1|3rd revolution of the cam.

Diameter of the cam shaft = 2″, least thickness of metal – 1″. The line of action of the tappet passes through the cam centre, and the cam rotates with uniform angular velocity.

[Ans. See example 11 page 243.]

(5) A Pratt Truss of 80 feet span and 12 feet high is divided into

eight equal panels and is loaded uniformly at 2,000 lbs. per foot run. Draw the stress diagram and show the members in compression.

[Ans. See example 9 page 63]

(6) A 30 feet girder is hinged at B and simply supported at A. The loading is as shown in fig. 372. Draw the B. M. and S. F diagrams.

[Ans. See example 27 page 38.]

1927.

(1) A beam is 25 feet long and carries a uniform load of 1·5 tons per foot over 20 feet of its length from the right hand end. One of the reactions is at the left hand end of the beam and the other reaction is at a point 5 feet from the right hand end of the beam. Draw by the usual graphical methods the diagrams of "bending moment" and "shearing force" and state their scales

(2) Draw to a scale of one inch equals 10,000 pounds and one inch equals one cubic foot an "indicator diagram" to the following data and find its area in foot-pounds:—

Initial pressure	20,000 pounds
Cut-off	at $\frac{1}{3}$ of stroke.
Back pressure	2000 pounds.
Exhaust closes	0·8 of back stroke.
Clearance	0·5 cubic feet.
Piston displacement	3·0 „ „

(3) The main rafters of a roof truss are each 30 feet long, and inclined to the horizontal at 30°. Each has at its centre and at right-angles to it a member 7 feet long. The other ends of these are connected to each other and to the outer ends of the rafters. The vertical loads on the upper joints are 5000; 10,000; 10,000; 10,000 and 5000 pounds. At right-angles to the right rafter there are in addition at the three joints loads of 6,000; 12,000 and 6000 pounds. The left end of the truss is fixed and the right is "free". Find and tabulate all the internal forces.

(4) An ordinary eccentric sheave 6 inches in diameter and of 1·5 inches throw is used as a cam. The line of action of the push rod passes through the centre of rotation of the cam. There is no roller on the push rod. Its end which presses against the cam is a straight bar at right-angles to the push rod and the motion of the push rod, Plot

curves for one complete stroke, of displacement, velocity and acceleration on a distance base. Two of them are straight lines. The cam makes 600 revolutions per minute.

1928.

(1) Two projections of a solid are shown in fig. 373. Show the development in a single piece, of one of the symmetrical halves of the sloping surface. Draw the plan of the mid-section SS, determine its area a and also a_1 and a_2 the areas of the ends of the solid.

Measure the height and calculate the volume of the soild by any method with which you are acquainted. Give the answer in cubic inches.

[Ans. $a=2{\cdot}62$ □″, $a_1=4{\cdot}84$ □″, $a_2=1{\cdot}32$ □″, volume $=4{\cdot}45$ cubic inches.]

(2) Fig. 374 shows a circular cam AB fixed to a shaft whose axis is at C; it rotates uniformly and gives a vertical motion to the tappet rod and roller D.

Draw on a time base a full size diagram showing the displacement of the centre of the roller D from its lowest position for one revolution of the cam. Take a base line 6 inches long to represent the time of one revolution.

[Ans. See example 13 page 245]

(3) A gas engine has a stroke of 14 inches, a connecting rod 4 feet long and runs at 250 revs. per minute. Find by vector diagrams the velocity and acceleration of a point on the connecting rod one foot from the crank pin, when the crank has turned 60 degrees from the inner dead centre.

(4) Refer to fig. 375. Prick off this figure onto your drawing

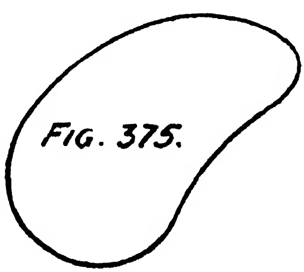


FIG. 375.

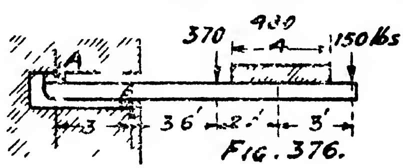


FIG. 376.

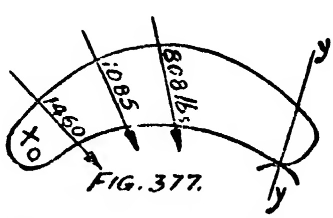


FIG. 377.

paper and find its area in square inches. Find also the position of its centre of area and mark the point clearly on your diagram.

The method of solution must be shown. No marks will be given for guess work.

[Ans. Area = 20·4 □″.]

(5) A cantilever is loaded and supported as shown in fig. 376. Draw diagrams of shearing force and bending moment and measure the maximum values of these quantities. State the magnitude of the supporting forces at A and B.

[Ans. See example 18 page 24.]

(6) In fig 377 the three given forces are balanced by two other forces, one of which acts along YY and the other passes through X.

Find the magnitude of the balancing forces and the angle between them.

[Ans. 2620, 1080 lbs. and the angle 64°.]

1929.

(1) Fig. 378 shows the plan and part elevation of a dome, horizontal sections of which are regular octagons.

(*a*) Complete the elevation.

(*b*) Draw the development of one of the eight curved faces of the dome.

(*c*) Find the number of square feet of sheet lead which would be required to just cover the surface of the dome, the scale of the diagram being 1 inch to 5 feet.

[Ans (c) = 250 square feet.]

(2) Fig. 379 shows the outline of a buttress which has a uniform thickness perpendicular to the plane of the paper. Find the position of the centre of gravity of the buttress.

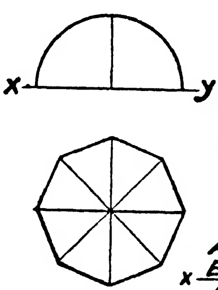


FIG. 378.

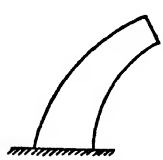


FIG. 379

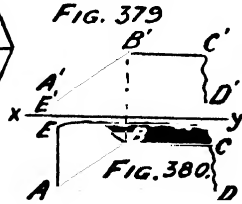


FIG. 380.

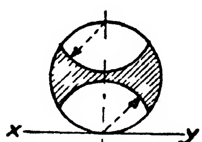
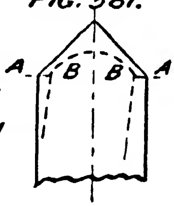


FIG. 381.



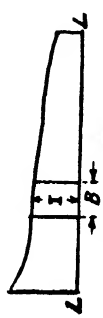


FIG. 383.

(3) Fig 380 shows a portion of a sloping roof by its plan and elevation.

(a) Find the inclination of each of the two sloping surfaces to the horizontal.

(b) Find the inclination of the hip joint AB to the horizontal.

(c) Find the dihedral angle between the two sloping surfaces.

[Ans. (a) 47°, 47°, and 37°. (b) 31°. (c) 86° and 122°.]

(4) A portion of $1\frac{1}{4}$ inch twist drill is shown in fig. 381 consisting of a cylinder with a conical end cut with two helical grooves each of 12 inch pitch and of the form shown in the given sectional elevation.

Complete the plan showing the curve BB and the helical grooves correctly projected for a distance of 6 inches from the line AA.

(5) Fig. 382 shows two views of a pedestal step, SS representing a vertical section plane. Draw a sectional elevation of the step on the given X′ Y′, the portion P of the step in front of the section plane being supposed removed.

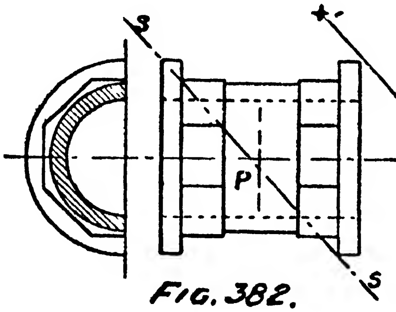


FIG. 382.

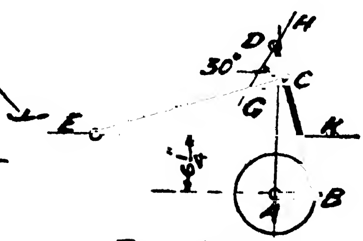


FIG. 384.

(6) In fig. 383, LL represents a horizontal beam and the diagram gives the moment of intertia I of any cross-section about the neutral axis. You are required to divide LL into eight parts such that the length of each part shall be approximately proportional to the mean moment of inertia of the part; this is, l|I is to be the same for all the segments. Give the value of the common ratio, l and I being measured on the same scale.

[Ans. See example 5 page 257.]

(7) In the mechanism in fig. 384, AB is 4 inches long, CD $3\frac{1}{2}$ inches, BD 16 inches and CE 20 inches long. AB rotates at 150 revolutions per minute and the point D is guided to move in the straight line EK. Find the velocity of the point E when the mechanism is in the configuration shown in the figure.

[Ans. See example 6, pages 288 and 289.]

(8) A valve and spindle are actuated by means of a cam, the axis of which is in the line of stroke. The valve is raised through 2 inches in 1/6th of a revolution of the cam, lowered in the following 1/6th and remains at rest during the remainder of the revolution.

Least thickness of metal, 1½ inch. Diameter of roller, 2 inches.

Design the cam so that the motion of the valve shall be simple harmonic.

[Ans. See example 14 page 246.]

1930.

(1) A vessel made of thin sheet metal has the form shown in fig. 385. The sloping surface can be divided into two triangles A, A′ and two portions of truncated oblique circular cones B, B′.

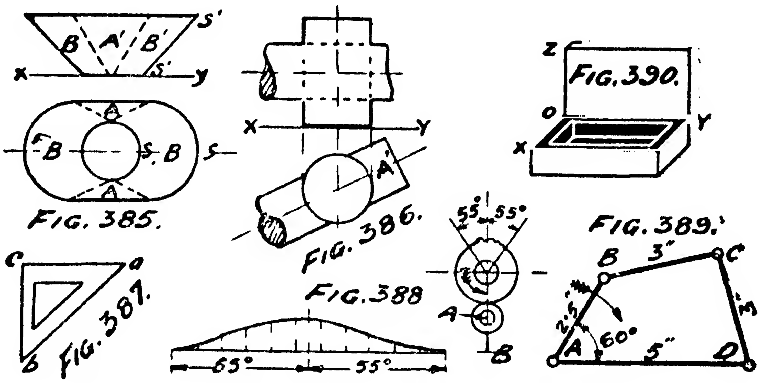
Draw a development of the piece of metal forming the curved surface, the joint being at SS. Omit all allowances for seams, etc.

(2) Two right circular cylinders shown in fig. 386, intersect at right angles.

(*a*) Draw the elevation of the line of interpenetration of the two cylinders.

(*b*) Develop the surface of the portion 'A' of the horizontal cylinder.

(3) Fig. 387 shows the plan of a 60 degree setsquare resting on its short edge BC.



(*a*) Find the height of the corner A and index its plan *a*, in inches.

(*b*) Determine a vertical plane which makes 19 degrees with the edge AB and draw an elevation of the setsquare on this plane.

a

[*Ans. (a) 3·6, 3·6.*]

(4) A cam rotating in the direction shown in fig. 388, has to move the centre of the tappet roller from A to B and back again to A while it rotates through an angle of 110 degrees. The diagram shows by its ordinates the displacement of A along AB in inches while the abscissæ represent the corresponding angles turned through by the cam. Draw, full size, the outline of the cam, its minimum radius being $1\frac{1}{2}$ inches and the roller diameter $1\frac{1}{4}$ inches.

[*Ans. See example 15 page 247.*]

(5) The bar AB of the mechanism shown in fig. 389 rotates at 10 revolutions per minute in the direction shown. When the mechanism is in the configuration indicated in the figure, find:—

(*a*) the angular velocity of CD.

(*b*) the angular acceleration of CD.

[*Ans. See example 3 page 283*]

(6) Fig. 390 shows a rectangular box with the lid open at right angles, dimensions parallel to OY and OZ being set off full size and parallel to OX half size. Draw an end elevation of the box with the lid open at 60 degrees to the horizontal, that is, an elevation on a vertical plane parallel to ZOX. From the elevation project a plan.

From the plan project a new elevation on a vertical plane which makes 45 degrees with the first vertical plane.

(7) The parabolic arched rib shown in fig. 391, is hinged at the crown and springings and carries a load of 2 tons at quarter span.

Find the reactions at the hinges and draw the line of thrust for the rib. What is the value of the maximum bending moment on the rib. ?

[*Ans.* $R_1 = 1{\cdot}95$, $R_2 = 1{\cdot}35$ *tons. The line of thrust is the centre line of the arch. Max: Bending moment = 10·5 tons feet.*]

(8) A beam has a square section of 3 inch side and is placed with its diagonals horizontal and vertical as shown in the fig 392. The ratio of the shearing stress q at any place BB to the mean stress q_0 over the whole section is given by the formula

$$\frac{q}{q_0} = \frac{24\,K}{BB\,(CC)^2}$$

where K is the moment of the shaded area ABB about CC.

Draw a diagram showing the value of $\frac{q}{q_0}$ at all points from A to O. Give the numerical value of $\frac{q}{q_0}$ at O.

[*Ans. ·99 at O*]

A person on the top of a tower 60 feet high, which rises from a horizontal plane, observes the angles of depression of two objects A and B on the plane to be 20 degrees and 30 degrees, the directions of A and B from the tower being west and south respectively. Find (*a*) the distance of A and B from the foot of the tower; (*b*) the distance apart of A and B; (c) the direction of B from A.

[Ans. (a) 166·5 feet, 104 feet. (b) 196 feet. (c) south East.]

1931.

(1) A valve spindle is actuated by a bell-crank lever the arms of which are at right angles. The line of motion of the valve spindle is 3 inches from the centre of oscillation of the bell-crank lever, and the centre of the camshaft is in the positoin given in Fig. 393.

Design the cam to give the valve spindle simple harmonic motoin.

Diameter of roller, 1 inch.

[*Ans.* *See example 16 page 248.*]

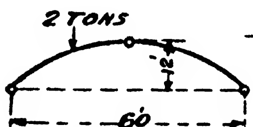
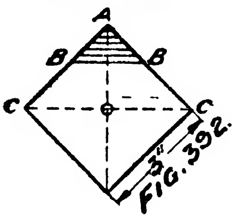
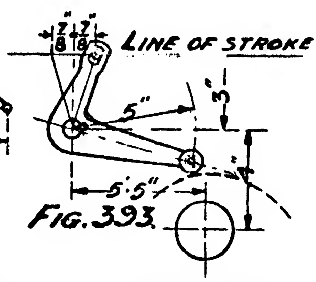


FIG. 391.





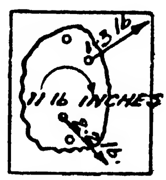
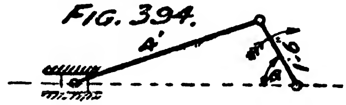


FIG. 395.

FIG. 394.



(2) Find the travel of an ordinary *D* slide valve with outside steam admission when the lead is $\frac{1}{4}$ inch, and the valve shuts off steam at 70 per cent of the outward stroke of the piston. The steam lap is $1\frac{1}{4}$ inches. The length of the connecting rod is 5 cranks. Neglect the effect of the obliquity of the eccentric rod.

(3) The crank of the steam engine mechanism, shown in Fig. 394, rotates at 120 revolutions per minute.

(*a*) Find the acceleration of the piston when θ is 0, 45, 90, 135, and 180 degrees.

(*b*) Plot the acceleration to a base of corresponding piston displacement, and hence find the point on the stroke at which the piston acceleration is zero, and the corresponding value of θ.

(4) A, B, and C are three points in a straight link which has a periodic plane motion and whose average position is vertical.

AB is 20 inches, AC is 6 inches, BC is 14 inches. The horizontal motion of A is simple harmonic, amplitude 4 inches, advance -60 degrees,

The horizontal motion of B is also simple harmonic, amplitude 3 inches, advance 25 degrees.

Find the horizontal motion of C and draw the harmonic image of the link.

(5) The magnitude F, the intercept x, and the angle θ are given in the table below for the wind forces on the joints of a roof truss referred to a horizontal line through one end.

F tons.	x feet.	θ degrees
0·92	0	-45
1·53	20	-60
1·04	31	-70
0·61	41	-78

Find the resultant force $x^F\theta$.

The reactions at the supports balancing this force being ${}_0r_{90}$ and ${}_{100}s\theta$, find r, s and θ.

[Ans. $x^F\theta = 23', 4$ tons, $-118°$. $r = 2·75$, $s = 2·1$ tons, $\theta = -157°$.]

(6) A card is temporarily pinned to a board. Forces of 1·3 ℔. and 2·2 ℔. are applied to the card as shown in fig. 395. A clockwise couple of 11 ℔ inches is also applied. The linear scale of the diagram being $\frac{1}{4}$, find the line of action and the magnitude of the force which acting on the card will produce equilibrium, so that if the pins are removed the card will remain at rest.

[Ans. 2·97 lbs.]

(7) Determine and measure the forces in the members A, B, C, D, E, and F of the given truss loaded as shown in the fig. 396.

The scale of the figure being $\frac{1}{4}$ inch to 1 foot, draw the diagram of bending moment for the horizontal beam, and measure the maximum bending moment.

[Ans. See example 17 page 274.]

(8) Fig. 397 shows a full sized section of a rolled steel rail. Find the "moment of inertia" of the section about an axis parallel to BB and passing through the centre of gravity of the section.

[Ans. 7·4 inch^4 units.]

(9) The two axles of a trolley are 5 feet apart and the loads on the front and rear axles respectively are 3 tons and 2 tons. The trolley slowly crosses a girder of 25 feet span which is freely supported at the ends. Determine the maximum bending moment on the girder and the position of the trolley which produces it. Draw the bending moment diagram for this position of the trolley.

[Ans. Max: Bending moment=26·63 tons feet and occurs at 13·5 feet from the support.]

1932.

(1) A valve spindle fitted with a roller $\frac{3}{4}$ inch diameter is actuated by a cam keyed to a shaft, the axis of which meets the line of stroke at right angles. The travel of the valve spindle is 2 inches and it is raised in $\frac{1}{4}$ revolution, kept fully raised for $\frac{1}{4}$, lowered in $\frac{1}{8}$, and remains at rest during the remainder of a complete revolution of the cam shaft. The least thickness of metal of the cam is $\frac{5}{8}$ inch, and the diameter of the cam shaft, which rotates uniformly, is 2 inches.

Design the cam to raise and lower the valve spindle with simple harmonic motion.

(2) In the mechanism shown in fig. 398, *AB* is a crank which rotates uniformly at 10 radiads per second Find for the given position, the velocity and acceleration of the slider *F*. AB = 1′, BC = 3·5, CD = 1·5, DE = 1′. AD = 3·5, EF = 5 feet.

[*Ans. Vel: 4 5 feet per sec. and Accl: 41·5 feet sec*^{2}.]

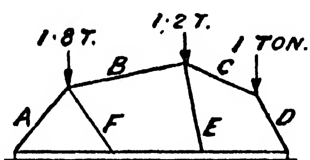


FIG. 396.

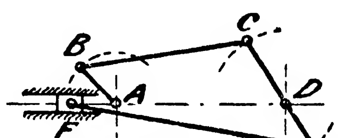


FIG. 398.

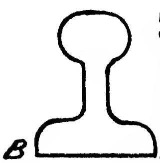


FIG. 397.

FIG. 399.

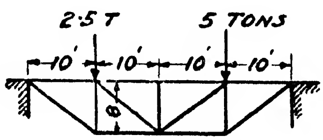


FIG. 400.

(3) Fg. 399 shows a full-sized section of a rolled steel bulb angle rail. Determine :—

(*a*) The 'centre of gravity' of the section.

(*b*) The area of the section in square inches.

(*c*) The weight of the rail in pounds per foot of its length.

Note:—The weight of a cubic inch of rolled steel may be taken as 0·284 lb.

[*Ans. (a) From top 2·01″ and from side ·65″. (b) 2·65 square inches. (c) 9·03 lbs. per foot.*]

(4) A beam *ABCD*, whose length *AD* is 40 feet, is supported at the end *A* and at a point *C*, which is 30 feet from *A*. The beam itself weighs 112 lb. per lineal foot, and carries a load of one ton at the overhanging end *D*. Find the reaction at the support *C*, and the bending moment at a point *B* midway between *A* and *C*.

Draw the diagrams of shearing force and bending moment.

[*Ans. Reaction at* $C = 2\frac{2}{3}$ *tons. Bending moment bet: A and* $C = -{\cdot}625$ *ton foot.*]

(5) A lattice girder of 40 feet span is of the form, and is loaded, as shown in fig. 400. Determine the forces on all the members of the girder, stating whether they are tensile or compressive.

[*Ans Top chord from left* $+3{\cdot}9, +4{\cdot}6, +4{\cdot}6 + {\cdot}53$ *tons. Bottom chord* $-5, -3{\cdot}9, -5{\cdot}3, -6{\cdot}9$ *tons. Web* $= +3{\cdot}125, -{\cdot}98, 0, -{\cdot}98, + 4{\cdot}375$ *tons.*]

(6) A circular cone is shown in fig. 401, in an inclined position, by its elevation.

(*a*) Draw the plan of the cone,

(*b*) Develop the surface of the portion of the cone which lies below *SS*.

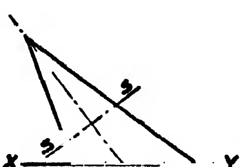
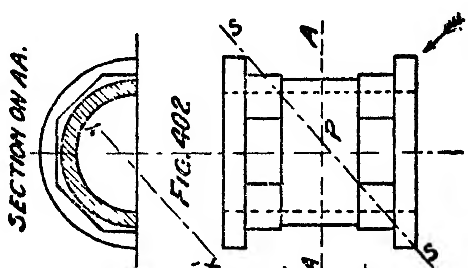


FIG. 401.
PLAN OF AXIS



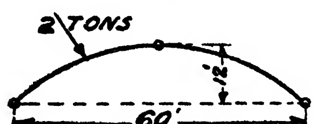


FIG. 404.

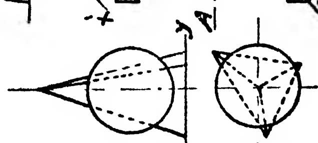


FIG. 403.

(7) Two views of a pedestal step are given in fig. 402. *SS*

represents a vertical section plane. Draw a sectional elevation of the step on $X'Y'$ the portion P of the step in front of the section plane being supposed removed.

(8) Determine the plan and elevation of the line of inter-penetration of the sphere and pyramid shown in fig. 403.

(9) The segmental arched rib, shown in fig. 404 is hinged at the crown and springings and is subjected to an inclined loading of 2 tons at quarter span.

Find the reactions at the hinges and draw the line of thrust for the rib.

Show where the bending moment on the rib is a maximum and give the value in tons feet.

[*Ans. Reactions 2·22, 1·55 tons. Max: Bending moment = 12·65 tons feet and is at the point where the load acts. The centre line of the rib is the line of thrust.*]

[*Hint:—Bending moment at any point in the rib is to be calculated (1) by taking the ordinate from the point parallel to the given load in linear scale and multiplying by pole distance in load scale. (2) By radial ordinate as shown in fig. 194. (3) By the curve of equilibrium as explained in the same fig. 194 pages 139–140.*]

(10) Find the travel and steam lap of a D slide valve admitting steam to the ports by its outside edges, when steam is cut off at 65 per cent. of the instroke of the piston. The connecting rod is four times tho length of the crank; maximum openning of the port to steam in one inch and the lead is $\frac{1}{4}$ inch.

Keeping the same travel, lap, and lead, find also the percentage of the outstroke of the piston at which the steam is cut off,

1933.

1. Two cranks attached to a common shaft are shown in Fig. 405. *OP* is the main crank of a single cylinder steam engine and *OV* is a crank operating an ordinary slide valve which cuts off steam at $\frac{3}{4}$ stroke of the piston. The obliquity of any rod connected to *OP* or *OV* is to be neglected.

Draw on a piston-displacement-base, 4 inches long, a diagram showing the opening of the port to steam for one stroke of the piston. State the amount in inches of the steam lap of the valve.

2. Fig. 406 shows the displacement-time diagram for the table of a planing machine with a quick return motion. Determine the speeds for all positions during the cutting and return strokes, and plot the values you obtain in the form of a curve showing the speed for any position of the table.

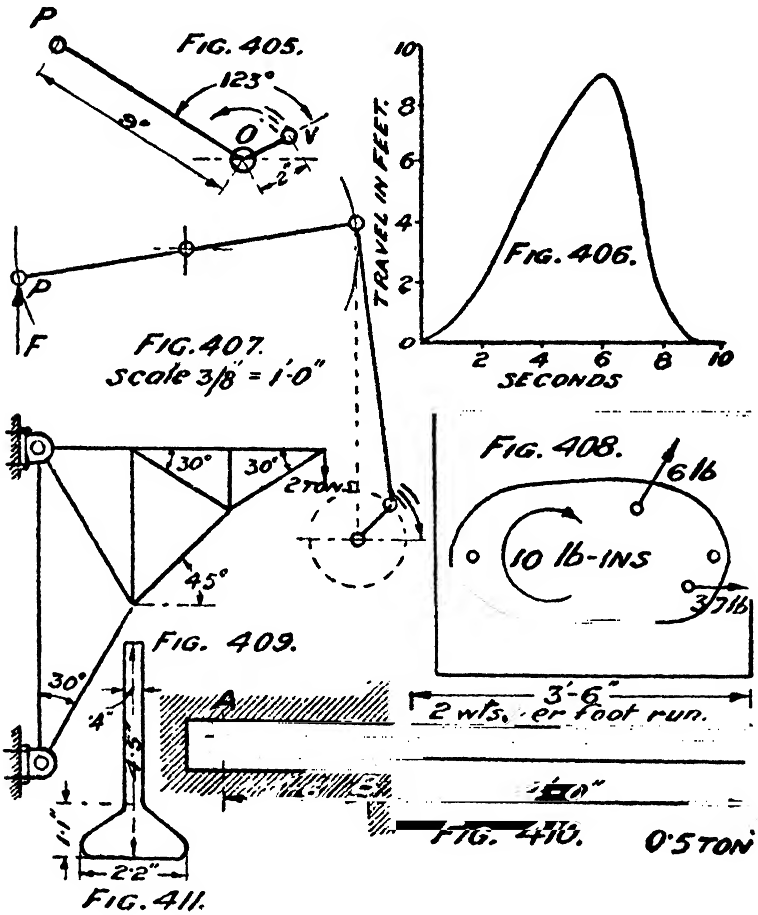


FIG. 405. FIG. 406. FIG. 407. FIG. 408. FIG. 409. FIG. 410. FIG. 411.

3. A rocking lever drives a crank as shown in Fig. 407. If the

vertical force F is 1,000 pounds, find the crank effort and the turning moment on the crank in the given position. Find also the ratio of the velocity of the point P to the velocity of the crank pin.

4. A card is temporarily pinned to a board. Forces are applied to the card as shown in Fig. 408. Find the line of action and the magnitude of the force which acting on the card will produce equilibrium, so that if the pins are removed the card will remain at rest.

5. Draw the force diagram or 'reciprocal figure' for the braced cantilever shown in Fig. 409. Measure the forces in the two bars CD and CE; give the answer in tons, distinguishing tension from compression.

6. Draw diagrams of shearing force and bending moment for the cantilever loaded and supported as shown in Fig. 410. Measure the maximum values of the shearing force and bending moment. Also state the magnitude of the supporting forces at A and B.

7. Find the '.centre of gravity' of the section shown in Fig. 411. Determine the 'moment of inertia' of the section about an axis through the centre of gravity and at right angles to the axis of symmetry.

8. A reinforced concrete beam is of rectangular section, width 12 inches, depth from upper surface to line of reinforcement, which consists of steel bars, is 22 inches. If the modulus of elasticity of steel is 10 times that of concrete and the ratio of maximum stress in steel and concrete is 24, find the distance of the neutral surface from the upper face of the beam.

Also find the total area of section of the steel bars and state the answer in square inches.

9. Two rolling loads of 4 tons and 6 tons respectively, 10 feet apart, traverse a girder of 60 feet span. Draw diagrams showing the maximum shearing force and maximum bending moment for all points in the span during the complete passage of the loads across the girder. State the scales employed.

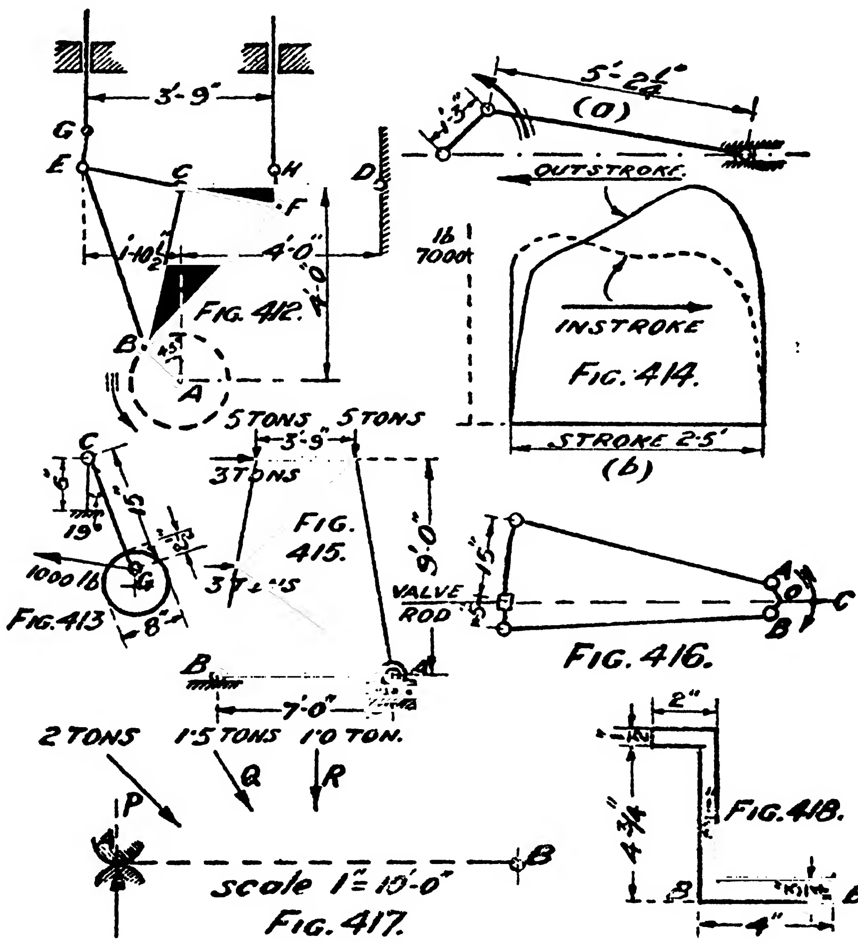
10. Draw, full size, the profile of a cycloidal tooth for a spur wheel, given the following particulars: width of tooth on pitch line, 1 inch; depth below pitch line, 4/7 inch; depth (or height) above pitch line, 3/7 inch; diameter of pitch circle, 18 inches; diameter of rolling circle, 6 inches.

1934.

1. A cam is required to rotate uniformly and give the following motion to a vertical spindle :—

A rise of $1\frac{1}{4}$ inches at a uniform rate during the first third of a revolution of the cam, a period of rest during $\frac{1}{6}$th revolution of the cam, then a sudden drop to its original position. The diameter of the shaft is 1 inch, the least thickness of metal round the shaft is $\frac{1}{2}$ inch, and the diameter of the roller is $\frac{1}{2}$ inch. Design the cam full size.

Does the cam really give the motion desired ? If not, indicate the portion of the cam which fails to give exactly the motion desired.



2. The crank AB of the mechanism shown in Fig. 412. rotates uniformly about the fixed centre A. The triangular connecting rod is maintained in position by means of a suspension link CD which oscillates about D.

The linear speed of the crank pin B is 10 feet per second. Find for the given position—

(1) the velocity of F.

(2) the velocity of G.

3. A lever and balance weight, centre of mass G. oscillates about an axis C. (Fig. 413.) The mass of the weight and lever is 100 lb. and its radius of gyration about G is 6 inches. When, in the configuration shown in Fig. 413. the force acting at G is 1,000 lb. in the direction indicated, and the angular acceleration of the weight and lever is 416 radians per second per second, clockwise—

(*a*) calculate the magnitude of the inertia couple ;

(*b*) combine the force and couple into a single force and indicate the force in the configuration diagram.

Adopt a linear scale of 1 inch to 0·5 foot.

4. A steam engine mechanism is shown in outline in Fig. 414. (a). After making allowance for the mass acceleration of the moving parts, the force usefully transmitted to the crosshead is shown for one revolution of the crank in Fig. 414. (*b*) vertical ordinates representing force on crosshead in pounds, horizontal ordinates representing stroke of piston in feet.

Draw a curve on a base of crank angles, showing the turning moment on the crank for one revolution.

Adopt the following scales :—

for turning moment, 1 inch to 7,000 lb. $\times$ $1\frac{1}{4}$ ft,
that is, 1 inch to 8,750 lb. ft.
for crank angle, 1 inch to 60 degrees.

Measure the maximum turning moment on the crank shaft in lb. ft.. and state at what position of the crank it occurs.

5. A framed pier has the form and is loaded and supported as shown in Fig. 415.

Determine the supporting forces at A and B, that at A being vertical.

Draw to a scale of 1 inch to 2 tons the force diagram for the pier, and state the nature and magnitude of the forces in the bars AC and CE.

6. A Stephenson link motion is shown in Fig. 416. Find the approximate equivalent eccentric when the gear is in the position shown in the figure, assuming there is no slipping of the valve rod block in the link DE.

OA = OB = 3 inches. Angle COA = Angle COB = 120°. AD = BE = 4 Feet. Radius of curved link = 4 feet.

7. A roof truss (not shown) is acted upon by three forces P, Q, and R which are shown in Fig. 417.

The supporting reactions pass through the points A and B, the reaction at A being vertical.

Find the magnitude of the two reactions and the angle which the reaction at B makes with the horizontal.

8. A rectangular pontoon, 40 feet long and 20 feet wide, floats in fresh water ; when loaded with two concentrated loads of 30 tons each at points 8 feet from the ends, the draught is 4 feet.

Draw the diagrams of shearing force and bending moment, and write down their maximum values.

9. The cross section of a rail is shown in Fig. 418. Find :—

(*a*) the position of the 'centre of gravity' of the section,

(*b*) the moment of inertia of the section about an axis passing through the centre of gravity and parallel to the line *BB*.

1935.

1. In Fig. 419 *OP* represents the main crank of a single cylinder steam engine and *OV* is a crank or eccentric operating an ordinary slide valve. It is required to cut off steam at $\frac{3}{4}$ stroke and to give a 'lead' of $\frac{1}{4}$ inch. Determine the value of α (the angle of advance) in degrees. Find also the steam lap of the valve in inches. Neglect obliquity of connecting rod and eccentric rod.

2. A block *S* is operated between guide bars by a crank *OC* and connecting rod *CS*, as shown in Fig 420. Find the velocity and the acceleration of the block *S* for the position shown. The crank is rotating at a uniform speed of 120 revs. per minute.

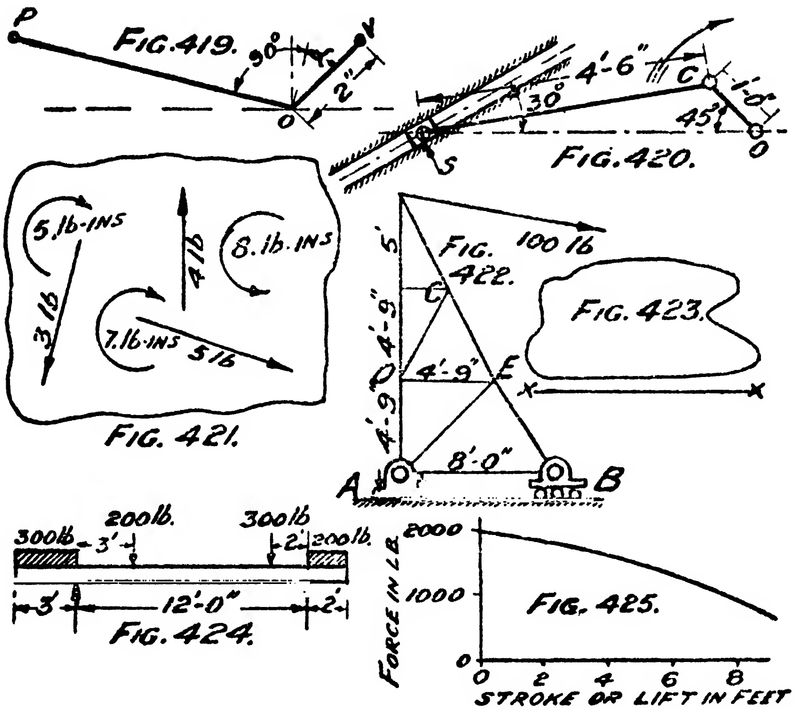
3. Determine the resultant of the system of forces and couples shown in Fig. 421, and indicate its position on the diagram.

4. A steel tower has the form and is loaded as shown in Fig. 422. The reaction at *B* is vertical. Determine the reaction at *A*.

Draw sufficient of the force diagram or 'reciprocal figure' to enable you to measure the forces in the two bars *CD* and *CE*.

State the magnitude of these forces in pounds.

5. Find the 'centre of gravity' of the section shown in Fig. 423. Determine the 'moment of inertia' of the section about an axis through the centre of gravity and parallel to the line *XX*.



6. Draw the diagrams of shearing force and bending moment for the beam loaded and supported as shown in Fig. 424. State clearly the scales employed. Measure the maximum value of the bending moment, and indicate where it occurs.

7. A cam which rotates uniformly is required to give the following motion to a vertical spindle : a rise of 2 inches with simple harmonic motion during 150 degrees rotation of cam, a period of rest during 60 degrees followed by descent at uniform velocity during the remaining 150 degrees.

The diameter of the shaft is 1 inch, least thickness of metal round the shaft $\frac{1}{2}$ inch, and the diameter of roller is $\frac{1}{2}$ inch.

8. The diagram (Fig. 425) shows the lifting force on a vertical pump rod during a portion of its stroke, starting from rest at the bottom of the stroke.

The dead weight of the pump rod and its attachments is 1,200 lb.; the frictional force is 200 lb. At what lift will the velocity be a maximum ? What is the acceleration when the lift is 4 feet ?

1936.

1. A train starts from rest and has a speed of 70 miles per hour after travelling a distance of 16 miles. At the end of each successive distance of two miles the speeds are 33, 48, $57\frac{1}{2}$, 63, 66, $68\frac{1}{2}$, $69\frac{1}{2}$, and 70 miles per hour taken in order. Determine the acceleration of the train at the end of the fourth mile.

Determine also the approximate time taken by the train to travel the distance of 16 miles.

2. The D-slide valve of a vertical engine has a travel of 6 inches and a lead at the top end of $\frac{5}{16}$ inch. The stroke of the piston is 36 inches and the connecting rod is 6 feet long. Determine the angle of advance, the outside and inside laps, and the lead at the bottom end of the valve, having given :—

Point of cut-off (down stroke)	...	73 per cent.
Point of cut-off (up stroke)	...	64 per cent.
Point of compression (up stroke)	...	89 per cent.
Point of compression (down stroke)	...	86 per cent.

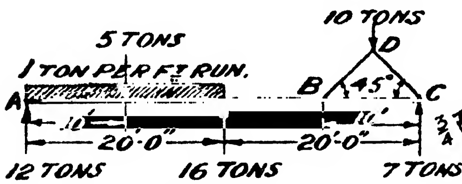


FIG. 426.

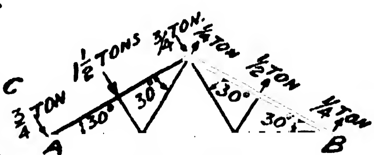
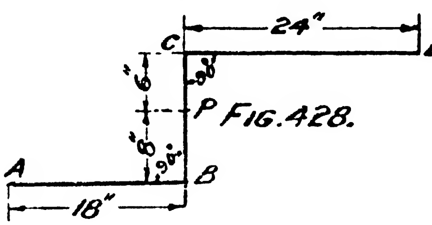
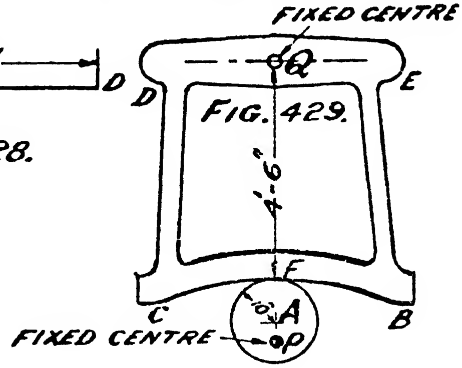


FIG. 427.





3. The vertical line of stroke of the valve spindle of an oil engine is 2″ from the centre line of the horizontal cam shaft. The minimum radius of the cam surface is $1\frac{1}{2}$″ and the maximum is 3″, and these parts of the cam surface are concentric with the cam shaft. The working face of the cam which lifts the valve spindle is part of an involute of a circle, 4″ diameter, which is

concentric with the cam shaft. There is a roller on the valve spindle 2″ diameter, which engages with the cam surface. Draw full size the profile of the working face of the cam, and determine (*a*) the maximum lift of the valve, and (*b*) the angle through which the cam rotates while lifting the valve.

4. A beam *AC* is subjected to the loads shewn in Figure 426. The load at *D* is carried on a truss *BDC* which is supported on the beam. Draw to scale the shearing force and bending moment diagrams, indicating clearly all the scales employed.

5. The forces due to wind pressure on a roof are snown in Fig. 427, those on the left being pressure and those on the right being suction. Assuming that the reaction at *B* is vertical, determine the reactions at *A* and *B*, and draw a diagram giving to scale the loads in the various members of the roof.

6. In the four bar chain *ABCD* (Fig. 428) the bars *AB* and *DC* oscillate about *A* and *D* respectively. Determine the linear velocity aud acceleration of the point *P* when *CD* has moved 10 degrees upwards from the given position and has an instantaneous angular velocity of 1·2 radians per second without angular acceleration.

7. The following particulars relate to a single cylinder horizontal engine :—

Length of stroke	... 10 inches.
length of connecting rod	... 18 inches.
Weight of reciprocating parts	... 112 lb.
Speed of engine	... 400 revolutions per minute.

Draw a diagram showing on a base of piston displacement the variations in the crank effort of the engine due only to the acceleration of the moving parts. Write down the maximum crank effort in pounds.

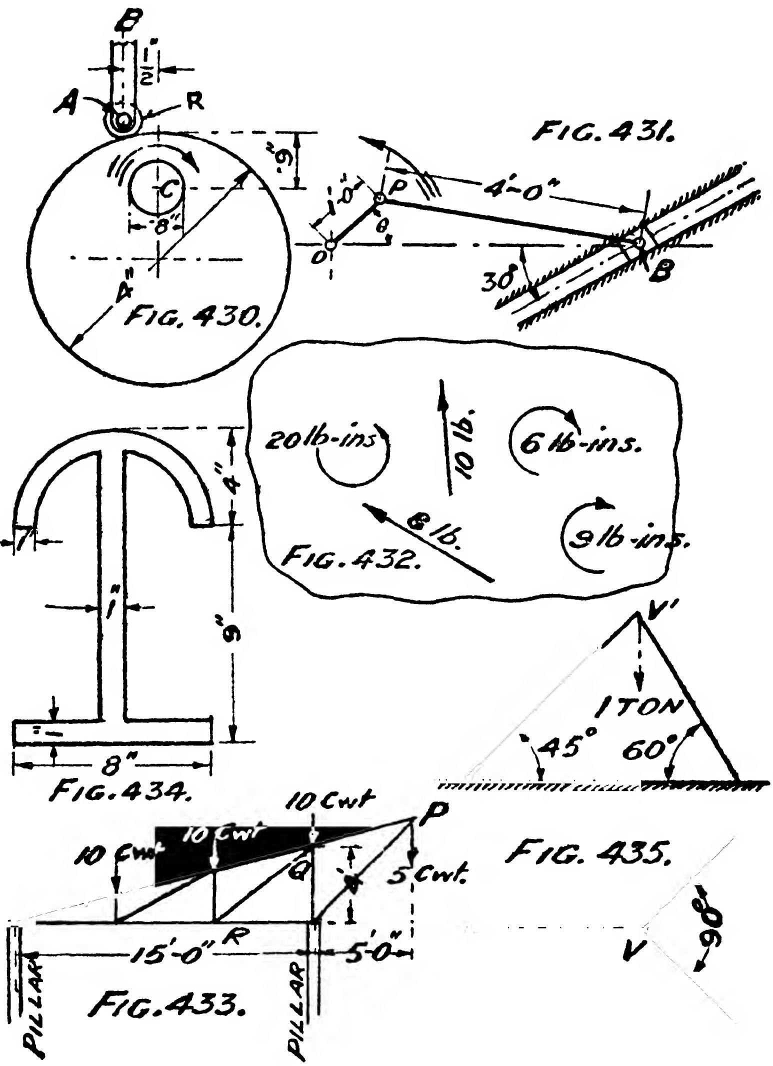
Note.—Acceleration force for reciprocating parts

$$= \frac{W}{g} \omega^2 r \left(\cos \theta + \frac{r}{l} \cos 2\theta \right).$$

8. In the Harfield steering gear (Fig. 429) the circular pinion *A* is keyed eccentrically to the driving shaft *P* and gears with the teeth on the rack *BC* which is integral with the steel casting *BCDE*. *BCDE* can rotate about the axis *Q*. The pinion *A* has a pitch diameter of 20 inches and its axis of rotation *P* is 5 inches from the centre of the pinion ; the minimum radins *QF* of the driven rack is $4\frac{1}{2}$ feet. Determine the shape of the pitch surface *CFB* of the rack.

1937.

1. The cam shown in Fig. 430 moves a rod, which is fitted with a roller R at the end, along a straight line in direction AB. The cam rotates about centre C with uniform angular velocity completing one revolution in 12 seconds.



On a time-base of ½ inch to one second, draw a diagram showing the displacement of the centre A of the roller R for one complete revolution of the cam.

2. A block B is moved between guide bras by a crank OP and connecting rod PB as shown in Fig. 431. Find the velocity and acceleration of the block B when the angle θ is 45 degrees.

Crank OP rotates with uniform angular velocity of 60 revolutions per minute.

3. Fig. 432 shows a system of forces and couples; the linear scale is full size. Determine the resultant, and indicate its position on the diagram.

4. A roof truss is shown diagrammatically in Fig. 433. Determine the supporting forces in the pillars and the forces in the bars PQ and QR, distinguishing pulls from thrusts.

5. A uniform rolling load of 5 cwt. per foot run passes over a bridge of 60 feet span from left to right: the load is longer than the span of the bridge.

Draw diagrams showing the maximum shearing force and bending moment at all positions of the span as the load passes completely across the bridge.

Choose your own scales for S. F. and B. M. but use a linear scale of one inch to 10 feet.

6. Find the moment of inertia of the plane section shown in Fig. 434. about a line through the 'centre of gravity' of the section, and perpendicular to the axis of symmetry, and lying in the plane of the section.

7. The geometrical form of a tripod is shown in plan and elevation in Fig. 435: the foot of each leg may be regarded as securely hinged to a base block in the ground.

Determine the thrust in each leg of the tripod when a load of one ton is suspended from the vertex V.

8. A symmetrical three-hinged arched rib is a circular arc with a span of 50 feet and rise of 10 feet. It carries a uniformly distributed load of one ton per foot run of the span on the left half only. Find the horizontal thrust and the bending moment at a horizontal distance of 12′-6″ from the left-hand support.

1938.

1. In Fig. 436, CD represents a crank which turns uniformly about a fixed centre C, its end D imparting a simple reciprocating motion to a slider EE moving between fixed guides FF. A point O in the slider EE is the centre of rotation of a second crank OP, which turns at half the speed of the first, and in the opposite direction, as indicated. Trace the complete locus of P.

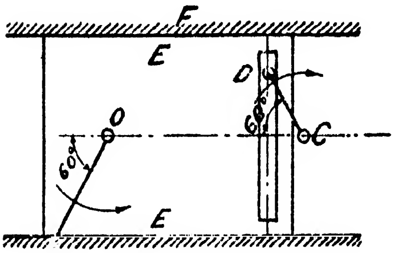
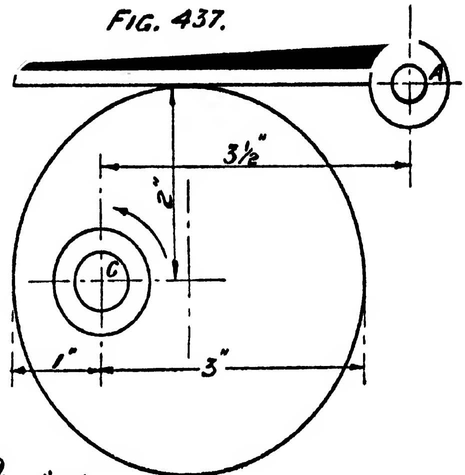


FIG. 436.

FIG. 437.



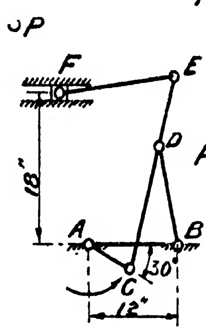
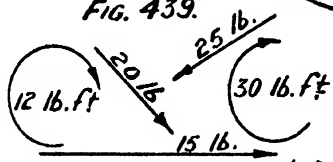


FIG. 438.

FIG. 439.



LINEAR SCALE 1" = 1'-0"

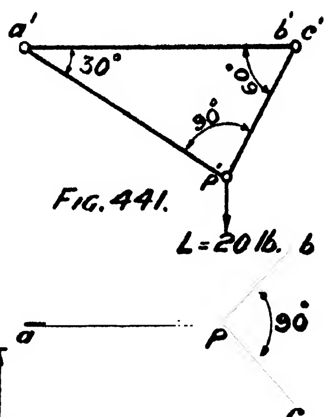


FIG. 441.

FIG. 440.

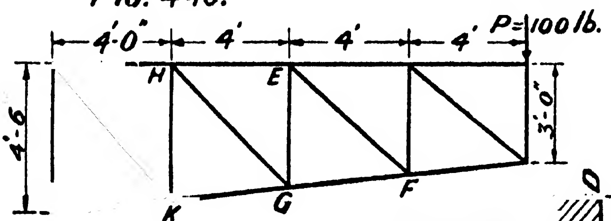
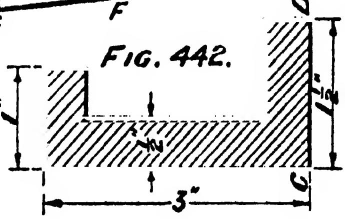


FIG. 442.



2. Fig. 437 shows a lever hinged at A and operated by a circular cam. The cam rotates about centre C with uniform angular velocity of 5 revolutions per minute. On a time base of

half an inch to one second, draw a diagram showing the angular displacement in radians of the lever from its mean position during one complete revolution of the cam.

Adopt a scale of 5 inches equal one radian angular displacement of lever.

3. Fig. 438 shows a linkage for operating a valve F. The crank AC rotates uniformly at 30 radians per second.

Draw the velocity image for the linkage in the given position, and state the velocity of the valve F in feet per second.

Adopt a velocity scale of 1 inch equals 10 feet per second.

4. Fig. 439 shows a system of forces and couples ; the linear scale is 1 inch equals one foot. Determine the resultant, and indicate its position relative to the given forces.

5 Fig. 440 shows the tail end of the fusilage of an aeroplane having only one load P at that end.

Draw the force diagram for the portion shown, and measure the forces in the members FE, HE, and GK.

6. A lamp L, weighing 20 lb., is suspended by a chain from a point P where three tension rods meet. These rods are supported at their upper ends by ceiling hooks ; the configuration of the rods is shown in plan and elevation in Fig. 441. Determine the force in each of the rods PA, PB, PC, and tabulate the results.

7. Fig. 442 shows a beam section. Determine—

(*a*) the neutral axis parallel to CD.

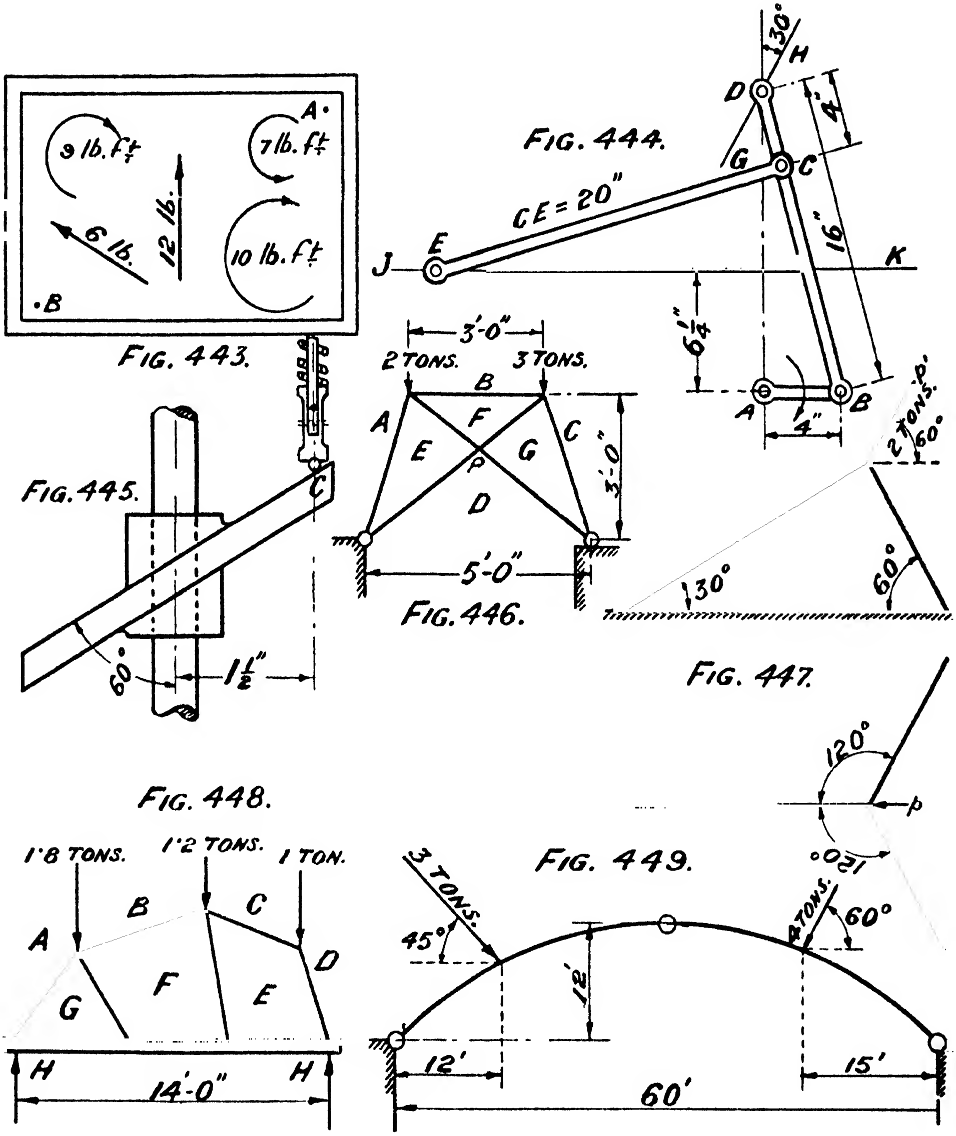
(*b*) the 'moment of inertia' of the section about the base CD.

8. A heavy chain is supported by its ends A and B which are 12 feet above the lowest point of the chain.

The horizontal distance between A and B is 66 feet and the weight of the chain is 20 lb. per foot of its horizontal projection. Draw to scale (10 feet to 1 inch) the shape of the chain, and find the force in the chain at its lowest point. What is the maximum force in the chain ?

1939

1. Fig. 443 shows a stiff card attached to a wooden backboard at the two points A and B by stout pins.



The linear scale of the figure is one inch to one foot. Forces and couples as shown are applied to the card and the pin B is then removed. What is the moment (in lb. ft.) of the system about A at the instant that pin B is removed ?

Determine the single force which must be applied to the card to maintain it in its original position, when both pins A and B are removed. State its magnitude in pounds and show its position and sense.

2. In the mechanism in Fig. 444 the crank $A B$ rotates at 150 revolutions per minute and the point D is guided to move in the straight line GH; E moves in the straight line JK.

When the mechanism is in the configuration shown in the figure, determine the velocity of the point E in feet per second.

3. An inclined cam-plate, mounted on a uniformly rotating shaft, operates a reciprocating tappet as shown in Fig. 445. Contact between ball and plate is to be considered as taking place at C, on the line of action of the tappet.

Draw full size the displacement diagram for the tappet. Is this a common geometrical curve? If so, give the name of the curve.

Take a base line 6 inches long to represent one revolution of the cam-plate.

4. Draw the complete force diagram (often called stress diagram) for the structure shown in Fig. 446. Indicate by thick lines the members which are in compression. Measure and state the magnitude of the force in the bar BF.

Assume the crossed members to be pinned at P, and that the structure is symmetrical about the vertical through the point P.

5. Fig. 447 shows plan and elevation of a tripod which is subject to an inclined load P of 2 tons. The lower ends of the tripod are prevented from sliding by being hinged in base blocks in the ground.

Determine the thrust in each leg of the tripod.

6. A braced horizontal girder is shown in Fig. 448.

(a) Determine the forces in the members GF and FE

(b) Draw the bending moment diagram for the girder.

(c) What is the magnitude of the maximum bending moment and where does it occur?

7. The circular arched rib shown in Fig. 449 is hinged at the crown and springings and carries the loads shown.

Find the reactions at the hinges and draw the line of thrust.

8. Two concentrated loads W_1 8 tons, and W_2 4 tons, centres 12 feet apart, pass over a girder 60 feet span from left to right, W_1 leading.

Draw the diagram of maximum bending moments for the complete passage of the loads along the girder.

What is the equivalent uniformly distributed dead load which gives the same maximum bending moment on the span?

Choose your own scale for bending moment, but use a linear scale of 1 inch to 10 feet.

1940

1. (*a*) A ball, weighing 100 lb, is suspended from the ceiling by a string 8 ft. long. Draw the force and vector diagrams, to find the force necessary to hold the weight 2 ft. from the vertical, by a horizontal pull.

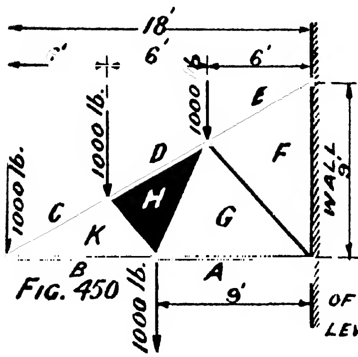


FIG. 450

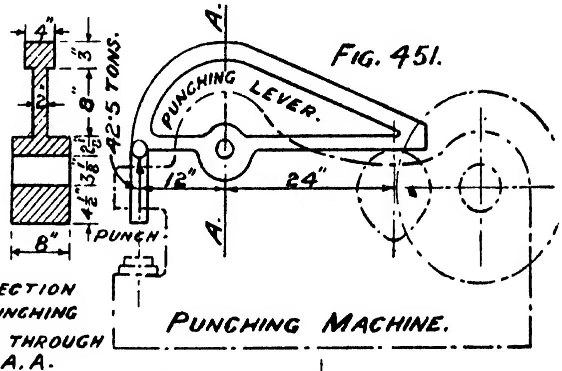
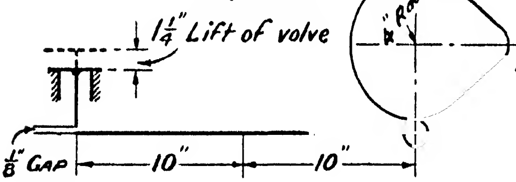


FIG. 451.

FIG. 452.



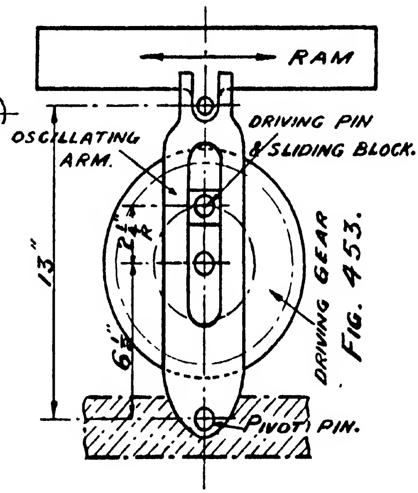


FIG. 453.

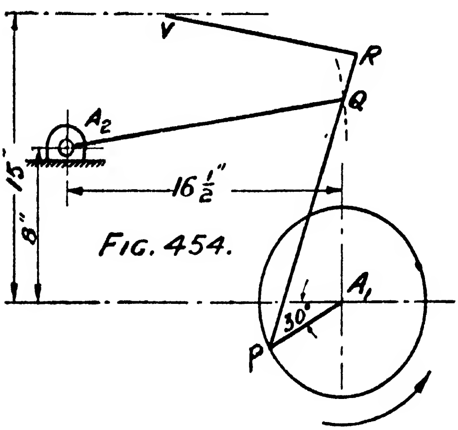


FIG. 454.

(*b*) Forces of 1, 2, and 3 lb. are parallel and act in the same direction at the corners *A*, *B*, and *C* respectively of an equilateral triangle of 4″ side. The force of 1 lb. makes an angle of 100° with

AB which is horizontal. Find graphically where the resultant acts.

2. A beam, 20 ft. span, carries loads $\frac{1}{2}$, 1, and $\frac{1}{2}$ tons at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of span respectively.

(*a*) Draw the bending moment and shearing force diagrams graphically. Take the space scale 1 in. = 5 ft., the load scale 1 in. = 1 ton, and the polar distance = $1\frac{1}{4}$ in.

(*b*) Give the scale for the bending moment diagram, and determine the maximum bending moment and the shearing force, and locate their positions.

Or,

Draw the stress diagram, and determine the stress in each member of the pent roof truss shown in Fig. 450. Distinguish members in compression.

3. Fig. 451 shows in outline a punching machine.

(*a*) Find out the position of the neutral axis and the moment of inertia about the same for the section of the punching lever given.

(*b*) Assuming the force required to punch a hole through a plate is 42·5 tons (i) calculate the maximum tensile stress to which the punching machine lever is subjected at the section A.A. ; (ii) calculate the bearing pressure per sq. in. at the axis pin.

4. Fig. 452 shows the exhaust valve cam and lever arrangement for a Diesel engine. The outline of the cam is made up of circular arcs and tangential straight lines so arranged that the cam rotates through 50° when opening the valve, 10° when keeping it fully open, and 50° when it is closing. Draw the profile of the cam, half size, taking the least radius of the cam as 4 in., the lift of the valve as $1\frac{1}{4}$ in. and the gap between the lever and valve stem as $\frac{1}{8}$ in. Show all the necessary dimensions.

5. Fig. 453 shows in outline the driving mechanism for the ram of a shaping machine. If the pin driving the sliding block is set to move in a circle of $2\frac{1}{4}$ in. radius, the length of the oscillating arm is 13 in., and the distance between the centres round

which the driving pin and the oscillating arm rotate is $6\frac{1}{2}$ in., draw the line diagram and find—

(*a*) the length of the stroke of the ram, and

(*b*) the ratio of times of cutting stroke and return stroke.

6. Fig. 454 shows the mechanism for a valve gear. The crank A_1P rotates at constant speed of 12 radians per second, and is pinned at P to the rod PR, the point Q in this rod being guided in a circular path by the oscillating link A_2Q, the centre of oscillation being at A_2. The valve rod is connected at V to R by the coupler VR.

For the given configuration, find the velocity of the valve rod V. Draw the velocity diagram to a scale 1 in. = velocity of 2 ft. per second.

$A_1P = 5$, $PQ = 14$, $PR = 16{\cdot}5$, $RV = 12$, and $QA_2 = 17$ in.

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